

Exam 9

Study Guide

ADVANCED RISK MANAGEMENT

Comprehensive study guide
with past CAS problems

Exam 9 Study Guide

2024 Sitting

Rising Fellow

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Introduction

How To Use This Guide

This guide is intended to **supplement** the syllabus readings. Although we believe it provides a thorough review of the exam material, the readings provide additional context that is invaluable. Please do NOT skip the syllabus readings.

This guide is meant to be used in conjunction with the Cookbook, Problem Pack, Flashcards and High-Level Summaries. We suggest you work through the Problem Pack as you make your way through the guide. As you near the exam date, the High-Level Summaries and Flashcards should help you focus in on key topics.

Original Practice Problems

There are no original practice problems included within this study guide. Instead, the guide was designed to be used in conjunction with the Problem Pack.

Past CAS Exam Problems

Past CAS exam problems & solutions are included for each paper. Note that these questions are solely owned by the CAS. They are included in the online course for student convenience. All past CAS problems are Excel-based and can be downloaded from the online course.

Feedback

We always working to improve the Exam 9 Study Guide and the rest of the Rising Fellow study material. Please send us an email at exam9@risingfellow.com if you have feedback about any of the following:

- ◇ Sections that are confusing or could be improved
- ◇ Errors (ex. formatting, spelling, calculations, grammar, etc.)

Note that errata will be posted on the Rising Fellow website on an as-needed basis.

Blank Pages

Since many students want a printed copy of the study guide, blank pages have been inserted throughout the guide to ensure that all outlines start on odd pages.

Bookmarks

Bookmarks have been added for easier PDF navigation.

Clark

Outline

I. Proportional Treaties

A **proportional treaty** is an agreement between a reinsurer and a ceding company (i.e., primary insurer, reinsured, cedant). Proportional treaties come in different flavors:

◇ Quota Share

- The reinsurer receives a flat percentage of the premium and assumes that same flat percentage of the losses
- The reinsurer pays the cedant a ceding commission which reflects differences in incurred underwriting expenses

◇ Surplus Share

- The reinsurer assumes a part of the risk in proportion to the amount that the insured value exceeds the retained line, up to a given limit. The given limit is expressed as a multiple of the retained line
- Surplus share treaties are common for property business

Example

Suppose a insurer enters into a surplus share treaty with a reinsurer. The contract has the following terms:

◇ Retained line: \$100,000

◇ 1st Surplus: 4 lines ($4 \times \$100,000 = \$400,000$)

Given the following for each risk subject to the treaty:

Risk	Insured Value (\$)	Gross Loss & ALAE (\$)
1	50,000	30,000
2	100,000	20,000
3	250,000	240,000
4	500,000	25,000
5	1,000,000	500,000
6	10,000,000	200,000

First, let's calculate the 1st surplus % (i.e., the ceded loss %):

Risk	Insured Value (\$)	Retained (\$)	Reinsured (\$)	1st Surplus %
1	50,000	50,000	0	0%
2	100,000	100,000	0	0%
3	250,000	100,000	150,000	60%
4	500,000	100,000	400,000	80%
5	1,000,000	600,000	400,000	40%
6	10,000,000	9,600,000	400,000	4%

Let's discuss the calculations in the table above:

- ◇ The maximum reinsured portion is the “number of lines \times retained line.” In this case, the maximum reinsured portion is $4 \times \$100,000 = \$400,000$
- ◇ For risks 1 – 2, the insured value is less than or equal to the retained line of \$100,000. Thus, the reinsured portion is \$0
- ◇ For risks 3 – 4, the insured value minus the retained line is less than or equal to the maximum reinsured portion. Thus, the reinsured portion is the insured value minus the retained line. For example, the reinsured portion of risk 3 is $\$250,000 - \$100,000 = \$150,000$
- ◇ For risks 5 – 6, the insured value minus the retained line is greater than the maximum reinsured portion. Thus, the reinsured portion is the maximum reinsured portion of \$400,000
- ◇ For each risk, the 1st surplus % is equal to $\frac{\text{Reinsured Portion}}{\text{Insured Value}}$. For example, the 1st surplus % for risk 3 is $\frac{150,000}{250,000} = 60\%$

Second, let's calculate the total ceded loss and total retained loss under the treaty:

Risk	1st Surplus %	Gross Loss & ALAE (\$)	Ceded Loss & ALAE (\$)
1	0%	30,000	0
2	0%	20,000	0
3	60%	240,000	144,000
4	80%	25,000	20,000
5	40%	500,000	200,000
6	4%	200,000	8,000
Total		1,015,000	372,000

For each risk, the ceded loss & ALAE is equal to the gross loss & ALAE multiplied by the 1st surplus %. The total ceded loss is \$372,000. The total retained loss is $\$1,015,000 - \$372,000 = \$643,000$.

When **pricing proportional treaties**, the following steps should be taken:

- (1) Compile the historical experience on the treaty:
 - ◇ If available, obtain the historical premium and incurred losses on the treaty for five or more years
 - ◇ If not available, obtain the historical premiums and incurred losses on a gross basis and “adjust the experience” as if the surplus share treaty were in effect
 - ◇ If the treaty is on a “losses occurring” basis (i.e., aggregate by loss occurrence year), earned premium and accident year losses should be used
 - ◇ If the treaty is on a “risks attaching” basis (i.e., aggregate by policy written year), written premium and the losses covered by those policies should be used
- (2) Exclude catastrophe and shock losses:
 - ◇ Cat losses are due to a single event that impacts a large number of risks (ex. hurricane)
 - ◇ Shock losses are any other losses that distort the overall results. Shock losses typically only impact a single risk
- (3) Adjust experience to ultimate level and project to future period:
 - ◇ First, we develop losses to their ultimate level
 - ◇ Second, we adjust historical premiums to the future level. This involves “on-leveling” to bring the historical premiums to the current level. If the premium base is inflation-sensitive (ex. insured value), then an exposure inflation factor would also need to be included when adjusting historical premiums
 - ◇ Third, we adjust historical losses to the future level using a loss trend analysis
- (4) Select the expected non-catastrophe loss ratio for the treaty:
 - ◇ Assuming the data used in step 3 is reliable, then the expected non-cat loss ratio is equal to the average of the historical loss ratios adjusted to the future level
 - ◇ We might want to compare the selected ratio to the cedant’s gross CY experience and to industry averages
- (5) Load the expected non-catastrophe loss ratio for catastrophes using one of the following approaches:
 - ◇ Historically, a reinsurers had priced catastrophe loads by “spreading” large losses over expected payback periods

- ◇ In today's world, cat loads are based on catastrophe models that incorporate the risk profile of the cedant

(6) Estimate the combined ratio given the ceding commission and other expenses:

- ◇ The "other expenses" include the reinsurer's general expenses and overhead, as well as brokerage fees

Let's look at an example of pricing a proportional treaty.

Example

Given the following historical AY experience and relevant factors for an insurer as of 09/30/22:

AY	Earned Premium (\$)	Incurred Losses & ALAE (\$)	On-Level Factor	LDF
2017	1,640,767	925,021	1.096	1.000
2018	1,709,371	2,597,041	1.086	1.000
2019	1,854,529	1,141,468	1.034	1.000
2020	1,998,751	1,028,236	0.992	1.000
2021	2,015,522	999,208	1.023	1.075
2022	1,550,393	625,830	1.028	1.600
Total	10,769,333	7,316,804		

A reinsurer's actuary would like to quote a property quota share treaty effective 1/1/2023 to the insurer. The following information also applies:

- ◇ The treaty is to be written on a losses occurring basis
- ◇ Selected loss trend = 4%
- ◇ Inflation = 3%
- ◇ The historical AY 2018 incurred losses & ALAE include \$1,582,758 due to a hurricane. There are no other catastrophe losses in the historical period
- ◇ A ceding commission of 30% has been suggested by the cedant
- ◇ Other expenses for the reinsurer are as follows (as a % of premium):
 - Brokerage fees = 5%
 - Administrative expenses = 1%
 - Unallocated expenses = 1%

First, let's exclude catastrophe losses from the historical losses:

AY	Unadjusted Incurred Losses & ALAE (\$)	Non-Cat Incurred Losses & ALAE (\$)
2017	925,021	925,021
2018	2,597,041	$1,014,283 = 2,597,041 - 1,582,758$
2019	1,141,468	1,141,468
2020	1,028,236	1,028,236
2021	999,208	999,208
2022	625,830	625,830
Total	7,316,804	5,734,046

Second, let's develop and project non-cat losses:

AY	Non-Cat Incurred Losses & ALAE (\$)	Trend		Developed, Projected Incurred Losses & ALAE (\$)
		Factor	LDF	
2017	925,021	1.265	1.000	1,170,152
2018	1,014,283	1.217	1.000	$1,234,382 = 1,014,283(1.217)(1.000)$
2019	1,141,468	1.170	1.000	1,335,518
2020	1,028,236	1.125	1.000	1,156,766
2021	999,208	1.082	1.075	1,162,229
2022	625,830	1.040	1.600	1,041,381
Total	5,734,046			7,100,427

In the table above, the trend factor is calculated as $(1.040)^{2023-AY}$. For example, the trend factor for AY 2018 is $(1.040)^{2023-2018} = 1.040^5 = 1.217$.

Third, let's annualize, on-level, and project the earned premium:

AY	Unadjust. Earn. Prem. (\$)	Annualization Factor	Inflation Factor	On-Level Factor	On-Leveled, Proj. Earn. Prem. (\$)
2018	1,709,371	1.000	1.159	1.086	$2,151,541 = 1,709,371(1.000)(1.159)(1.086)$
2019	1,854,529	1.000	1.126	1.034	2,159,198
2020	1,998,751	1.000	1.093	0.992	2,167,158
2021	2,015,522	1.000	1.061	1.023	2,187,654
2022	1,550,393	1.333	1.030	1.028	2,188,824
Total	10,769,333				13,001,522

In the table above, the inflation factor is calculated as $(1.03)^{2023-AY}$. For example, the inflation factor for AY 2018 is $(1.03)^{2023-2018} = 1.03^5 = 1.159$. Recall that AY 2022 is a partial year since the historical data is evaluated as of 9/30/22. On the loss side, the LDF for AY 2022 both develops

and annualizes the losses. To ensure we are comparing apples to apples, we need to annualize the partial year AY 2022 premium. Since AY 2022 is 75% exposed, the annualization factor for the premium is $\frac{1}{0.75} = 1.333$.

Fourth, let's calculate the expected non-cat loss ratio and load it for cats:

- ◇ Using the adjusted premium and losses, the expected non-cat loss ratio is $\frac{7,100,427}{13,001,522} = 54.6\%$
- ◇ If the cat losses from Hurricane Andrew had been included, the loss ratio would have been approximately 15% higher. As a result, we select a cat loading of 10% (as a % of premium). Note that it would be better to select a cat loading based on a cat model
- ◇ The projected loss ratio (including the cat loading) is $54.6\% + 10\% = 64.6\%$. We will use 65%

Fifth, let's calculate the projected combined ratio for the treaty:

- ◇ Projected combined ratio = projected loss ratio + ceding commission + other reinsurer expenses. Thus, the projected combined ratio is $65\% + 30\% + 5\% + 1\% + 1\% = 102\%$
- ◇ In these assumptions, the treaty appears to be unprofitable (combined ratio > 100%). The reinsurer's actuary might recommend a lower ceding commission as a result

Next, we will discuss the following special features of proportional treaties:

- ◇ Sliding scale commission
- ◇ Profit commission
- ◇ Loss corridor

Sliding Scale Commission

Unlike a flat ceding commission, a sliding scale commission is a percent of premium paid by the reinsurer to the cedant that **slides** with the actual loss experience, subject to set minimum and maximum amounts.

Example

Given the following commission terms:

- ◇ Minimum commission: 25% at a 65% loss ratio
- ◇ Sliding 1:1 to: 35% at a 55% loss ratio
- ◇ Sliding 0.5:1 to a maximum: 45% at a 35% loss ratio

Based on these terms, the final commissions for various actual loss ratios are as follows:

Actual Loss Ratio	Sliding Scale Commission
30% or Below	45.0%
35%	45.0%
40%	42.5%
45%	40.0%
50%	37.5%
55%	35.0%
60%	30.0%
65%	25.0%

Let's discuss the calculations in the table above:

- ◇ An actual loss ratio at 35% or below results in a maximum commission of 45%. Hence, the first two rows have a sliding scale commission of 45%
- ◇ An actual loss ratio of 40% sits between a loss ratio of 35% and a loss ratio of 55%. Thus, the sliding scale commission must sit between 35% and 45%. To calculate the sliding scale commission, we do $commission_{higher} + slide \times (L/R_{higher} - L/R_{actual}) = 35\% + 0.5(55\% - 40\%) = 42.5\%$. The notation " $commission_{higher}$ " refers to the commission at the higher loss ratio of the range. The higher loss ratio in this range is 55% and the commission at a 55% loss ratio is 35%. Actual loss ratios of 45% and 50% are calculated in the same way
- ◇ An actual loss ratio of 55% clearly maps to an sliding scale commission of 35%
- ◇ An actual loss ratio of 60% sits between a loss ratio of 55% and 65%. Thus, the sliding scale commission must sit between 25% and 35%. To calculate the sliding scale commission, we do $commission_{higher} + slide \times (L/R_{higher} - L/R_{actual}) = 25\% + (65\% - 60\%) = 30\%$
- ◇ An actual loss ratio of 65% clearly maps to a sliding scale commission of 25%

In the example above, we determined the adjusted commission **after** the actual loss ratio was known. If we are pricing a proportional treaty, we must treat the loss ratio as a random variable. Thus, for pricing purposes, we are interested in the expected loss ratio. Now, it may be tempting to simply map the expected loss ratio to the adjusted commission as we did in the prior example. But this fails to consider the fact that the expected loss ratio is an average of all possible outcomes. Hence, we need to consider each possible loss ratio separately, calculate the adjusted commission for each of those possible loss ratios, and then take the average to get an expected commission. Clark provides **two approaches** for doing this:

- (1) Estimate the expected commission based on the historical loss ratios (including cat and shock losses), adjusted to the future level. This is not the preferred approach since it may be distorted by historical cats or years with low premium volume

- (2) Use an aggregate loss distribution model to create a probability distribution for the loss ratio. This is the preferred approach

Example

Given the following results from an aggregate loss distribution model:

Range of Loss Ratios	Average in Range	Probability Loss Ratio is in Range
0% – 35%	31.5%	0.025
35% – 55%	46.9%	0.311
55% – 65%	59.9%	0.222
65% or above	82.2%	0.442
Total	65.0%	1.000

First, we calculate the sliding scale commission for each range using the same sliding scale from the prior example:

- ◇ 0% – 35%: The average loss ratio of 31.5% clearly maps to the maximum commission of 45% since $31.5\% < 35\%$
- ◇ 35% – 55%: The average loss ratio of 46.9% sits between a loss ratio of 35% and a loss ratio of 55%. Thus, the sliding scale commission must sit between 35% and 45%. To calculate the sliding scale commission, we do $commission_{higher} + slide \times (L/R_{higher} - L/R_{actual}) = 35\% + 0.5(55\% - 46.9\%) = 39.1\%$
- ◇ 55% – 65%: The average loss ratio of 59.9% sits between a loss ratio of 55% and a loss ratio of 65%. Thus, the sliding scale commission must sit between 25% and 35%. To calculate the sliding scale commission, we do $commission_{higher} + slide \times (L/R_{higher} - L/R_{actual}) = 25\% + (65\% - 59.9\%) = 30.1\%$
- ◇ 65% or above: The average loss ratio of 82.2% clearly maps to the minimum commission of 25% since $82.2\% > 65\%$

Second, we calculate the expected sliding scale commission using the range probabilities:

- ◇ The expected commission is $45.0\%(0.025) + 39.1\%(0.311) + 30.1\%(0.222) + 25\%(0.442) = 31.0\%$

Once we have the expected commission, we can calculate the expected **technical ratio**, which is the sum of the expected loss ratio and the expected commission. For this example, the expected technical ratio is $65.0\% + 31.0\% = 96.0\%$. Note that the expected loss of 65% is found by calculating the sum-product of the average loss ratios for each range and the corresponding probabilities (i.e., the expected value). Of course, in this example, it was provided to you in the data table.

