Exam 9 High-Level Summaries

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Capital Allocation to Risky Assets

Overview

Investors must make decisions between risk and return when creating a portfolio. These decisions are usually driven by the investor's own appetite for risk. The amount of risk that an investor chooses to bear will also impact the expected return received. As with most financial exercises, the goal is to maximize return while minimizing risk.

Risk and Risk Aversion

Most assets in the marketplace include an amount of risk. In return for accepting that risk, a risk premium is required. **Risk premium** is the return on an investment above the risk-free rate. However, different investors may require a greater risk premium due to their **aversion** to risk.

When considering the **utility** of a portfolio, the investor must consider his own risk aversion as well as the risk of the portfolio. These are represented by A and σ, respectively, below:

$$\boxed{U = E(r) - \tfrac{1}{2} A\sigma^2}$$

There are three types of investors in the marketplace:

1. **Risk averse:** A > 0. These investors penalize utility of a risky asset.
2. **Risk neutral:** A = 0. These investors judge assets solely on the expected return.
3. **Risk lover:** A < 0. These investors adjust expected returns upwards in the presence of risk.

The utility of a risky asset is also called the **certainty equivalent rate**. This is the rate that a risk-free investment would offer to provide the same utility as the risky asset. Risk-free assets, by definition, have σ equal to 0. If a risky asset creates a certainty equivalent return greater than the risk-free alternative, it is desirable.

For risk-averse investors, an increase in risk implies a required increase in return.

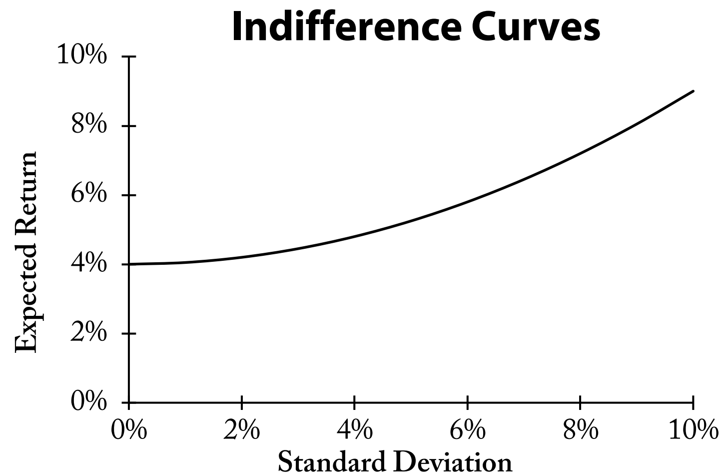
Portfolio A is said to **dominate** portfolio B if:

1. $E(r_A) \geq E(r_B)$
2. $\sigma_A \leq \sigma_B$

This is called the **mean-variance (M-V) criterion** (note: one inequality must be absolute). Since the risk-averse investor can get a higher return in exchange for less risk, portfolio A will be chosen every time.

It is possible for two (or more) different risky assets to produce the same utility for an investor. These assets will lie on the same **indifference curve**, named as such because the investor is indifferent in his selection of either asset.

The following indifference curve shows a utility of U = 4% for an investor with risk aversion A = 10:



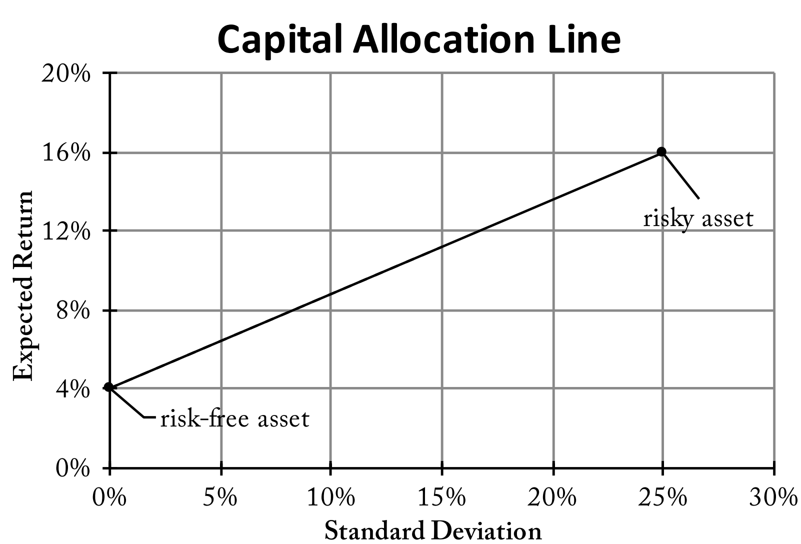
Capital Allocation across Risky and Risk-Free Portfolios

Investors create portfolios of securities from all asset classes, which can vary in risk. In order to control some of that risk, an investor can include risk-free assets (such as Treasury bills) as a portion of his portfolio. This is the **capital allocation decision** and is considered by many the most important step in constructing a portfolio.

If we construct a **complete portfolio** consisting of a single risky portfolio and a risk-free asset, we can determine the expected return and overall risk.

$$\mathrm{E}\left[r_c\right] = y \cdot \mathrm{E}\left[r_p\right] + (1 - y) \cdot r_f \qquad \sigma_c = y \cdot \sigma_p$$

We can graph the expected return of the complete portfolio against the associated risk to determine the **capital allocation line (CAL)**. This line is the combination of all risk-return combinations an investor is able to select:



- The risk-free asset will always lie on the y-axis, since its standard deviation is zero. The return is the risk-free rate.
- The risky asset is equal to the complete portfolio only if the investor allocates 100% of his position to the risky asset (y = 1.0).
- If investors can borrow at the risk-free rate, the line will continue when y > 1.0 More likely, however, is that the borrowing rate is greater than risk-free, and the CAL will "kink" at the risky asset.