We can complicate the sliding scale commission calculation by allowing **carryforward provisions**. Suppose that past loss ratios have been above the loss ratio corresponding to the minimum commission. A carryforward provision allows the “excess loss amount” to be included with the current year’s loss in the estimate of the current year’s commission. The **intent of the carryforward provision** is to smooth results over time.

There are **two approaches** for pricing the impact of carryforward provisions:

- (1) Estimate the impact on the current year only
 - ◇ With this approach, we shift the slide by the amount of the carryforward
 - ◇ The issue with this approach is that it ignores the potential for carryforward beyond the current year
- (2) Estimate the impact on a block of years (ex. the next five years)
 - ◇ With this approach, the variance of the aggregate distribution is reduced since we assume that individual bad years will be smoothed by individual good years
 - ◇ The reduced variance is captured by putting higher probabilities in the ranges closer to the expected loss ratio
 - ◇ One issue with this approach is that the method for reducing the variance is not obvious
 - ◇ Another issue with this approach is that it ignores the possibility that the contract may not renew the following year

Let’s look at an example of the first approach for incorporating a carryforward provision.

Example

Given the following tables from an aggregate loss distribution model:

Range of Loss Ratios	Average in Range	Probability Loss Ratio is in Range
0% – 35%	31.5%	0.025
35% – 55%	46.9%	0.311
55% – 65%	59.9%	0.222
65% or above	82.2%	0.442
Total	65.0%	1.000

Assume the following:

- ◇ The treaty includes a carryforward provision
- ◇ The cedant’s loss ratio in the prior year for the book of business underlying the treaty was 70%

Range of Loss Ratios	Average in Range	Probability Loss Ratio is in Range
0% – 30%	27.4%	0.006
30% – 50%	43.0%	0.221
50% – 60%	55.1%	0.222
60% or above	78.3%	0.551
Total	65.0%	1.000

- ◇ The sliding scale commission terms are as follows:
 - Minimum commission: 25% at a 65% loss ratio
 - Sliding 1:1 to: 35% at a 55% loss ratio
 - Sliding 0.5:1 to a maximum: 45% at a 35% loss ratio

First, we calculate the carryforward provision:

- ◇ The loss ratio associated with the minimum commission is 65%. The carryforward provision is the excess loss amount from the prior year. In this case, the carryforward provision is $70\% - 65\% = 5\%$

Second, we calculate the shifted sliding scale based on the carryforward provision:

- ◇ Minimum commission: 25% at a 60% loss ratio, where $60\% = 65\% - 5\%$
- ◇ Sliding 1:1 to: 35% at a 50% loss ratio, where $50\% = 55\% - 5\%$
- ◇ Sliding 0.5:1 to a maximum: 45% at a 30% loss ratio, where $30\% = 35\% - 5\%$

Third, we calculate the sliding scale commission for each range using the sliding scale terms:

- ◇ We must use the second table produced by the aggregate loss distribution model since the ranges match the shifted sliding scale ranges
- ◇ 0% – 30%: The average loss ratio of 27.4% clearly maps to the maximum commission of 45% since $27.4\% < 30\%$
- ◇ 30% – 50%: The average loss ratio of 43.0% sits between a loss ratio of 30% and a loss ratio of 50%. Thus, the sliding scale commission must sit between 35% and 45%. To calculate the sliding scale commission, we do $commission_{higher} + slide \times (L/R_{higher} - L/R_{actual}) = 35\% + 0.5(50\% - 43.0\%) = 38.5\%$
- ◇ 50% – 60%: The average loss ratio of 55.1% sits between a loss ratio of 50% and a loss ratio of 60%. Thus, the sliding scale commission must sit between 25% and 35%. To calculate the sliding scale commission, we do $commission_{higher} + slide \times (L/R_{higher} - L/R_{actual}) = 25\% + (60\% - 55.1\%) = 29.9\%$

- ◇ 60% or above: The average loss ratio of 78.3% clearly maps to the minimum commission of 25% since $78.3\% > 60\%$

Fourth, we calculate the expected sliding scale commission using the range probabilities:

- ◇ The expected commission is $45.0\%(0.006) + 38.5\%(0.221) + 29.9\%(0.222) + 25\%(0.551) = 29.2\%$

Fifth, we calculate the expected technical ratio:

- ◇ Expected technical ratio = expected loss ratio + expected commission = $65.0\% + 29.2\% = 94.2\%$

Profit Commission

The profit commission is much simpler than the sliding scale commission. Here are the steps for calculating a profit commission **after** the actual loss ratio is known:

- ◇ Subtract the actual loss ratio, ceding commission and a margin for expenses from 1.00 to determine the reinsurer's profit as a percentage of premium
- ◇ Return a specified percentage of the reinsurer's profit as a profit commission

Example

Given the following:

- ◇ Actual loss ratio = 55%
- ◇ Ceding commission = 25%
- ◇ Margin = 10%
- ◇ The reinsurer returns 50% of its profit back to the cedant

The reinsurer's profit is $100\% - 55\% - 25\% - 10\% = 10\%$. Thus, the profit commission is $50\%(10\%) = 5\%$. If the actual ceded premium was \$1,000,000, then the dollar amount of the profit commission back to the cedant would be $\$1,000,000(5\%) = \$50,000$.

In this example, we have already observed the actual loss ratio. If we wanted to include the profit commission while pricing the treaty, we would need to calculate an expected profit commission similar to the sliding scale commission examples. In this case, for each range from an aggregate loss distribution model, we would calculate the profit commission based on the average loss ratio in the range. Then, we would calculate the expected profit commission using the range probabilities. An example of this is provided in the Cookbook.

Loss Corridor

Under a loss corridor, the cedant re-assumes a portion of the reinsurer's liability for a specified loss ratio layer.

Example

Given the following table from an aggregate loss distribution model:

Range of Loss Ratios	Average in Range	Probability Loss Ratio is in Range
0% – 80%	64.1%	0.650
80% – 90%	84.7%	0.156
90% or above	103.9%	0.194
Total	75.0%	1.000

The treaty includes a loss corridor with the following terms:

- ◇ The cedant re-assumes 75% of the 80% to 90% loss ratio layer

First, let's calculate the reinsurer's loss ratio net of the loss corridor for each loss ratio range:

- ◇ 0% – 80%: Since the average loss ratio of 64.1% is below 80%, the loss corridor does not apply. Thus, the reinsurer's loss ratio net of the loss corridor is still 64.1%
- ◇ 80% – 90%: Since the average loss ratio of 84.7% is above 80%, the loss corridor applies. Under the loss corridor terms, 75% of (84.7% – 80%) is re-assumed by the cedant. Thus, the reinsurer's loss ratio net of the loss corridor is $84.7\% - 0.75(84.7\% - 80\%) = 81.2\%$
- ◇ 90% or above: Since the average loss ratio of 103.9% is above 80%, the loss corridor applies. Under the loss corridor terms, 75% of (90% – 80%) is re-assumed by the cedant. Thus, the reinsurer's loss ratio net of the loss corridor is $103.9\% - 0.75(90\% - 80\%) = 96.4\%$

Second, let's calculate the reinsurer's expected loss ratio net of the loss corridor using the range probabilities:

- ◇ The reinsurer's loss ratio net of the loss corridor is $64.1\%(0.650) + 81.2\%(0.156) + 96.4\%(0.194) = 73.0\%$
- ◇ As we can see, the loss ratio reduces the reinsurer's expected loss ratio by two points (from 75% to 73%)
- ◇ This example also highlights why we can't simply look at an expected loss ratio. The expected loss ratio before applying the loss corridor is 75%. Even though this is below the corridor layer of 80% – 90%, the reinsurer still benefits because we must consider each possible outcome (not just the expected outcome)

II. Property Per-Risk Excess Treaties

Property per-risk excess treaties apply on a per-risk basis and provide a limit of coverage in excess of the cedant's retention. Some important reminders about property per-risk excess treaties are as follows:

- ◇ The treaty premium is set as a percent of a subject premium base
- ◇ For losses occurring policies, the subject premium is called the “gross net earned premium income” (GNEPI)
- ◇ For risks attaching policies, the subject premium is called the “gross net written premium income” (GNWPI)
- ◇ For both policy types, the “net” refers to the fact that the subject premium is net of any other reinsurance inuring to the benefit of the per-risk treaty (i.e., reinsurance that is applied before the per-risk treaty is applied). The “gross” refers to the fact the subject premium is gross of the per-risk treaty being priced

Per-risk excess treaties are priced using either **experience rating** or **exposure rating**.

Experience Rating

The steps for experience rating are as follows:

- (1) Assemble the subject premium and historical losses for several years
- (2) Adjust the subject premium to the future level (i.e., on-leveling and inflation)
- (3) Apply loss trend factors to the historical large losses and determine the amount included in the excess layer being analyzed. If ALAE applies pro-rata with losses, it should be added individually for each loss
- (4) Apply excess development factors to the summed excess losses for each historical period
- (5) Divide the trended and developed layered losses by the adjusted subject premium to calculate loss costs for each historical period and in total

If the projected loss costs for each year are increasing or decreasing over time, then the model assumptions are not met and should be re-examined (ex. the trend or LDFs may be too high or too low, there may have been shifts in the types of business written by the cedant, etc.).

Example

A reinsurer's actuary is pricing a property per-risk excess treaty with the following terms as of 12/31/2022:

- ◇ Effective Date: 1/1/23
- ◇ Treaty Limit: \$400,000
- ◇ Attachment Point: \$100,000
- ◇ Loss Trend: 4%
- ◇ Exposure Trend: 2%
- ◇ The treaty is on a losses occurring basis
- ◇ No other reinsurance policies apply to this book of business

The historical subject premium and corresponding on-level factors are as follows:

AY	Gross Net Earned Premium Income (\$)	On-Level Factor
2018	1,910,507	1.02
2019	1,936,665	1.03
2020	1,963,617	1.04
2021	2,081,886	1.01

The historical large losses are as follows:

Accident Date	Untrended Total Loss
2/23/18	324,298
4/30/18	100,549
9/22/18	75,475
1/1/19	171,885
5/18/19	94,218
8/19/19	170,297
8/15/20	87,133
7/12/21	771,249

The following excess LDFs apply to the \$400,000 xs \$100,000 layer:

	12-Ult	24-Ult	36-Ult	48-Ult	60-Ult
CDF	2.25	1.30	1.15	1.05	1.01

First, we adjust the historical subject premium (i.e., GNEPI) to the future level:

AY	GNEPI	On-Level Factor	Trend Factor	Adjusted Subject Prem.
2018	1,910,507	1.02	1.104	$2,151,384 = 1,910,507(1.02)(1.104)$
2019	1,936,665	1.03	1.082	2,158,336
2020	1,963,617	1.04	1.061	2,166,734
2021	2,081,886	1.01	1.040	2,186,813

The trend factors are calculated as $(1 + \text{Exposure Trend})^{2023 - \text{AY}}$. For example, the trend factor for AY 2021 is $(1.02)^{2023 - 2021} = (1.02)^2 = 1.040$.

Second, we trend the losses and calculate the excess layer losses:

Accident Date	Untrended Total Loss	Years of Trend	Trend Factor	Trended Loss	Treaty Layer Loss
2/23/18	324,298	5.36	1.234	400,184	300,184
4/30/18	100,549	5.17	1.225	123,173	23,173
9/22/18	75,475	4.78	1.206	91,024	0
1/1/19	171,885	4.50	1.193	205,059	105,059
5/18/19	94,218	4.12	1.175	110,706	10,706
8/19/19	170,297	3.87	1.164	198,226	98,226
8/15/20	87,133	2.88	1.119	97,589	0
7/12/21	771,249	1.97	1.080	832,949	400,000

Let's discuss the table above:

- ◇ For losses, we **trend from the accident date to the average prospective accident date**. For example, the first loss must be trended from 2/23/18 to 7/1/23. The second loss must be trended from 4/30/18 to 7/1/23. To calculate the trend period in years, we can use the "YEARFRAC(Start Date, End Date)" function in Excel. Thus, the trend period for the 2/23/18 accident is "YEARFRAC(2/23/18, 7/1/23)" = 5.36. Then, the trend factor for this loss is $(1 + \text{Loss Trend})^{\text{Trend Period}} = (1.04)^{5.36} = 1.234$
- ◇ The trended total loss is equal to the untrended total loss multiplied by the trend factor
- ◇ The loss in the treaty layer is equal to MIN(MAX(Trended Total Loss – Attachment Point, 0), Treaty Limit). Thus, for the 2/23/18 accident, the loss in the treaty layer is MIN(MAX(400,184 – 100,000, 0), 400,000) = 300,184

Third, we aggregate the treaty layer losses by AY and develop them using the excess LDFs:

AY	Trended Treaty Layer Losses	Excess LDF	Trended, Developed Treaty Layer Losses
2018	323,356	1.01	326,590
2019	213,991	1.05	224,690
2020	0	1.15	0
2021	400,000	1.30	520,000

In the table above, AY 2021 is 24 months old at 12/31/22. Thus, we multiply the aggregated, trended treaty layer losses by the 24-Ultimate excess LDF of 1.30. The other years are calculated in a similar manner.

Fourth, we divide the trended, developed treaty layer losses by the adjusted subject premium for each AY and in total:

AY	Trended, Developed Treaty Layer Losses	Adjusted Subject Premium	Loss Cost
2018	326,590	2,151,384	15.2%
2019	224,690	2,158,336	10.4%
2020	0	2,166,734	0.0%
2021	520,000	2,186,813	23.8%
Total	937,347	8,663,267	12.4%

Using experience rating, we arrive an expected treaty loss ratio of 12.4%.

Exposure Rating

Unlike experience rating, exposure rating models the current risk profile rather than using past risk profiles. This is an **advantage** of exposure rating. For a portfolio of risks, exposure rating involves the following steps:

- (1) Determine the distribution of premium by different ranges of insured values (known as the **limits profile**)
- (2) Select an exposure curve P , which represents the amount of loss capped at a given percent p of the insured value relative to the total value of the loss. Mathematically:

$$P(p) = \frac{\int_0^{p \times IV} [1 - F(x)] dx}{E(x)}$$

where $F(x)$ is the CDF of the individual loss dollar amount. Note that the exposure curve P is normally provided to you for Clark problems.

- (3) Calculate the portion of the expected loss on the risk which falls in the treaty layer. This is known as the **exposure factor**. Mathematically:

$$\text{Exposure Factor} = P\left(\frac{\text{Retention} + \text{Limit}}{\text{Insured Value}}\right) - P\left(\frac{\text{Retention}}{\text{Insured Value}}\right)$$

- (4) Multiply the exposure factor in the treaty layer for each insured value range by the expected loss for the range to determine the expected treaty losses (i.e., the expected losses for the reinsurer)

Example

Given the following for the pricing of a losses occurring Property Per-Risk treaty:

- ◇ Treaty Limit = \$400,000
- ◇ Attachment Point = \$100,000
- ◇ Expected Loss Ratio = 65%
- ◇ The limits profile is as follows:

Range of IVs (\$000)	Subject Premium (\$000)
20 – 100	682
100 – 250	161
250 – 1,000	285
1,000 – 2,000	1,156

- ◇ The exposure curve P is as follows:

Percent of IV	Exposure Factor
0%	0%
10%	37%
20%	49%
30%	57%
40%	64%
50%	70%
60%	76%
70%	81%
80%	85%
90%	89%
100%	93%
110%	97%

120% | 100%

First, let's calculate the exposure factor in the treaty layer \$400,000 xs \$100,000:

Range of IVs (\$000)	Midpoint	$\frac{R+L}{IV}$	$\frac{R}{IV}$	$P\left(\frac{R+L}{IV}\right)$	$P\left(\frac{R}{IV}\right)$	Exposure Factor
20 – 100	60	833%	167%	100%	100%	0%
100 – 250	175	286%	57%	100%	74%	26%
250 – 1,000	625	80%	16%	85%	44%	41%
1,000 – 2,000	1,500	33%	7%	59%	25%	34%

Let's discuss the table above:

- ◇ We use the midpoint of the range as the IV for each range
- ◇ The $\frac{R+L}{IV}$ and $\frac{R}{IV}$ columns are simple to calculate. For example, for the 20 – 100 range, $\frac{R+L}{IV} = \frac{100+400}{60} = 833\%$ and $\frac{R}{IV} = \frac{100}{60} = 167\%$
- ◇ For the $P\left(\frac{R+L}{IV}\right)$ and $P\left(\frac{R}{IV}\right)$ columns, we use the exposure curve P . For values that cannot be pulled directly from the table, we use **linear interpolation**. For example, for the 100 – 250 range, $\frac{R+L}{IV} = 57\%$, which sits between 50% and 60% on the exposure curve. Thus, $P(57\%) = 70\% + \frac{76\%-70\%}{60\%-50\%} \times (57\% - 50\%) = 74\%$
- ◇ The exposure factor column is equal to $P\left(\frac{R+L}{IV}\right) - P\left(\frac{R}{IV}\right)$

Second, let's calculate the expected losses for the reinsurer for each IV range:

Range of IVs (\$000)	Sub. Prem. (\$000)	ELR	Expos. Fac.	Exp. Loss for Reins. (\$000)
20 – 100	682	65%	0%	0
100 – 250	161	65%	26%	27.209
250 – 1,000	285	65%	41%	75.953
1,000 – 2,000	1,156	65%	34%	255.476
Total	2,284			358.638

The expected loss for the reinsurer for the treaty layer of \$400,000 xs \$100,000 is \$358,638. Thus, the reinsurer's expected loss cost is $\frac{358,638}{2,284,000} = 15.7\%$. Similar to experience rating, we would finish out by loading this expected loss cost for expenses and a margin.

In the example above, we used the same exposure curve regardless of the size of the insured value. In reality, the exposure curve may differ by insured value (ex. large commercial risks). The Cookbook includes an example of using multiple exposure curves based on the size of insured value.

Once expected loss costs are estimated using the experience and exposure rating models, the actuary must reconcile the results and **select a final expected loss cost**. Clark discusses three additional issues that must be considered before we can complete the reconciliation process:

- ◇ Free Cover
- ◇ Credibility
- ◇ Inuring Reinsurance

Free Cover

When using experience rating, it's possible that no losses "trend into" the higher portion of the layer being priced. For example, suppose we are comparing the price for the layer \$750,000 xs \$250,000 with the price for the layer \$250,000 xs \$250,000. If the largest trended loss is \$500,000, the prices for each layer will be identical since the loss never pierces the top \$500,000 of the first layer. As a result, the top \$500,000 of the first layer is essentially free coverage (i.e., free cover).

We can address the issue for free cover as follows:

- ◇ Use experience rating as a basis for the lowest portion of the layer
- ◇ Use relativities in the exposure rating to project the higher portion of the layer

Example

Once again, we want to price two layers:

- ◇ \$250,000 xs \$250,000
- ◇ \$750,000 xs \$250,000

Given the following experience and exposure rating loss costs:

Layer	Exper. Rating Loss Costs	Expos. Rating Loss Cost
\$250,000 xs \$250,000	16%	20%
\$500,000 xs \$500,000	0%	10%
\$750,000 xs \$250,000	16%	30%

Let's implement the experience rating approach described above:

- ◇ We have a reasonable experience rating loss cost for the \$250,000 xs \$250,000 layer (i.e., 16%)
- ◇ The experience rating loss cost for the top portion (i.e., \$500,000 xs \$500,000) of the \$750,000 xs \$250,000 layer is 0%. Let's replace the 0% with $8\% = 16\% \left(\frac{10\%}{20\%}\right)$. We obtain the 8% by multiplying the lower layer experience rating loss cost by the exposure rating relativity between the low portion and top portion of the \$750,000 xs \$250,000 layer

- ◇ Then, the final selected loss cost for the \$750,000 xs \$250,000 layer is simply $16\% + 8\% = 24\%$

Credibility

Clark mentions two measures of credibility:

- (1) Expected number of claims. If the expected number of claims is not available, we can use the expected loss dollars based on exposure rating instead
- (2) Year-to-year variation in the projected loss cost from each of the historical periods. This measure is nice because credibility can still be high even if the number of expected claims is small (as long as the year-to-year variation in the projected loss cost is small)

Inuring Reinsurance

When a reinsurance policy applies before another reinsurance policy, we say that the reinsurance policy inures to the benefit of the treaty that is applied second.

When using experience rating, the only way to handle inuring reinsurance is to restate the historical loss experience net of the inuring reinsurance.

When **using exposure rating**, we apply the exposure rating steps directly to a risk profile adjusted to reflect the inuring reinsurance. Suppose the inuring reinsurance is a surplus share treaty. If multiple exposure curves varying by insured value are used, we must do the following:

- ◇ The curves should be selected based on the insured value **BEFORE** the surplus share is applied
- ◇ The exposure factor should apply to the subject premium **AFTER** the surplus share is applied

Example

Suppose a cedant decides to purchase a surplus share treaty with a retained line of \$200,000 on any one risk. On the “net of surplus share retention,” the cedant purchases a per-risk cover with the following details:

- ◇ Treaty Limit = \$100,000
- ◇ Attachment Point = \$100,000
- ◇ Expected Loss Ratio = 65%

◇ The limits profile is as follows:

Range of IVs (\$000)	Subject Premium (\$000)
20 – 100	682
100 – 250	161
250 – 1,000	285
1,000 – 2,000	1,156

◇ The exposure curve P is as follows:

Percent of IV	Exposure Factor
0%	0%
10%	37%
20%	49%
30%	57%
40%	64%
50%	70%
60%	76%
70%	81%
80%	85%
90%	89%
100%	93%
110%	97%
120%	100%

First, let's calculate the insured values and subject premium net of the surplus share:

Range of IVs (\$000)	IV Midpoint	IV Midpoint	Subject Premium (\$000)	Subject Premium Net S/S (\$000)
20 – 100	60	60	682	682
100 – 250	175	175	161	161
250 – 1,000	625	200	285	91.2
1,000 – 2,000	1,500	200	1,156	154.133

Let's discuss the table above:

◇ The first two IV ranges are not impacted by the S/S treaty since the IV midpoint is below the retained line. Thus, the IV midpoint and subject premium net of the S/S treaty are unchanged

- ◇ For the 250 – 1,000 range, the IV midpoint net of the S/S treaty is 200 since the retained line under the S/S treaty is 200. Thus, the reinsurer is only exposed to 200 of the total IV of 625. To determine the subject premium net of the S/S treaty, we multiply the unadjusted subject premium by the ratio of the net IV to the gross IV. Thus, the subject premium net of the S/S treaty is $285 \left(\frac{200}{625}\right) = 91.2$
- ◇ The 1,000 - 2,000 range works in the same way as the 250 - 1,000 range

Second, let’s calculate the exposure factor in the treaty layer \$100,000 xs \$100,000 based on the IVs net of the S/S treaty:

Range of IVs (\$000)	Midpoint Net of S/S	$\frac{R+L}{IV}$	$\frac{R}{IV}$	$P\left(\frac{R+L}{IV}\right)$	$P\left(\frac{R}{IV}\right)$	Exposure Factor
20 – 100	60	333%	167%	100%	100%	0%
100 – 250	175	114%	57%	98%	74%	24%
250 – 1,000	200	100%	50%	93%	70%	23%
1,000 – 2,000	200	100%	50%	93%	70%	23%

Third, let’s calculate the expected losses for the reinsurer for each IV range:

Range of IVs (\$000)	Sub. Prem. Net of S/S (\$000)	Exposure ELR	Exposure Factor	Exp. Loss for Reins. (\$000)
20 – 100	682	65%	0%	0
100 – 250	161	65%	24%	25.116
250 – 1,000	91.2	65%	23%	13.634
1,000 – 2,000	154.133	65%	23%	23.043
Total	1,088.333			61.793

The expected loss for the reinsurer for the treaty layer of \$100,000 xs \$100,000 is \$61,793. Thus, the reinsurer’s expected loss cost is $\frac{61,793}{1,088,333} = 5.7\%$.

III. Casualty Per-Occurrence Excess Treaties

Casualty per-occurrence excess treaties are often separated into **three categories**:

(1) Working layer

- ◇ Low attachment point
- ◇ Expected to be pierced multiple times annually
- ◇ The working layer is often retained by the cedant

(2) Exposed excess

- ◇ Excess layer with attachment point below some policy limits

- ◇ These losses are less frequent than a working layer as there will be some years where no losses pierce the attachment point

(3) Clash cover

- ◇ High attachment point above any single policy limit
- ◇ Clash covers are penetrated due to losses on multiple policies from a single occurrence OR from extra-contractual obligations (i.e., bad faith claims)

Similar to property per-risk excess treaties, casualty lines use experience and exposure rating.

Experience Rating

The **experience rating steps** are the same as those shown for property per-risk excess treaties, with the following **additional complications**:

- ◇ When compiling historical loss information, ALAE must be captured separately from losses
- ◇ For general liability and auto liability losses, the underlying policy limit should be captured
- ◇ For workers' compensation losses, the losses should be captured on an undiscounted basis
- ◇ The loss inflation factors should vary by line and should be derived from unlimited large losses. Using losses capped at the policy limit may understate the final results
- ◇ The trended losses must be capped at applicable policy limits
 - If we apply the historical policy limit to each loss, we fail to recognize that the policy limits will generally increase over time
 - As an alternative, we could apply the trend factor to the historical loss without applying a policy limit cap. In doing so, we are implicitly assuming that the policy limit increases at the same rate as inflation. If we use this approach, then the subject premium must be increased to a level that matches the inflation-adjusted policy limits
- ◇ Once loss and ALAE amounts are trended, we must determine the portion of each in the treaty layer. For ALAE, this is handled in one of two ways:
 - Pro-rata with loss: ALAE in the layer is estimated in proportion to losses
 - Included with loss: ALAE is added to the loss and the treaty limit applies to the sum

Example

This example highlights how to handle losses when performing experience rating for casualty per-occurrence excess treaties. Given the following as of 12/31/22 for the pricing of an excess of loss treaty for a general liability book of business:

- ◇ Treaty Limit = \$400,000

- ◇ Attachment Point = \$100,000
- ◇ Effective Date = 1/1/23
- ◇ Large Loss Trend = 4%
- ◇ ALAE is included pro-rata with loss in the treaty
- ◇ ALAE has the same trend as large losses
- ◇ Large loss experience is as follows:

Accident Date	Untrended Total Loss (\$000)	Untrended ALAE (\$000)	Underlying Policy Limit (\$000)
7/1/18	850	145	750
7/1/19	100	20	100
10/1/19	75	25	100
4/1/20	175	19	250
10/1/20	80	15	250
1/1/21	350	60	500
7/1/21	425	70	500

First, let's determine the trended and capped loss:

Accident Date	Untrended Total Loss (\$000)	Policy Limit (\$000)	Years of Trend	Trend Factor	Trended Loss (\$000)	Trended and Capped Loss (\$000)
7/1/18	850	750	5.00	1.217	1,034.45	750
7/1/19	100	100	4.00	1.170	117.00	100
10/1/19	75	100	3.75	1.158	86.85	86.85
4/1/20	175	250	3.25	1.136	198.80	198.80
10/1/20	80	250	2.75	1.114	89.12	89.12
1/1/21	350	500	2.50	1.103	386.05	386.05
7/1/21	425	500	2.00	1.082	459.85	459.85

Let's discuss the table above:

- ◇ The Years of Trend and Trend Factor columns are calculated in the same way we showed for the property per-risk example (note that the loss trend is 4% in this example)
- ◇ The Trended and Capped Loss column is calculated as $\text{MIN}(\text{Trended Loss}, \text{Policy Limit})$

Second, let's calculate the trended ALAE:

Accident Date	Untrended ALAE (\$000)	Years of Trend	Trend Factor	Trended ALAE (\$000)
7/1/18	145	5.00	1.217	176.465
7/1/19	20	4.00	1.170	23.400
10/1/19	25	3.75	1.158	28.950
4/1/20	19	3.25	1.136	21.584
10/1/20	15	2.75	1.114	16.710
1/1/21	60	2.50	1.103	66.180
7/1/21	70	2.00	1.082	75.740

In the table above, the ALAE is also trended with a 4% annual trend.

Third, let's calculate the portion of loss and ALAE in the treaty layer of \$400,000 xs \$100,000:

Accident Date	Trended and Capped Loss (\$000)	Trended ALAE (\$000)	Treaty Loss (\$000)	Treaty ALAE (\$000)
7/1/18	750	176.465	400.000	94.115
7/1/19	100	23.400	0	0
10/1/19	86.85	28.950	0	0
4/1/20	198.80	21.584	98.800	10.727
10/1/20	89.12	16.710	0	0
1/1/21	386.05	66.180	286.050	49.037
7/1/21	459.85	75.740	359.850	59.269

Let's discuss the table above:

- ◇ The Treaty Loss column is calculated as $\text{MIN}(\text{Treaty Limit}, \text{MAX}(\text{Trended and Capped Loss} - \text{Attachment Point}, 0))$
- ◇ Since ALAE is included pro-rata with losses, the Treaty ALAE column is calculated as $(\text{Trended ALAE}) \left(\frac{\text{Treaty Loss}}{\text{Trended and Capped Loss}} \right)$. For example, the Treaty ALAE for the 7/1/18 accident is $176.465 \left(\frac{400}{750} \right) = 94.115$

At this point, we aggregate the treaty losses and treaty ALAE by accident year and apply excess LDFs to project the aggregate figures to ultimate. Then, we adjust the subject premium as shown in the per-risk experience rating example. Finally, we calculate loss costs as shown before.

Clark mentions that actuaries can use industry information from the **Reinsurance Association of America (RAA) to obtain excess development factors**. However, he offers up some **cautions** in doing so:

- ◇ The reporting lag from the occurrence of an event to the establishment of a reinsurer's case reserve may vary by company
- ◇ The mix of attachment points and limits is not cleanly broken out in the RAA studies. This is problematic since development varies by attachment point
- ◇ The RAA data might contain Asbestos and Environmental claims which could distort development patterns
- ◇ The RAA workers' compensation data might not include a consistent handling of tabular discounts on large claims

Exposure Rating

Exposure rating for casualty per-occurrence excess treaties is similar to what we saw earlier for property per-risk treaties. We want to estimate a loss cost based on the premium and limits expected to be exposed during the treaty period. For **general liability** and **auto liability**, we generally use a severity distribution to calculate **increased limits factors (ILFs)** and then use those ILFs to calculate exposure factors. For **workers' compensation**, we generally use a severity distribution to calculate **excess loss factors (ELFs)** and then use those ELFs to calculate exposure factors.

Regardless of the approach used, the following is needed:

- ◇ x = random variable for size of loss (i.e., severity)
- ◇ $E(x)$ = unlimited expected value of x
- ◇ $E(x; L) = E[\min(x, L)]$ = expected value of losses capped at L

The *ILF* from limit L to limit U is defined as follows:

$$ILF_{L,U} = \frac{E(x; U)}{E(x; L)}$$

The *ELF* for a limit L is defined as follows:

$$ELF_L = \frac{E(x) - E(x; L)}{E(x)}$$

Notice that the numerator of ELF_L is calculating a difference. This is the "excess" part.

Now, let's zoom in on **general liability** and **auto liability**:

- ◇ The industry practice is to use a truncated Pareto distribution for loss severity
- ◇ Since these two lines use ILFs for exposure rating, the final **exposure factor** is calculated as follows:

$$\begin{aligned} \text{Exposure Factor} &= \frac{E(x; \min(PL, AP + Lim)) - E(x; \min(PL, AP))}{E(x; PL)} \\ &= \frac{ILF(\min(PL, AP + Lim)) - ILF(\min(PL, AP))}{ILF(PL)} \end{aligned}$$

where PL = cedant policy limit, AP = treaty attachment point, and Lim = treaty limit. Notice that we cap $AP + Lim$ and AP at PL in the two terms in the numerator. The reason why the two pieces of the formula above are equivalent is because we can divide the top and bottom by $E[x; B]$, where B is the basic limit. In doing so, we obtain ILF s with respect to the basic limit.

- ◇ If the treaty includes ALAE in proportion to losses, then we apply the exposure factor to the subject premium times an expected loss and ALAE ratio
- ◇ If the ALAE is included with losses, then we use a **modified exposure factor** as follows:

$$\text{Exposure Factor} = \frac{E(x; \min(PL, \frac{AP+Lim}{1+e})) - E(x; \min(PL, \frac{AP}{1+e}))}{E(x; PL)}$$

where PL = underlying policy limit applying to the loss only, AP = treaty attachment point applying to "ALAE + loss capped at policy limit," Lim = treaty limit applying to "ALAE plus loss capped at policy limit," and e = ALAE as a percent of loss capped at policy limit. Note that this formula assumes that ALAE varies directly with the capped loss. In reality, ALAE is probably not a constant percent of any loss. Another **major limitation** of this modified formula is that the allocated expenses may actually exceed the percent of capped losses in some cases.

Let's look at a general liability example.

Example

An actuary is pricing a 7.5M xs 2.5M excess of loss treaty for a general liability book of business. Given the following information:

- ◇ Expected Loss Ratio = 65%
- ◇ Expected ALAE Ratio (as % of Premium) = 5%

- ◇ ALAE is included pro-rata in the treaty
- ◇ The exposure profile for the book is as follows:

Policy Limit (\$)	Subject Premium (\$)
5M	15,000
10M	45,000
15M	80,000

- ◇ You are also given the following increased limit factors:

Limit (\$)	ILF
1M	1.0
2.5M	1.8
5M	2.0
10M	2.4
15M	2.8

First, let's calculate the exposure factor for each policy limit:

Policy Limit (\$)	$AP + Lim$ Capped at PL (\$)	AP Capped at PL (\$)	Exposure Factor
5M	5M	2.5M	10.0%
10M	10M	2.5M	25.0%
15M	10M	2.5M	21.4%

As an example calculation, let's consider the 5M policy limit. $AP + Lim$ capped at PL is $\text{MIN}(5M, 2.5M + 7.5M) = 5M$. AP capped at PL is $\text{MIN}(5M, 2.5M) = 2.5M$. Thus, the exposure factor is $\frac{ILF(\min(PL, AP+Lim)) - ILF(\min(PL, AP))}{ILF(PL)} = \frac{ILF(5M) - ILF(2.5M)}{ILF(5M)} = \frac{2.0 - 1.8}{2.0} = 10\%$.

Second, let's calculate the reinsurer's expected loss and ALAE for each policy limit:

Policy Limit (\$)	Subject Premium (\$)	Expected Loss & ALAE Ratio	Exposure Factor	Expected Reinsurer Loss & ALAE (\$)
5M	15,000	70% = 65% + 5%	10.0%	1,050 = 15,000(0.70)(0.10)
10M	45,000	70%	25.0%	7,875
15M	80,000	70%	21.4%	12,000
Total	140,000			20,925

The loss cost is $\frac{20,925}{140,000} = 14.9\%$. Recall that ALAE was included pro-rata with losses in this example. The Cookbook shows a small example of using the modified exposure factor when ALAE is included with losses.

Now, let's zoom in on **workers' compensation**:

- ◇ Unlike general liability or auto liability, workers' compensation uses special NCCI severity curves
- ◇ The NCCI curves vary by state and hazard group and can be approximated by an inverse power curve as follows:

$$\begin{aligned} ELF_L &= \frac{E(x) - E(x; L)}{E(x)} \\ &= aL^{-b} \end{aligned}$$

for some parameters a and b .

- ◇ Since workers' compensation uses ELF s and does not incorporate policy limits, the final exposure factor is as follows:

$$\boxed{\text{Exposure Factor} = ELF_{AP} - ELF_{AP+Lim}}$$

Example

An actuary is pricing a \$750,000 xs \$250,000 treaty on a workers' compensation book of business. Given the following information:

- ◇ Alabama (AL) Expected Loss Ratio = 70%
- ◇ New Jersey (NJ) Expected Loss Ratio = 85%
- ◇ The exposure profile for the book is as follows:

State	Hazard Group (HG)	Standard Premium (\$)
AL	II	100,000
AL	III	100,000
NJ	II	100,000
NJ	IV	100,000

- ◇ You are provided with the following limited severities (\$000):

Limit	HG II		HG III		HG IV	
	AL	NJ	AL	NJ	AL	NJ
250k	58.20	69.75	62.40	78.00	68.00	76.50
1,000k	59.64	73.50	64.48	79.50	69.00	82.03
Unlimited	60.00	75.00	65.00	80.00	70.00	85.00

First, let's calculate the exposure factor for each state and hazard group:

State	HG	ELF_{250k}	$ELF_{1,000k}$	Exposure Factor
AL	II	0.030	0.006	0.024
AL	III	0.040	0.008	0.032
NJ	II	0.070	0.020	0.050
NJ	IV	0.100	0.035	0.065

As an example calculation, let's consider AL and HG II. The ELF's are $ELF_{AP} = ELF_{250k} = \frac{E(x) - E(x; 250k)}{E(x)} = \frac{60.00 - 58.20}{60.00} = 0.030$ and $ELF_{AP+Lim} = ELF_{1,000k} = \frac{E(x) - E(x; 1,000k)}{E(x)} = \frac{60.00 - 59.64}{60.00} = 0.006$. Then, the exposure factor is $ELF_{AP} - ELF_{AP+Lim} = ELF_{250k} - ELF_{1,000k} = 0.030 - 0.006 = 0.024$.

Second, let's calculate the expected reinsurer loss in the treaty layer:

State	HG	Subject Premium (\$)	Exposure ELR	Exposure Factor	Expected Reinsurer Loss (\$)
AL	II	100,000	70%	0.024	$1,680 = 100,000(0.70)(0.024)$
AL	III	100,000	70%	0.032	2,240
NJ	II	100,000	85%	0.050	4,250
NJ	IV	100,000	85%	0.065	5,525
Total		400,000			13,695

The loss cost is $\frac{13,695}{400,000} = 3.4\%$.

Similar to property per-risk excess treaties, casualty excess treaties can have additional complications such as the following:

- ◇ Including Umbrella Policies
- ◇ Handling ALAE Included with Loss (already discussed)
- ◇ Loss Sensitive Features
- ◇ Workers' Compensation Experience Rating

Including Umbrella Policies

Umbrella policies apply on top of the primary policy written by the cedant. Clark goes into some detail on how to handle umbrella policies under exposure rating. **Ignoring drop-down coverage**, we modify the exposure factor as follows:

$$\text{Exposure Factor} = \frac{E(x; \min(UL + PL, UL + AP + Lim)) - E(x; \min(UL + PL, UL + AP))}{E(x; UL + PL) - E(x; UL)}$$

where UL = limit of underlying policy (also equal to the attachment point of the umbrella policy), PL = policy limit on the umbrella policy, AP = treaty attachment point, and Lim = treaty limit.

Example

A ceding company sells an umbrella policy for \$1M xs \$1M and is pricing a treaty that covers the umbrella loss layer of \$500k xs \$500k. In this case, the exposure factor for the treaty layer is calculated as follows:

$$\begin{aligned} \text{Exposure Factor} &= \frac{E(x; \min(UL + PL, UL + AP + Lim)) - E(x; \min(UL + PL, UL + AP))}{E(x; UL + PL) - E(x; UL)} \\ &= \frac{E(x; \min(\$2M, \$2M)) - E(x; \min(\$2M, \$1.5M))}{E(x; \$2M) - E(x; \$1M)} \\ &= \frac{E(x; \$2M) - E(x; \$1.5M)}{E(x; \$2M) - E(x; \$1M)} \end{aligned}$$

Some umbrella policies have a **drop-down provision** that covers losses from the first dollar up to the umbrella limit when an underlying aggregate limit is exhausted. If a drop-down provision is present, we modify the exposure factor to the following:

$\text{Exposure Factor} = \frac{[E(x; UL + AP + Lim) - E(x; UL + AP)](1 - \phi) + [E(x; AP + Lim) - E(x; AP)]\phi}{[E(x; UL + PL) - E(x; UL)](1 - \phi) + [E(x; PL)]\phi}$
--

where each term in the numerator is capped at $UL + PL$ as shown when a drop-down provision was not present.

Loss Sensitive Features

For lower loss layers (i.e., working layers), ceding companies may be willing to retain more of the losses. One way to accomplish this is through the use of an **aggregate annual deductible (AAD)**. When an AAD is present, the cedant retain all losses in the layer up until the aggregate losses reach the AAD. All losses above the AAD are covered by the reinsurer.

An **excess charge factor** for a given AAD is defined as follows:

$$\Phi_{AAD} = \frac{\int_{AAD}^{\infty} (y - AAD)g(y)dy}{E(y)}$$

where $g(y)$ is the distribution of aggregate losses in the layer. If we multiply the excess charge factor Φ_{AAD} by the loss cost for the layer gross of the AAD, we obtain the net loss cost for the reinsurer.

A second type of loss sensitive program is the **swing plan**. Under a swing plan, the actual layer losses are loaded for expenses and the result is charged back to the cedant, subject to maximum and minimum constraints. Similar to a sliding scale commission, an aggregate loss distribution model should be used to properly price a swing plan.

Example

Given the following range of loss costs from an aggregate loss distribution model for a swing plan:

Range of Loss Costs	Average Loss	
	Cost in Range	Probability
0% – 8%	6%	0.120
8% – 24%	18%	0.630
24% and above	40%	0.250

- ◇ The retro premium is equal to the actual layer losses $\times 100/80$
- ◇ Provisional Rate (as % of Subject Premium) = 15%
- ◇ Maximum Premium (as % of Subject Premium) = 30%
- ◇ Minimum Premium (as % of Subject Premium) = 10%

Using the information above, we create the following table:

Range of Loss Costs	Average Loss		Loaded Loss Cost	Capped Premium
	Cost in Range	Probability		
0% – 8%	6%	0.120	7.5%	10.0%
8% – 24%	18%	0.630	22.5%	22.5%
24% and above	40%	0.250	50.0%	30.0%
Total	22.1%		27.6%	22.9%

Let's discuss the table above:

- ◇ Since the retro premium is equal to the actual layer losses $\times 100/80$, it means that the retro premium targets an ELR of 80%
- ◇ The loaded loss cost column is equal to the average loss cost column divided by the target ELR. For example, the loaded loss cost for the 0% – 8% range is $7.5\% = \frac{6.0\%}{0.80}$
- ◇ The capped premium column is equal to $\text{MAX}(\text{Min. Prem.}, \text{MIN}(\text{Loaded Loss Costs}, \text{Max. Prem.}))$. For example, the capped premium for the 0% – 8% range is $10.0\% = \text{MAX}(10\%, \text{MIN}(7.5\%, 30\%))$

After accounting for the maximum and minimum premiums, the ELR is $\frac{22.1\%}{22.9\%} = 96.5\%$. Notice that this is not equal to the target ELR of 80%. Thus, we say that the swing plan is NOT balanced. Also, note that the provisional rate of 15% is well below the expected ultimate swing plan premium rate of 22.9%. This difference creates a **cash flow advantage for the cedant** which must be included in the final pricing evaluation.

Workers' Compensation Experience Rating

Experience rating for workers' compensation can be distorted due to how tabular discounts are handled. This distortion can be avoided by collecting individual claimant information and projecting each claimant's expected costs into the treaty layer. The claimant information needed includes the following:

- ◇ Claimant's current age
- ◇ Claimant's sex
- ◇ Estimate of annual indemnity cost including escalation
- ◇ Estimate of annual medical cost
- ◇ Amounts paid to date

IV. Aggregate Distribution Models

There are a number of different aggregate distribution models. We will look at the following:

- ◇ Empirical Distribution
- ◇ Single Distribution Model
- ◇ Recursive Calculation of the Aggregate Distribution
- ◇ Other Collective Risk Models

Empirical Distribution

For any of the adjustable features we have covered in the paper so far (ex. sliding scale commission, loss sensitive features, etc.), historical experience can be used to estimate the impact of the feature.

An **advantage** of the empirical approach is that it is easy to calculate.

Three **disadvantages** of the empirical approach are as follows:

- (1) It does not account for all possible outcomes and relies on the historical average. The actual result may differ greatly from the historical average
- (2) It may not properly reflect changes in business volume or mix of business

- (3) If losses were developed with the BF or Cape Cod method, historical losses may under-represent the true future volatility

Single Distribution Model

The single distribution model assumes that aggregate treaty losses follow a known distribution. A common single distribution model is the **lognormal distribution**.

An **advantage** of the single distribution model is that it is relatively simple to use, even when the source data is limited.

Two **disadvantages** of the single distribution model are as follows:

- (1) It does not allow for the loss free scenario. For example, the lognormal distribution is not defined at $y = 0$, where Y is lognormally distributed
- (2) There is no easy way to reflect the impact of changing per occurrence limits on the aggregate losses

Recursive Calculation of the Aggregate Distribution

The recursive formula is a type of collective risk model that works well for low frequency scenarios. The **frequency distribution** is assumed to be Poisson, negative binomial, or binomial. The **severity distribution** is defined in discrete steps. For the rest of this section, we will assume a Poisson frequency distribution with mean λ . It has the following form:

$$Pr(n) = \frac{\lambda e^{-\lambda}}{n!}$$

If we calculate the first few terms, we find that:

$$\begin{aligned} \diamond Pr(0) &= e^{-\lambda} \\ \diamond Pr(1) &= \frac{\lambda e^{-\lambda}}{1!} = \left(\frac{\lambda}{1}\right) Pr(0) \\ \diamond Pr(2) &= \frac{\lambda^2 e^{-\lambda}}{2!} = \left(\frac{\lambda}{2}\right) Pr(1) \end{aligned}$$

We can generalize this to $Pr(n) = \left(\frac{\lambda}{n}\right) Pr(n-1)$.

For the severity distribution, the discrete severity steps must be equally spaced apart. Also, the largest severity may be set equal to the per occurrence limits on an excess treaty OR to the limit times a loading for ALAE. For example, suppose the per occurrence limit on an excess treaty is 1,000. Then, one possible discrete severity distribution is as follows:

Notation	Severity	Probability
S_1	250	0.40
S_2	500	0.15
S_3	750	0.10
S_4	1,000	0.35

Notice that each severity is “250” apart from the preceding severity (i.e., equally spaced) and that the probabilities sum to 1. Also, please note that S_1 , S_2 , S_3 , and S_4 represent the **severity probabilities**, not the severities themselves. Thus, the probability of a 250 severity claim is $S_1 = 0.40$.

Now that we have frequency and severity distributions, let’s calculate a few aggregate loss figures assuming:

◇ Aggregate Loss = 0

- Let $A_0 = Pr(AggLoss = 0)$
- We obtain an aggregate loss of 0 by having no claims. Thus, $A_0 = Pr(0)$

◇ Aggregate Loss = 250

- Let $A_1 = Pr(AggLoss = 250)$
- We obtain an aggregate loss of 250 by having a single claim with a severity of 250. Thus, $A_1 = Pr(1) \cdot S_1 = \binom{\lambda}{1} Pr(0) \cdot S_1 = \binom{\lambda}{1} \cdot S_1 \cdot A_0$

◇ Aggregate Loss = 500

- Let $A_2 = Pr(AggLoss = 500)$
- We can have one loss of severity 500 or two losses of severity 250. Thus, $A_2 = Pr(1) \cdot S_2 + Pr(2) \cdot S_1 \cdot S_1 = \binom{\lambda}{2} \cdot (S_1 \cdot A_1 + 2 \cdot S_2 \cdot A_0)$

If we keep increasing the aggregate loss, we find the following recursive formula for A_k :

$$A_k = \sum_{i=1}^k \binom{\lambda}{i} i S_i A_{k-i}$$

Note that the formula above only holds for a Poisson frequency distribution. We must generalize it if we want to use it with the negative binomial or binomial distributions.

Assuming a Poisson frequency distribution, we calculate the **mean and variance of the aggregate losses** as follows:

$$E(\text{AggLoss}) = E(\text{Freq.}) \cdot E(\text{Sev.}) = \lambda \cdot E(\text{Sev.})$$

$$\text{Var}(\text{AggLoss}) = E(\text{Freq.}) \cdot \text{Var}(\text{Sev.}) + \text{Var}(\text{Freq.}) \cdot E(\text{Sev.}^2) = \lambda E(\text{Sev.}^2)$$

Two **advantages** of the recursive aggregate loss model are as follows:

- (1) It is simple to work with
- (2) It provides an accurate handling of low frequency scenarios

Two **disadvantages** of the recursive aggregate loss model are as follows:

- (1) The calculation is more intensive for higher frequencies
- (2) Only a single severity distribution can be used

Example

Suppose frequency is Poisson distributed with mean $\lambda = 3$. The severity distribution is as follows:

Notation	Severity	Probability
S_1	250	0.40
S_2	500	0.15
S_3	750	0.10
S_4	1,000	0.35

First, let's calculate the aggregate loss distribution through 1,250:

Aggregate Loss	k	Probability (A_k)
0	0	0.050
250	1	0.060
500	2	0.058
750	3	0.056
1,000	4	0.096
1,250	5	0.094

The calculations for the table above are as follows:

$$\diamond A_0 = Pr(\text{AggLoss} = 0) = e^{-3} = 0.050$$

$$\diamond A_1 = Pr(\text{AggLoss} = 250) = \binom{\lambda}{1} [1 \cdot S_1 \cdot A_0] = \binom{3}{1} [1 \cdot 0.40 \cdot 0.050] = 0.060$$

$$\diamond A_2 = Pr(AggLoss = 500) = \left(\frac{\lambda}{2}\right) [1 \cdot S_1 \cdot A_1 + 2 \cdot S_2 \cdot A_0] = \left(\frac{3}{2}\right) [1 \cdot 0.40 \cdot 0.060 + 2 \cdot 0.15 \cdot 0.050] = 0.058$$

$$\diamond A_3 = Pr(AggLoss = 750) = \left(\frac{\lambda}{3}\right) [1 \cdot S_1 \cdot A_2 + 2 \cdot S_2 \cdot A_1 + 3 \cdot S_3 \cdot A_0] = \left(\frac{3}{3}\right) [1 \cdot 0.40 \cdot 0.058 + 2 \cdot 0.15 \cdot 0.060 + 3 \cdot 0.10 \cdot 0.060] = 0.056$$

$$\diamond A_4 = Pr(AggLoss = 1,000) = \left(\frac{\lambda}{4}\right) [1 \cdot S_1 \cdot A_3 + 2 \cdot S_2 \cdot A_2 + 3 \cdot S_3 \cdot A_1 + 4 \cdot S_4 \cdot A_0] = \left(\frac{3}{4}\right) [1 \cdot 0.40 \cdot 0.056 + 2 \cdot 0.15 \cdot 0.058 + 3 \cdot 0.10 \cdot 0.060 + 4 \cdot 0.40 \cdot 0.050] = 0.096$$

$$\diamond A_5 = Pr(AggLoss = 1,250) = \left(\frac{\lambda}{5}\right) [1 \cdot S_1 \cdot A_4 + 2 \cdot S_2 \cdot A_3 + 3 \cdot S_3 \cdot A_2 + 4 \cdot S_4 \cdot A_1] = \left(\frac{3}{5}\right) [1 \cdot 0.40 \cdot 0.096 + 2 \cdot 0.15 \cdot 0.056 + 3 \cdot 0.10 \cdot 0.058 + 4 \cdot 0.40 \cdot 0.060] = 0.094$$

Second, let's calculate the mean and variance of the aggregate losses:

$$\diamond \text{Mean} = \lambda \cdot E(Sev.) = 3[250(0.40) + 500(0.15) + 750(0.10) + 1,000(0.35)] = 1,800$$

$$\diamond \text{Variance} = \lambda \cdot E(Sev.^2) = 3[250^2(0.40) + 500^2(0.15) + 750^2(0.10) + 1,000^2(0.35)] = 1,406,250$$

As we can see above, the calculations of A_k can get a bit complicated. Take a look at the Cookbook Clark Excel file for an efficient way of performing these calculations in Excel.

Other Collective Risk Models

The recursive aggregate loss model assumes a single, discrete severity distribution. If a more complicated severity distribution(s) are used, then simulation or numerical methods will probably be needed to describe the aggregate distribution. Although collective risk models are generally the best way to price the treaties covered in this paper, **there are some things to keep in mind**:

- ◇ Collective risk models can be complex and feel like a “black box”
- ◇ Collective risk models assume that each occurrence is independent of the others and that the frequency and severity distributions are independent. This may not be accurate in all cases
- ◇ Some collective risk models have a large error for low frequency scenarios
- ◇ The aggregate distribution reflects the process variance of losses, but does not reflect the full parameter variance

V. Property Catastrophe Covers

Property catastrophe covers typically provide protection for catastrophic events on an occurrence basis. The cover usually applies to the cedant's exposure net of surplus share, per-risk excess treaties, and facultative reinsurance. In other words, other reinsurance typically inures to the benefit of a catastrophe cover.

Catastrophe covers apply on an excess basis and have large limits. For example, a catastrophe cover may cover the \$10M xs \$30M layer. Catastrophe covers have limited **reinstatements**. A reinstatement refers to when coverage is replenished after the layer is exhausted. If the catastrophe cover applied to the \$10M xs \$30M layer and no reinstatements were provided, then the catastrophe cover would stop providing protection once \$10M of loss has been reinsured.

Reinstatements are available in two flavors:

- (1) Pro-Rata as to Amount
- (2) Pro-Rata as to Time

Example

Given the following:

- ◇ A catastrophe cover provides protection for the layer that runs from \$30M to \$40M for the treaty year from January 1, 2022 to December 31, 2022
- ◇ A catastrophe produces losses of \$34.5M on July 31, 2022
- ◇ The annual premium of the catastrophe cover is \$2,000,000
- ◇ The reinstatement provision is “110%, pro-rata as to amount”

In this case, the protected layer is $\$10M = \$40M - \$30M$ and \$4.5M of that \$10M has been exhausted. The reinstatement premium is $\$2M(1.10) \left(\frac{4.5}{10}\right) = \$990,000$.

Suppose the reinstatement provision was “110%, pro-rata as to time.” In that case, the reinstatement premium would be $\$2M(1.10) \left(\frac{5}{12}\right) = \$916,667$. Notice that the proportionality factor is calculated as the amount of time from the occurrence date to the treaty expiration date divided by the treaty length. Treaties with this type of reinstatement premium are no longer common.

When **pricing catastrophe covers**, the most common approach is to utilize a **cat model**:

- ◇ Cat models estimate property damage from a large bank of simulated events. The damage estimates are translated into insured loss estimates for the cedant
- ◇ The final output of a cat model is a distribution of possible losses on the subject business. The expected amount in the treaty layer can be calculated along with its standard deviation. This can be used as a starting point for a loss cost on the cover

A catastrophe model requires the following information:

- ◇ Measure of exposure – this should be insured values, construction types, occupancies, etc., along with attachment points for excess contracts
- ◇ Terms of the insurance policies – this should include deductible and coinsurance provisions of the original policies

- ◇ Geographical information – if possible, property address information is converted into latitude and longitude coordinates. If latitude/longitude information is not available, insured values may be summarized by zip code or state
- ◇ Details of inuring reinsurance – if a surplus share treaty inures to the benefit of the catastrophe cover, features such as occurrence caps or loss corridors will affect the catastrophe exposure and should be considered

Once results are produced by a catastrophe model, the following should be considered:

- ◇ Workers' compensation losses may be included within the cover (ex. earthquake occurs during working hours)
- ◇ The inuring reinsurance terms may not be calculable by the model
- ◇ Even if earthquake coverage is not offered by the cedant, there might still be exposure due to fires following an earthquake
- ◇ The reinsurer must understand other important coverage terms, such as the portion of policyholders that have replacement cost coverage vs. actual cash value

The point is that **catastrophe models might not be able to capture every impact from a catastrophe**. Some of these items may need to be tacked on subjectively.

It's also important to understand whether the catastrophe cover applies on a **losses occurring** or **risks attaching** basis. If the cover applies on a risks attaching basis, it's possible for the reinsurer to pay twice on the same loss event:

- ◇ A treaty renews on 1/1/22 for a layer of \$10M and is written on a risks-attaching basis
- ◇ A loss event takes place on 3/1/22
- ◇ An annual policy written on 7/1/21 is covered by the 1/1/21 treaty since the policy is still in effect on 3/1/22
- ◇ An annual policy written on 1/1/22 is covered by the 1/1/22 treaty since the policy is still in effect on 3/1/22
- ◇ Two treaties with \$10M limits are triggering at the same time due to a single event. Thus, it's possible for the reinsurer to pay \$20M for the same catastrophe in this case
- ◇ If the treaty was written on a losses occurring basis, the 1/1/21 treaty would not trigger because the loss occurs in 2022

The rest of this section focuses on finite reinsurance or **finite risk**. Using Clark's definition, finite risk refers to property catastrophe covers for which the maximum loss amount is reduced relative to traditional covers.

Finite risk covers have the following two characteristics:

- (1) Multiple year features
- (2) Loss sensitive features such as profit commissions and additional premium formulas

By including these types of features, the downside risk on the contract may be substantially reduced. In doing so, the contract may no longer qualify as reinsurance.

Under statutory accounting, **a contract can only be considered reinsurance if two conditions are met:**

- (1) The reinsurer assumes significant insurance risk under the reinsured portions of the underlying insurance contracts
- (2) It is reasonably possible that the reinsurer may realize a significant loss from the transaction

We should also note that features such as profit commissions can also reduce the upside of the contract from the reinsurer's perspective (a high profit commission in a loss free year might mean a large amount of money back to the cedant).

To price a finite risk cover, it's useful to understand how it compares to an equivalent traditional cover. Let's look at an example.

Example

Given the following terms on a finite basis:

- ◇ Annual Premium = \$2,500,000
- ◇ Occurrence Limit = \$10,000,000
- ◇ Profit Commission = 80% after 10% Margin on Annual Premium
- ◇ Additional Premium = 50% of (Loss + Margin - Annual Premium)

Ignoring expenses, the underwriting results for the reinsurer for a loss free year and one full loss year are as follows:

	Loss Free Year	One Full Loss Year
Premium	\$2,500,000	\$2,500,000
Loss	\$0	\$10,000,000
Profit Commission	\$1,800,000	\$0
Additional Premium	\$0	\$3,875,000
UW Result for Reinsurer	\$700,000	-\$3,625,000

Let's discuss the table above:

- ◇ In the loss free year, the reinsurer's profit is $\$2,500,000 - \$0 - 0.10(\$2,500,000) = \$2,250,000$. There is no additional premium. We return 80% of the reinsurer's profit back to the cedant. Thus, the profit commission is $\$2,250,000(0.80) = \$1,800,000$. The final UW result for the reinsurer $\$2,500,000 - \$1,800,000 = \$700,000$
- ◇ In the one full loss year, the reinsurer's profit is clearly negative. Hence, there is no profit commission. However, there is an additional premium of $0.50(\$10,000,000 + 0.10(\$2,500,000) - \$2,500,000) = \$3,875,000$. The final UW result for the reinsurer $\$2,500,000 - \$10,000,000 + \$3,875,000 = -\$3,625,000$

Now, let's consider a traditional cover with the following terms:

- ◇ Annual Premium = \$1,618,500
- ◇ Occurrence Limit = \$10,000,000
- ◇ Reinsurer's Share = 43.25%

Ignoring expenses, the underwriting results for the reinsurer for a loss free year and one full loss year are as follows:

	Loss Free Year	One Full Loss Year
Premium	$\$700,000 = \$1,618,500(0.4325)$	$\$700,000$
Loss	$\$0$	$\$4,325,000 = \$10,000,000(0.4325)$
UW Result for Reinsurer	$\$700,000$	$-\$3,625,000$

The nominal rate on line for the finite risk cover is $\frac{2.5M}{10M} = 25\%$. The rate on line for the traditional cover is $\frac{1.6185M}{10M} = 16.2\%$. To understand if the finite risk cover is appropriately priced, the reinsurer should ask itself if it would be willing to offer the same coverage on a traditional basis at a 16% rate on line. If not, then the finite risk cover pricing is probably inadequate.

One thing to be aware of is that the finite risk cover assumes that additional premium will be collected in certain cases to reduce the downside risk of the contract. This creates a **credit risk** for the reinsurer since it is relying on the cedant to provide the additional premium if and when the time comes.

VI. Calculating the Final Price

After the treaty loss cost is calculated, it must be loaded with the **reinsurer's expenses** and account for the **risk assumed by the stockholder** to calculate the final price.

The **reinsurer's expenses can be broken into three types**:

- (1) Expenses Varying with Premium (i.e., ceding commission, brokerage fees, federal excise tax)
- (2) Fixed Expenses (i.e., general overhead costs, UW and claims audit expenses)
- (3) Expenses Varying with Losses (i.e., reinsurer's ULAE)

Using the standard pricing formula, we calculate the **final price** as follows:

$$Premium = \frac{Loss\ Cost \times (1 + ULAE\%) + Fixed\ Expense}{1 - Variable\ Expense\%}$$

where the Loss Cost is the Selected Loss Cost % times the Subject Premium.

Example

Given the following information as of December 31, 2022 for the pricing of a Casualty Liability Excess of Loss Treaty:

- ◇ Subject Premium = \$100,000
- ◇ Loss Cost = 8.5%
- ◇ ULAE (as % of Reinsurer Loss) = 5%
- ◇ Ceding Commission = 12%
- ◇ Brokerage Fees = 3%
- ◇ Fixed Expense = \$1,500

The loss cost (in dollars) is $0.085(\$100,000) = \$8,500$. Using the standard pricing formula, the premium is $\frac{Loss\ Cost \times (1 + ULAE\%) + Fixed\ Expense}{1 - Variable\ Expense\%} = \frac{8,500(1 + 0.05) + 1,500}{1 - 0.12 - 0.03} = \$12,265$. The premium rate (as a % of the subject premium) is $\frac{12,265}{100,000} = 12.3\%$. The **issue with the standard pricing formula above** is that it fails to consider the following:

- ◇ The timing of cash flows
- ◇ Risk elements (including a risk load)
- ◇ Any adjustable features such as a swing plan premium or sliding scale commission
- ◇ Profit load provisions

Bernegger

Outline

When pricing non-proportional per risk reinsurance treaties, we want to utilize both loss experience and actual exposure to price each risk. To do this, we first assign each risk to a risk band based on the risk's maximum possible loss (MPL). Then, we allocate the expected loss for each risk band between the insurer (this is the retained portion) and the reinsurer (this is the ceded portion).

In this paper, we introduce special exposure curves to help us perform this allocation. These exposure curves are known as the MBBEFD curves.

I. Distribution Function and Exposure Curve

This paper focuses on ratios rather than raw values. We introduce the following definitions and notation:

- ◇ D is the cedant's maximum retention under a non-proportional reinsurance treaty
- ◇ M is the MPL
- ◇ X is the gross loss, where $X \leq M$
- ◇ $d = \frac{D}{M}$ is the ratio of the cedant's retention to the MPL. This is known as the normalized deductible
- ◇ $x = \frac{X}{M}$ is the ratio of the gross loss to the MPL. This is known as the normalized loss
- ◇ $L(d) = E[\min(d, x)]$. This is the limited expected value of the ratio x

If we **ignore the possibility of a reinsurance limit**, the percentage of the pure risk premium (i.e., the expected losses) retained by the cedant is $G(d)$, where $G(d)$ is defined as follows:

$$\begin{aligned} G(d) &= \frac{L(d)}{L(1)} \\ &= \frac{\int_0^d (1 - F(y)) dy}{E(x)} \end{aligned}$$

The derivative of $G(d)$ is $G'(d) = \frac{1-F(d)}{E(x)}$. We can use $G'(d)$ to define the CDF of $x = \frac{X}{M}$ as follows:

$$F(x) = 1 - \frac{G'(x)}{G'(0)}$$

where $0 \leq x < 1$. For $x = 1$, $F(x) = 1$.

Since there is a finite maximum possible loss, there is a probability mass at $x = 1$. This probability mass is the **probability of a total loss** and is notated as p . We define p as follows:

$$p = 1 - F(1^-) = \frac{G'(1)}{G'(0)}$$

In the case where there is NOT a maximum possible loss, it's not possible to divide X or D by M . Instead, we set $x = \frac{X}{X_0}$ and $d = \frac{D}{X_0}$, where X_0 is an arbitrary reference loss. Then, $p = \frac{G'(1)}{G'(0)}$ is the probability p of having a loss X that exceeds the reference loss X_0 .

II. The MBBEFD Class of Two-Parameter Exposure Curves

In this section, the author introduces the MBBEFD distribution class and associated exposure curves. The general exposure curve is given by the following:

$$G(x) = \frac{\ln(a + b^x) - \ln(a + 1)}{\ln(a + b) - \ln(a + 1)}$$

As shown earlier, we can use the derivative to calculate the CDF $F(x) = 1 - \frac{G'(x)}{G'(0)}$. This simplifies to the following:

$$F(x) = 1 - \frac{(a + 1)b^x}{a + b^x}$$

where $0 \leq x < 1$. For $x = 1$, $F(x) = 1$.

Now, to ensure desirable mathematical properties are met, we can re-parameterize the curves by replacing a with a new parameter. Let $g = \frac{a+b}{(a+1)b} = \frac{1}{p}$, which is the inverse of the total probability of loss p . If we solve for a , we find that $a = \frac{(g-1)b}{1-gb}$. If we replace a with $\frac{(g-1)b}{1-gb}$ in the MBBEFD exposure curve above, we obtain the re-parameterized exposure curve $G_{b,g}(x)$.

Before we provide the final formula of $G_{b,g}(x)$, we must consider the following cases separately:

$$\diamond b = 1 \text{ and } g > 1$$

- ◇ $g = 1$ or $b = 0$
- ◇ $bg = 1$ and $g > 1$

If treat the three cases above separately, we arrive at the following:

$$G_{b,g}(x) = \left\{ \begin{array}{ll} x, & (g = 1) \vee (b = 0) \\ \frac{\ln(1+(g-1)x)}{\ln(g)}, & (b = 1) \wedge (g > 1) \\ \frac{1-b^x}{1-b}, & (bg = 1) \wedge (g > 1) \\ \frac{\ln\left(\frac{(g-1)b+(1-gb)b^x}{1-b}\right)}{\ln(gb)}, & (b > 0) \wedge (b \neq 1) \wedge (bg \neq 1) \wedge (g > 1) \end{array} \right\}$$

Note that “ \vee ” means “or” and “ \wedge ” means “and.” In our opinion, it’s probably not worth memorizing these four special cases of $G_{g,b}(x)$ as past exam problems have always provided the formula.

Once you have the $G_{b,g}$ curves, we can calculate the derivative $G'(x)$. Then, we can use the derivative to calculate the CDF $F(x) = 1 - \frac{G'(x)}{G'(0)}$ or the expected value $\mu = E(x)$, defined as follows:

$$\begin{aligned} \mu &= E(x) \\ &= \frac{E(X)}{M} \\ &= \frac{1}{G'(0)} \end{aligned}$$

We can also calculate the density function $f(x) = F'(x)$.

Similar to the what we saw before, we can also consider situations where there is not a finite maximum possible loss. In this case, we compare X and D to a reference loss X_0 . Then, $x = \frac{X}{X_0}$ and $d = \frac{D}{X_0}$. The formulas for $G_{b,g}(x)$ must be modified to account for these new definitions of x and d (again, probably not worth memorizing).

Example

An actuary is pricing a non-proportional treaty with a \$750,000 attachment point and no reinsurance limit. Given the following information about the risk:

- ◇ Gross Premium = \$12,000
- ◇ Maximum Possible Loss = \$10,000,000
- ◇ Expected Loss Ratio = 70%

The actuary uses the following MBBEFD exposure curve to price the treaty:

$$\diamond G(x) = \frac{\ln\left(\frac{(g-1)b+(1-gb)b^x}{1-b}\right)}{\ln(gb)}$$

◇ $g = 25$

◇ $b = 31$

First, let's calculate the exposure factor using $G(d)$:

- ◇ Recall that $G(d)$ is the percentage of the overall pure risk premium retained by the cedant. Thus, $1 - G(d)$ must be the percentage of the overall pure risk premium ceded to the reinsurer. We call this the **exposure factor**

◇ Exposure factor = $1 - G(d) = 1 - G\left(\frac{750,000}{10,000,000}\right) = 1 - \frac{\ln\left(\frac{(25-1)31+(1-25(31))31^{0.075}}{1-31}\right)}{\ln(25(31))} = 0.677$

Second, let's calculate the portion of the overall pure risk premium that is retained by the risk and the portion of the overall pure risk premium that is ceded to the reinsurer:

- ◇ The overall pure risk premium is $\text{Premium(ELR)} = 12,000(0.70) = 8,400$
- ◇ The portion of the overall pure risk premium retained by the risk is $8,400(0.323) = \$2,713.20$
- ◇ The portion of the overall pure risk premium ceded to the reinsurer is $8,400(0.677) = \$5,686.80$

Now, **suppose the treaty included a reinsurance limit of \$2,000,000**. How does this impact the expected ceded loss?

- ◇ In this case, the exposure factor (i.e., the percentage of the overall pure risk premium ceded to the reinsurer) changes from $[1 - G(d)]$ to $\left[G\left(\frac{\text{Attach. Point} + \text{Limit}}{MPL}\right) - G(d)\right]$
- ◇ Thus, the exposure factor is $G(0.275) - G(0.075) = 0.560 - 0.323 = 0.237$
- ◇ The portion of the overall pure risk premium ceded to the reinsurer is now $8,400(0.237) = \$1,990.80$

III. Curve Fitting

Suppose we want to fit a MBBEFD curve to price a non-proportional reinsurance treaty. The steps to fit the curve are as follows:

- ◇ Determine the g parameter as $g = \frac{1}{p}$. This requires the actuary to make an assumption about the total probability of loss p
- ◇ Calculate $\mu = \frac{E(X)}{M} = \frac{1}{G'(0)}$
- ◇ Determine b based on special cases or the general case:
- Special Cases
 - ◇ If $\mu = 1$, then $b = 0$
 - ◇ If $\mu = \frac{g-1}{\ln(g)g}$, then $b = \frac{1}{g}$

- ◇ If $\mu = \frac{\ln(g)}{g-1}$, then $b = 1$
- ◇ If $\mu = \frac{1}{g}$, then $b = \infty$

- General Case

- ◇ If μ is not one of the four special cases above, then we determine b iteratively using the following equation:

$$\mu = \frac{\ln(gb)(1-b)}{\ln(b)(1-gb)}$$

- ◇ We simply use trial and error to find the b that reproduces the calculated μ

Example

An actuary is pricing a non-proportional treaty. Given the following assumptions about the risk:

- ◇ The probability of total loss is 4%
- ◇ The expected loss severity is \$500,000
- ◇ The maximum possible loss is \$5,000,000

The actuary wants to fit a two-parameter MBBEFD curve based on these assumptions.

First, let's calculate g :

$$\diamond g = \frac{1}{p} = \frac{1}{0.04} = 25$$

Second, let's calculate μ :

$$\diamond \mu = \frac{E(X)}{M} = \frac{500,000}{5,000,000} = 0.10$$

Third, let's calculate b :

- ◇ Under the general case, we know that $\mu = \frac{\ln(gb)(1-b)}{\ln(b)(1-gb)}$. Since we know that $g = 25$ and $\mu = 0.10$, we simply need to find the b such that $0.10 = \frac{\ln(25b)(1-b)}{\ln(b)(1-25b)}$. A bit of trial and error in Excel gives $b = 4.15$

Now that we have both parameters, we can proceed with allocating the overall pure risk premium between the risk and the reinsurer as shown in the first example.

The last things covered in this paper are the **Swiss Re Y_i property exposure curves**, where $i = 1, 2, 3, \text{ or } 4$. These are special curves used by non-proportional property underwriters. They can be approximated by a subclass of one-parameter MBBEFD exposure curves.

The subclass has the following exposure curve formula:

$$G_c(x) = G_{b_c, g_c}(x)$$

where $b_c = e^{3.1-0.15(1+c)c}$ and $g_c = e^{(0.78+0.12c)c}$. Notice that b_c and g_c are completely determined by the single parameter c . In practice, the following values of c are commonly used for different purposes:

- ◇ $c = 0$: Represents a distribution of total losses only
- ◇ $c = \{1.5, 2.0, 3.0, 4.0\}$: Coincides with the Swiss Re Curves $\{Y_1, Y_2, Y_3, Y_4\}$
- ◇ $c = 5.0$: Coincides with a Lloyd's curve used for the rating of industrial risks

Example

An actuary is pricing a non-proportional treaty and would like to use a one-parameter MBBEFD curve to approximate the Swiss Re Y_2 curve.

The g_c and b_c parameters are calculated as follows:

- ◇ Since the actuary is approximating the Swiss Re Y_2 curve, $c = 2.0$
- ◇ Thus, $g_2 = e^{(0.78+0.12(2))2} = 7.691$ and $b_2 = e^{3.1-0.15(1+2)2} = 9.025$

Now that we have g_2 and b_2 , we can proceed with allocating the overall pure risk premium between the risk and the reinsurer as shown in the first example.

Grossi & Kunreuther

Ch. 2 (Introduction to Cat Models)

Outline

I. Structure of Catastrophe Models

There are **four basic components** of a catastrophe model:

(1) Hazard

- ◇ Describes the natural catastrophe (i.e., hazard)
- ◇ The description might include items such as earthquake epicenter location, projected hurricane path, hurricane wind speed, etc.

(2) Inventory

- ◇ Describes the portfolio of properties at risk
- ◇ The description might include items such as the location of each exposed property, the construction type of a building, the number of stories of a building, etc.
- ◇ The location description is often based on geocoding, which assigns a latitude and longitude to the exposed property

(3) Vulnerability

- ◇ Combines the hazard with the exposed properties to calculate the physical impact of the hazard on the properties. In other words, this module determines the severity of the impact on the property

(4) Loss

- ◇ Determines the direct and indirect losses of the hazard on the exposed properties
- ◇ Direct losses include the cost to repair and/or replace a structure
- ◇ Indirect losses include business interruption impacts and relocation costs of residents forced to leave their homes

II. Uses of a Catastrophe Model for Risk Management

Catastrophe models are used in a variety of ways by various stakeholders:

- ◇ Insurers use model output to understand what level of reinsurance protection is needed to ensure solvency in the event of a catastrophe
- ◇ Reinsurers use model output to price catastrophe covers
- ◇ Capital markets use model output to price catastrophe bonds
- ◇ Emergency management agencies (ex. FEMA) use model output to understand where the largest concentration of loss will occur in the event of a specific catastrophe (ex. large earthquake)

One valuable model output to insurers and reinsurers is the **exceedance probability (EP) curve**. An EP curve is a graphical representation of the probability that a certain level of loss will be surpassed in a given time period. Insurers and reinsurers use EP curves in the following ways:

- ◇ To determine the size and distribution of their portfolios' potential losses
- ◇ To determine the types and locations of buildings they would like to insure, what coverage to offer, and what price to charge
- ◇ To determine what proportion of their risk needs to be transferred to either a reinsurer or the capital markets

The term “exceedance probability” can be used in three ways:

- (1) **Occurrence Exceedance Probability (OEP)** – the probability that at least one loss exceeds the specific loss amount
- (2) **Aggregate Exceedance Probability (AEP)** – the probability that the sum of all losses during a given period exceeds some point
- (3) **Conditional Exceedance Probability (CEP)** – the probability that the amount of a single event exceeds a specific loss amount

For this paper, the term “exceedance probability” is specifically referring to an OEP. We will use “OEP” from now on.

An OEP curve plots losses on the x -axis and occurrence exceedance probabilities on the y -axis. Suppose an insurer offering earthquake coverage sets \$10M as an acceptable level of loss at a 1% (1-in-100) probability of exceedance. If the loss amount at a 1% occurrence exceedance probability (i.e., the x value) is greater than \$10M, then the insurer should look for ways to 1) reduce its portfolio, 2) transfer the excess amount above \$10M to a reinsurer, or 3) purchase a catastrophe bond to cover the excess amount above \$10M.

III. Derivation and Use of an Occurrence Exceedance Probability Curve

To generate an OEP curve, we need to define and assume some things:

- ◇ There is a set of natural disaster events, E_i
- ◇ Each event has an annual probability of occurrence, p_i
- ◇ Each event has an associated loss, L_i
- ◇ The events are assumed to be independent Bernoulli random variables. Each event occurs at most once with the **probability mass function** defined as:

$$\begin{aligned} Pr(E_i \text{ occurs}) &= p_i \\ Pr(E_i \text{ does not occur}) &= 1 - p_i \end{aligned}$$

- ◇ The **expected loss for a given event** is as follows:

$$E(L) = p_i L_i$$

- ◇ The **average annual loss (AAL)** is the sum of the expected losses for each event in a given year and is defined as follows:

$$AAL = \sum_i p_i L_i$$

- ◇ Assuming that each event occurs at most once in a given year, the **OEP** for a given level of loss, denoted $OEP(L_i)$, is defined as follows:

$$\begin{aligned} OEP(L_i) &= P(L > L_i) \\ &= 1 - P(L \leq L_i) \\ &= 1 - \prod_{j=1}^{i-1} (1 - p_j) \end{aligned}$$

The OEP is the annual probability that at least one loss exceeds the given value. *Note that the formulas and wording above have been adjusted to reflect the section 2.4 errata for the book. As stated in the Exam 9 Content Outline, students are responsible for the errata from this section.*

An insurer can also use an OEP curve to determine its **probable maximum loss (PML)**. The PML is the loss amount that corresponds to an acceptable OEP. PMLs are often framed in terms of the return period, which is simply the reciprocal of the OEP.

Example

This example comes from the textbook. Since there are a number of errors in the table, we will recreate the entire table with the corrected values. A catastrophe model produces a set of 14 events from an insurer's book of business. Here are the events, along with associated probabilities and loss amounts. Note that the events have already been sorted in descending order by loss amount:

Event (E_i)	Annual Probability of Occurrence (p_i)	Loss (L_i)
1	0.002	\$25M
2	0.005	\$15M
3	0.010	\$10M
4	0.020	\$5M
5	0.030	\$3M
6	0.040	\$2M
7	0.050	\$1M
8	0.050	\$800k
9	0.050	\$700k
10	0.070	\$500k
11	0.090	\$500k
12	0.100	\$300k
13	0.100	\$200k
14	0.100	\$100k

First, let's calculate the OEPs and AAL:

Event (E_i)	Annual Probability of Occurrence (p_i)	Loss (L_i)	$OEP(L_i)$	$E(L) = p_i L_i$
1	0.002	\$25M	0.000	\$50k
2	0.005	\$15M	0.002	\$75k
3	0.010	\$10M	0.007	\$100k
4	0.020	\$5M	0.017	\$100k
5	0.030	\$3M	0.037	\$90k
6	0.040	\$2M	0.065	\$80k
7	0.050	\$1M	0.103	\$50k
8	0.050	\$800k	0.148	\$40k
9	0.050	\$700k	0.190	\$35k
10	0.070	\$500k	0.231	\$35k
11	0.090	\$500k	0.285	\$45k

12	0.100	\$300k	0.349	\$30k
13	0.100	\$200k	0.414	\$20k
14	0.100	\$100k	0.473	\$10k
Total				\$760k

Let's discuss the table above:

- ◇ For each event E_i , $E(L) = p_i L_i$. Thus, for E_1 , $E(L) = p_1 L_1 = 0.002(\$25M) = \$50k$. If we do this calculation for each event and sum them up, we obtain **AAL = \$760,000**
- ◇ For each event E_i , $OEP(L_i) = 1 - \prod_{j=1}^{i-1} (1 - p_j)$. Let's look at a few of these to get the pattern down:

- Event 1: $OEP(L_1) = 1 - \prod_{j=1}^0 (1 - p_j) = 1 - 1 = 0$. The OEP for Event 1 is always 0 since there is no loss amount bigger than it assuming we have **sorted the events in descending order**. If you are given un-sorted events, sort them in descending order first and label them starting with "1"
- Event 2: $OEP(L_2) = 1 - \prod_{j=1}^1 (1 - p_j) = 1 - (1 - 0.002) = 0.002$
- Event 3: $OEP(L_3) = 1 - \prod_{j=1}^2 (1 - p_j) = 1 - (1 - 0.002)(1 - 0.005) = 0.007$
- Event 4: $OEP(L_4) = 1 - \prod_{j=1}^3 (1 - p_j) = 1 - (1 - 0.002)(1 - 0.005)(1 - 0.010) = 0.017$

Second, let's calculate the PML assuming the insurer's acceptable risk level is the 1-in-250 year PML:

- ◇ A 1-in-250 return period corresponds to an OEP of $\frac{1}{250} = 0.004$
- ◇ If we look at our OEP curve, an OEP of 0.004 corresponds to a loss amount between \$10M (corresponds to an OEP of 0.007) and \$15M (corresponds to an OEP of 0.002)
- ◇ We linearly interpolate to obtain the PML. Thus, **PML** = $\$15M + \frac{\$10M - \$15M}{0.007 - 0.002} \times (0.004 - 0.002) = \$13M$

We can also use the OEP curve to distribute the losses between stakeholders. As a simple example, consider a home portfolio where each insured has a deductible on their policy. Further suppose that the portfolio is reinsured. In this case, we have three stakeholders or participants: 1) Homeowners, 2) Insurer, and 3) Reinsurer. The reinsurance and deductible might be structured such that the first \$5M is covered by the homeowners, the losses between \$5M and \$30M are covered by the insurer, and losses in excess of \$30M are covered by the reinsurer.

Suppose that the portfolio of homes is characterized by the OEP curve from the prior example. At first glance, it would seem that the reinsurer has a 0% chance of a positive loss amount since the highest loss amount from the OEP table is \$25M. But this fails to recognize that each event's loss amount is stochastic and has a range around it! In other words, the loss amounts in the OEP curve are the mean losses for each event. If we create a range of possible outcomes for each event, then we can build accurate OEP curves for each stakeholder.

IV. Insurability of Catastrophe Risks

A risk must satisfy two conditions to be considered **insurable**:

- (1) The ability to identify and quantify the chances of the event occurring (i.e., frequency) and the extent of the losses likely to be incurred (i.e., severity)
 - ◇ The estimates of frequency and severity can be based on past data, cat modeling, and expert opinion
- (2) The ability to set premiums for each potential customer or class of customers

A risk can be **insurable and not profitable**. This happens when the insurer is unable to specify a rate for which there is enough demand and incoming revenue to cover costs such as development, marketing, and claims processing.

Regarding the second condition above, there are many **factors that influence the rate charged by the insurer**:

- ◇ Uncertainty of Losses
 - If insurers are unable to produce precise estimates of the risk, they might set higher premiums to account for the additional uncertainty. Loss uncertainty is particularly challenging for catastrophes given the nature of the events
- ◇ Supply Shortages
 - If the capacity of the insurance industry is reduced due to recent large losses (ex. large hurricane), then insurers might charge higher premiums
- ◇ Highly Correlated Losses
 - Clearly, natural catastrophes result in correlated losses given that a single event can result in multiple losses
 - When an insurer issues a large number of independent policies, then the losses from the policies should follow the law of large numbers. Under the law of large numbers, the variance around the mean of the random variables decreases as the number of variables increases

- Since natural catastrophes result in higher correlated losses, they do not follow the law of large numbers
- ◇ Adverse Selection
 - Adverse selection occurs when an insurer fails to distinguish between the expected losses for different risk groups. In this case, the price does not differ between the groups and the better performing risks will shop for coverage from insurers who recognize their better performance
 - Adverse selection is not a major issue for catastrophe risks given that catastrophes can affect anyone at anytime
- ◇ Moral Hazard
 - Moral hazard refers to an increase in the expected loss caused by the behavior of the policyholder (ex. an insured intentionally “loses” a covered wedding ring)
 - Moral hazard is not a major issue for catastrophe risks

When determining whether or not to **offer coverage for catastrophe risks**, insurers look to maximize expected profit subject to a **survival constraint**. An insurer satisfies the survival constraint by choosing a portfolio of risks with an overall expected probability of insolvency less than some threshold p_1 . Mathematically, the survival constraint is as follows:

$$Pr(\text{Total Loss} > (n \cdot z + A)) < p_1$$

where n = maximum number of policies satisfying constraint, z = insurance premium for each policy, and A = surplus. Note that customer demand comes into play here. Larger values of z can reduce demand, which may result in the insurer leaving the market if it cannot generate a positive expected profit.

V. Framework to Integrate Risk Assessment with Risk Management

A framework for integrating catastrophe risk assessment with risk management can be summarized as follows:

- ◇ Combine all four modules of a catastrophe model (hazard, inventory, vulnerability, and loss) to assess the risk
- ◇ Stakeholder digests the risk assessment information from the catastrophe model and uses a decision rule to develop risk management strategies
- ◇ Stakeholder implements appropriate risk management strategies

As an example, assume that the stakeholder is an insurer. The insurer uses catastrophe model output to assess catastrophe risk. The insurer’s decision rule for developing risk management

strategies is to maximize expected profits subject to meeting the survival constraint. The insurer determines that risk transfer through reinsurance is an appropriate risk management strategy to maximize expected profits while keeping the probability of insolvency at an acceptable level. Note that the book mentions that another broad risk management strategy is risk reduction (ex. mitigation).

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Ch. 3 (Risk Assessment Process)

Outline

I. Introduction

Unlike automobile or fire insurance policies, actuaries cannot use traditional techniques of estimating future losses for catastrophe events due to the variability and scarcity of historical loss data. Instead, a **probabilistic approach** is the most appropriate way to model catastrophe losses. This is the approach taken by catastrophe models:

- ◇ The hazard module estimates the probability that the physical parameters of the hazard will exceed various levels
- ◇ The vulnerability module estimates the probability that structure damage will exceed various levels as a result of the hazard
- ◇ These two modules comprise what is traditionally known as **probabilistic risk analysis**

The final result of the catastrophe model is the OEP curve, which is also based in probability.

II. Hazard Module

The hazard module addresses the following three elements:

- (1) The most likely locations of future events
- (2) The frequency of future events
- (3) The severity of future events

There are numerous variables that define the three elements above. Probability distributions are developed for each variable based on historical data. Then, we sample from each variable to simulate a specific event. Once an event is simulated, we estimate the local intensity of the event for each location within the affected area of the event. We will now look at each element in detail.

Locations of Future Events

The idea here is to identify the region over which the hazard applies. Obviously, this differs by hazard type:

- ◇ Earthquakes

- In general, we should identify faults and seismic source zones that could have a significant impact on the building inventory being analyzed
 - ◇ If the model domain was southern California, the San Andreas Fault would be identified as a hazard source
 - ◇ Since not all earthquakes occur on known faults, seismic source zones should also be identified. Catastrophe models use smoothing techniques to allow simulated earthquakes to occur anywhere within a seismic source zone with a given probability (not just at the spots where earthquakes have occurred in the past)

- ◇ Hurricanes

- Specific approaches used to quantify the geographical distribution of hurricanes include defining various parameters such as storm tracks, landfall location, and track angle at landfall
- Storm tracks show where the storm is going to go. Since catastrophe models use a probabilistic approach, each movement of the hurricane is based on a probability (ex. what is the probability that a hurricane sitting in the middle of the atlantic will turn north?). In addition, a probabilistic approach means that any storm track is possible, not just the ones observed historically
- Regarding landfall locations, we can easily plot historical landfalls. Although this historical data gives us some indication of where a hurricane is more likely to land, “gaps” in landfall do not necessarily mean that a hurricane will not make landfall in that spot. It may just mean that the historical data is too immature. Thus, smoothing techniques are used to ensure there is a positive probability of landfall at any given spot on a coastline

Frequency of Future Events

- ◇ Earthquakes

- Historical data can provide an estimate of event frequency. For example, large earthquakes on the San Andreas Fault occur every 150 years, which suggest an annual occurrence probability of 0.67%

- It is also common to model the relationship between the frequency of earthquake occurrence and their magnitude. As expected, larger magnitude losses occur less often
- ◊ Hurricanes
 - The frequency of weather hazards tend to reflect the regional climate
 - Hurricanes form where there is a large area of warm water and there is a lack of vertical wind shear

Severity of Future Events

To estimate the damage potential of natural hazards, a cat model must estimate their physical parameters at two spots:

- ◊ The source
- ◊ The sites of the affected building inventory (i.e., local intensity parameters)

For **earthquakes**, source parameters includes things like earthquake magnitude and fault-rupture characteristics and local intensity parameters includes things like seismic wave amplitude which is impacted by the local terrain.

For **hurricanes**, source parameters include things like forward speed and barometric pressure and local intensity parameters include things like local windfields which is also impacted by the local terrain.

III. Vulnerability Module

The vulnerability module estimates the level of building damage expected for different levels of severity of the external forces imposed, such as earthquake ground motion or high winds. Although engineering methods can provide an accurate assessment of building damage for a specific building at a specific location as a result of hazard, they are not practical for large insurance portfolios. To deal with large portfolios, we divide the building inventory into broad classes. Then, we choose a typical building from each class and analyze it using structure-specific engineering methods. We assume that each building in that class will have the same response to the hazard.

Clearly, we should not expect every building within a class to perform in the same exact way. However, this method generally produces accurate estimates of mean damage on a portfolio basis.

In summary, there are **two major steps** in the application of a engineering-based vulnerability approach to insurance portfolios:

- (1) Identify and define typical buildings in the modeled region
- (2) Calculate the building performance to ground motion or winds of different intensities

These two steps are referred to as **vulnerability analysis**. Let's dig deeper into each one.

Identification of Typical Buildings

In this step, we want to define as many building classes as needed to represent the statistical population of structures in the region. The building classes are identified by the most important factors that **affect the structural response to the hazards being analyzed**. These factors include the following:

- ◇ Building material (ex. steel, reinforced concrete)
- ◇ Structural system (ex. moment frame vs. braced-frame)
- ◇ Building height

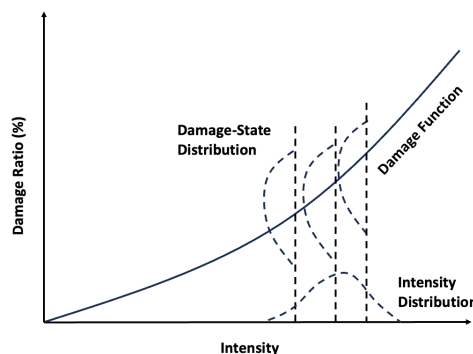
Then, we sub-divide each building class based on secondary modifiers such as roof or foundation type.

Evaluation of Building Performance

In this step, we want to estimate the damage to a building as a result of the hazard.

Damage to buildings from **earthquakes** is often both structural (ground shaking can lead to full building collapse) and non-structural. Damage to buildings from **hurricanes** is typically non-structural and localized to one area of the building. However, extreme winds can cause building collapse if the building is poorly engineered.

In insurance portfolio risk assessment, a **damage function** is often used to relate the structural damage to the event intensity. A damage function plots the **damage ratio** against the event intensity. The damage ratio ranges from 0% to 100%, with 100% being a total loss. Each point on the damage function is an expectation with a distribution around it. This distribution is known as the **damage-state distribution**. Thus, for each intensity level on the x -axis, there is a range of possible damage-states for the building. Here is an example of a damage function:



Notice that there is a range around the intensity level as well. This distribution is known as the **intensity distribution**. It represents the uncertainty in the intensity of the particular hazard being assessed.

Assume that the damage function above is for the entire building. To build each point on the curve, we consider the sum of the damage ratios for all building components and contents. The expected sum would be plotted on the curve.

IV. Loss Module

In general, the loss module uses cost models to translate estimates of physical damage from the vulnerability module into estimates of monetary loss. This includes the cost of repair or replacement for each damaged structural and non-structural component. It also includes loss estimates due to loss of use of the building.

Once total losses are determined, we apply policy terms and conditions to the total losses to determine the **insured losses**.

Some cat modelers have tried to link ground motion or wind intensity directly to the level of monetary loss. In these cases, damage functions are based on the opinions of experts rather than actual engineering analysis of building types. The main **issue with this approach** is that the damage functions based on expert opinion cannot be easily updated to reflect new construction techniques, building codes, repair costs, or information gained in the aftermath of actual events.

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Ch. 4 (Uncertainties & Cat Modeling)

Outline

I. Classifications of Uncertainty

For the purposes of cat models, there are **two types of uncertainty**:

(1) **Aleatory Uncertainty**

- ◇ The uncertainty due to the inherent randomness associated with natural hazard events
- ◇ This uncertainty cannot be reduced by the collection of additional data
- ◇ This uncertainty is reflected via probability distributions

(2) **Epistemic Uncertainty**

- ◇ The uncertainty due to lack of information or knowledge of the hazard
- ◇ This uncertainty can be reduced by the collection of additional data

II. Sources of Uncertainty

This section provides examples of aleatory and epistemic uncertainty.

◇ **Examples of Aleatory Uncertainty**

- The frequency of a hazard occurrence (we cannot know the exact time of occurrence)
- The fragility of a building (we cannot know the precise level of structural damage)

◇ **Examples of Epistemic Uncertainty**

- The lack of historical data describing earthquake or hurricane occurrence make it more difficult to predict where they might occur. Obviously, as more data is collected, those predictions become less uncertain
- The lack of information regarding repair costs and business interruption costs affect the accuracy of the loss module
- The lack of available data to create the Geographic Information Systems (GIS) databases within modeling software. GIS databases include maps of hazard sources and geologic features. An incomplete description of a hazard source or the local geology (such as soil condition) can lead to inaccurate cat model output

- Partial information on a structure’s characteristics can result in an inaccurate estimate of future damage
- Lack of accurate data on true market values of the inventory properties can lead to inaccurate estimates of the insured loss

III Representing and Quantifying Uncertainty

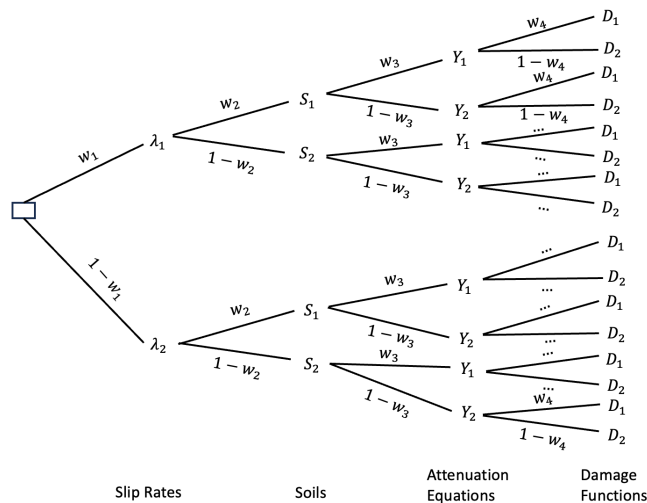
The **most common methods for incorporating uncertainty into cat modeling** are as follows:

- ◇ Logic Trees
- ◇ Simulation Techniques

Logic Trees

In the logic tree approach, alternative parameter values or mathematical relationships are identified and assigned various weights. The tree splits at each parameter or mathematical relationship creating more possible paths that the final cat model calculation can take. At the end of the tree, we are left with a number of possible combinations of parameters or mathematical relationships, each with a different weight.

Here is an example of a logic tree:



Let’s discuss the tree:

- ◇ We have an earthquake fault that generates an event
- ◇ The event is estimated using a model with two alternatives for the fault’s slip rate, λ_1 and λ_2 . λ_1 and λ_2 are assigned weights of w_1 and $1 - w_1$, respectively

- ◇ There is a single structure in the inventory being assessed. There is some uncertainty around the soil at the site of the structure. Thus, two soil parameters, S_1 and S_2 , are considered with weights w_2 and $1 - w_2$, respectively
- ◇ Once the slip rate and soil parameters are determined, attenuation equations are used to model seismic wave amplitudes. But there are two different equations that could be used. Thus, we treat each of those equations like parameters. We call them Y_1 and Y_2 and give them weights of w_3 and $1 - w_3$, respectively
- ◇ Once we have the Y equation, we can calculate a damage ratio using damage function D_1 or damage function D_2 . These have weights of w_4 and $1 - w_4$, respectively
- ◇ We end up with 16 different calculations of structure damage to the assessed building. The weight of the first calculation would be $w_1(w_2)(w_3)(w_4)$. The weight of the second calculation would be $w_1(w_2)(w_3)(1 - w_4)$

Regarding the weights, there are numerous ways to determine them. Here are a few approaches:

- ◇ Equal weights
- ◇ Weights based on some comparison of previously assessed estimates with actual outcomes
- ◇ Weights proportional to the ranking of alternative parameter paths

We can make the logic tree more complicated by allowing more possibilities for each parameter.

Simulation Techniques

Unlike a logic tree that is based on simplifying assumptions, simulation methods can model more complex processes. To do so, we assume a distribution for each uncertain parameter (this is in contrast to the logic tree example where each parameter only had two possible values). Then, we sample from each parameter distribution and simulate an event based on those sampled parameters. If we do this thousands of times, we build a range of possible outcomes which can be used to understand the uncertainty for the hazards. This is Monte Carlo simulation.

As a **simple example**, suppose the damage state is described by the following discrete distribution:

Damage State	Damage State Probability	Cumulative Probability
None	0.05	0.05
Minor	0.24	0.29
Moderate	0.48	0.77
Severe	0.16	0.93
Collapse	0.07	1.00

To simulate a damage state, we do the following:

- ◇ Randomly draw a value u from a uniform random variable on $(0, 1)$
- ◇ If $u \leq 0.05$, then the damage state is “None”
- ◇ If $0.05 < u \leq 0.29$, then the damage state is “Minor”
- ◇ And so on and so forth

In Chapter 2, we talked extensively about OEP curves. We can **create an OEP curve by combining a logic tree with Monte Carlo simulation**. Here’s a summary of how it works:

- ◇ Suppose that the cat model requires five different parameters and/or mathematical calculations
- ◇ We let each branch of a logic tree represent a different set of assumptions for the five different parameters. These assumption sets are based on samples from the various probability distributions (this is the simulation component)
- ◇ We assume that we can produce an exhaustive list of all possible assumptions. Since we have an exhaustive list, we assign each assumption set a weight and those weights sum to 1
- ◇ For each assumption set, we create an OEP curve (i.e., $Pr(\text{Loss} > L)$ for each assumption set)
- ◇ We can calculate the mean, median, standard deviation, etc. of the OEP curves using the assumption set weights

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Ch. 5 (Use of Cat Models in Ratemaking)

Outline

I. Actuarial Principles

This chapter starts with a review of the actuarial Principles on Ratemaking. Although unlikely to be tested, let's briefly review them:

- ◇ Principle 1: A rate is an estimate of the expected value of future costs
- ◇ Principle 2: A rate provides for all costs associated with the transfer of risk
- ◇ Principle 3: A rate provides for the costs associated with an individual risk transfer
- ◇ Principle 4: A rate is reasonable and not excessive, inadequate, or unfairly discriminatory if it is an actuarially sound estimate of the expected value of all future costs associated with an individual risk transfer

The ASB also suggests that the **determination of an appropriate exposure unit or premium basis is essential** and that the units should vary with the hazard and be practical and verifiable.

II. Use of Catastrophe Models in Ratemaking

Cat models are essential for calculating two things:

- (1) The Average Annual Loss (AAL)
- (2) The Surplus Cost

Under the authors' simple ratemaking model, the premium that the insurer should charge to policyholders is defined as follows:

$$\text{Premium} = \text{AAL} + \text{Risk Load} + \text{Expense Load}$$

Here are the details for each component:

- ◇ $\text{AAL} = \sum_i p_i L_i$. We showed how to calculate the AAL in Chapter 2
- ◇ The Risk Load reflects the insurer's concern with the survival constraint and the need for additional surplus capital. It is determined by the uncertainty surrounding the AAL. One

approach for measuring this uncertainty is the standard deviation σ of the OEP curve. Mathematically, σ is calculated as follows:

$$\sigma = \sqrt{\sum_i (L_i^2 p_i) - AAL^2}$$

Once we have σ , we can set the Risk Load equal to some function of σ .

- ◇ The Expense Load reflects the administrative costs involved in insurance contracts (ex. LAE, premium taxes, commissions)

Example

An earthquake model produces the following loss distribution for a \$1,000,000 home:

Loss	Probability
\$100,000	0.15
\$400,000	0.07
\$950,000	0.02

Given the following information:

- ◇ The insurer's expense load is 20% of premium
- ◇ The risk load is set to 15% of the standard deviation of the loss

First, let's calculate the AAL:

$$\diamond AAL = 100(0.15) + 400(0.07) + 950(0.02) = \$62,000$$

Second, let's calculate the risk load:

$$\diamond \sigma = \sqrt{\sum_i (L_i^2 p_i) - AAL^2} = \sqrt{(100^2)(0.15) + (400^2)(0.07) + (950^2)(0.02) - 62^2} = \$164,030$$

$$\diamond \text{The risk load is } 0.15(164,030) = \$24,605$$

Third, let's calculate the premium for the home:

$$\diamond \text{Premium} = AAL + \text{Risk Load} + \text{Expenses. Since the expenses vary with premium, this is } \frac{62+24.605}{1-0.20} = \$108,256$$

The **two most critical factors for differentiating risks for ratemaking** in a catastrophe setting are as follows:

- (1) The structure attributes of the portfolio (i.e., the building inventory)
- (2) The location attributes of the portfolio (i.e., the proximity to a hazard)

Structure Attributes

The following structure attributes differentiate risks for the purposes of charging equitable rates:

- ◇ Construction Materials
 - Wood frame construction is better for earthquake resistance since it is lightweight and flexible
 - Masonry construction is better for hurricanes due to its mass and resistance to damage from projectiles
- ◇ Building Codes
 - Buildings constructed in older years with less robust building codes are more prone to damage and loss
- ◇ Building Occupancy
 - Building occupancy refers to the purpose of the building
 - Different purposes leads to different building layouts
 - Building occupancy shows the greatest risk differentiation when estimating business interruption losses. As an example, a processing plant that requires significant amounts of water is “interrupted” if the water supply is cut off by an earthquake. This leads to a larger business interruption loss since water is critical to the processing plant’s operations

Location Attributes

The following location attributes differentiate risks for the purposes of charging equitable rates:

- ◇ Flood Plains
 - Is the building in a 100-year flood plain? If so, the building is more prone to flood damage
- ◇ Proximity to Known Earthquake Faults or the Coastline
 - Buildings close to a known fault are more prone to earthquake damage
 - Buildings close to the coast are more prone to hurricane damage
- ◇ Local Soil Conditions
 - If the soil condition shows a higher propensity for a landslide, then the earthquake risk is higher

III. Regulation and Catastrophe Modeling

In general, catastrophe models serve as a “double-edged sword” for regulators:

- ◇ On one hand, cat models provide a scientific approach for quantifying an insurer’s risk. The models also allow rates to be based on all possible events, rather than limited historical data
- ◇ On the other hand, regulators may view cat models as a means for insurers to justify higher rates

Catastrophe models can be difficult for regulators to assess for the following reasons:

- ◇ They require specialized expertise to evaluate the model
- ◇ Given that cat models are built by a number of competing firms, they contain proprietary information that firms may be hesitant to share
- ◇ Differences in model assumptions across different firms can lead to significantly varying loss outcomes

IV. Open Issues for Using Catastrophe Models to Determine Rates

There are a number of open issues regarding the use of cat models in ratemaking:

- ◇ Regulatory Acceptance – proprietary and complex cat models are difficult to assess by regulators due to the required technical expertise
- ◇ Public Acceptance – public acceptance has been low due to the resulting rate increases due to cat model output
- ◇ Actuarial Acceptance – since a pricing actuary’s goal is to determine fair and equitable rates, the use of a cat model makes sense because it is an improvement over using limited historical data. However, since cat models include inputs from a number of experts in other fields, actuaries are responsible for having a basic understanding of the components underlying a cat model and determining if the model is appropriate for the intended application
- ◇ Model-to-Model Variance – as mentioned above, models from competing firms can produce vastly different loss estimates. This is due to the uncertainty in the understanding of various catastrophes and the fact that different experts have different opinions on model parameters