The slope of the line is called the **reward-to-volatility ratio**, or **Sharpe ratio**:

$$S_{y\leq 1} = \frac{\mathrm{E}\left[r_p\right] - r_f}{\sigma_p} \qquad S_{y>1} = \frac{\mathrm{E}\left[r_p\right] - r_b}{\sigma_p}$$

In most exam questions, the first equation is used. However, it is worth noting that if an investor's position is such that he borrows money at a rate greater than risk-free (y > 1.0), the slope of the CAL will decrease.

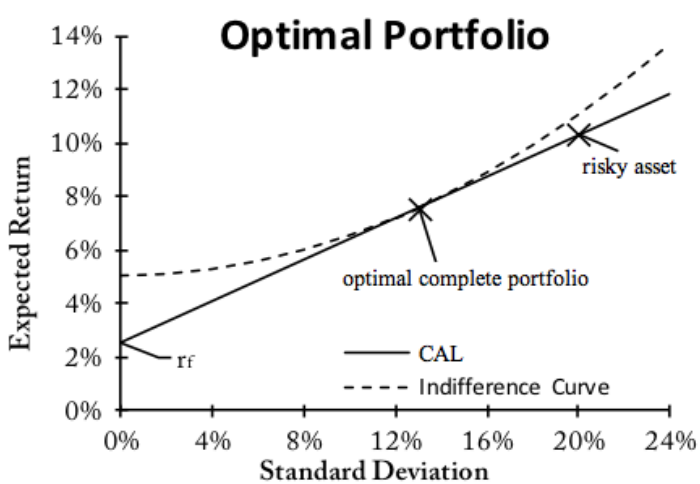
Risk Tolerance and Asset Allocation

As investors differ in their risk aversion, so too will they differ in their asset allocation. We can determine the optimal position for an investor in a risky asset (**optimal complete portfolio**) that maximizes utility:

$$y^* = \frac{\mathrm{E}\left[r_p\right] - r_f}{A\sigma^2}$$

The greater the risk aversion, the less weight will be placed on the risky portfolio.

Graphically, the optimal complete portfolio is found where the CAL is tangent to the indifferent curve with the maximum utility for the investor.



Capital Market Line & Passive Strategies

Risky portfolios can be created using many different security classes.

- If an investor is involved directly in analyzing which securities should make up the risky portfolio, he is said to have an **active** strategy.
- If instead the investor prefers to take a **passive** approach, he would choose the portfolio as a whole without analysis on individual assets within the portfolio.

Popular portfolios in a passive approach include broad indices of common stocks.

Benefits of a Passive Strategy

- Active strategies require time, effort, and in many cases, money (in the form of fees).
- **Free-rider benefit**: Essentially letting the active investors buy and sell such that most assets will be fairly priced. In this case, the passive approach may be just as good as an active approach.

BKM notes that passive index funds have outperformed most actively managed funds.

If the investor takes a passive approach, the risky asset in the graph above is replaced with the index portfolio, and the CAL becomes the **CML (Capital Market Line)**.

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Optimal Risky Portfolios

Overview

In BKM 6, the optimal complete portfolio consisted of a risky portfolio and a risk-free asset. BKM 7 focuses on not only the capital allocation between these two items, but also the asset allocation of the risky portfolio. This introduces a third asset consideration in portfolio construction, and while BKM 6 assumes no correlation between the risky portfolio and the risk-free asset, we now must consider correlation between the assets in the risky portfolio.

Diversification and Portfolio Risk

Different factors can influence the risk of an asset. Macroeconomic factors include the economic factors such as inflation or unemployment rates. These factors impact the return of all assets (though possibly in different ways). Firm-specific factors such as management changes or poor reputation can impact the return of a single asset while not impacting the return of others.

Diversifying a portfolio allows an investor to minimize **firm-specific** risk to a negligible level. This risk is also called **unique, nonsystematic,** or **diversifiable risk**.

The **market risk** is the risk that remains after diversification, as the market itself is subject to volatility. It is considered **nondiversifiable risk** since it cannot be diversified away. It is also called **systematic risk** as it impacts all assets in the system.

Portfolios of Two Risky Assets

Diversification can be achieved using any combination of two or more assets. The greater the number of assets, the less firm-specific risk the investor is exposed to. The key is to diversify efficiently to reduce the risk to the lowest possible acceptable level for any given level of expected returns.

$$\mathrm{E}\left[r_p\right] = \sum w_i \cdot \mathrm{E}\left[r_i\right]$$

$$\sigma_p^2 = \sum \sum w_i w_j \operatorname{Cov}\left(r_i, r_j\right)$$

The variance of the portfolio is a weighted sum of the covariances. For a two-asset portfolio, the variance is found using the following formula:

$$\sigma_p^2 = w_D^2 \sigma_D^2 + w_E^2 \sigma_E^2 + 2 w_D w_E \operatorname{Cov}\left(r_D, r_E\right)$$

As can be seen above, the greater the covariance between the two assets, the greater the resulting portfolio variance.

Special Cases

If D and E are perfectly correlated, the formula simplifies to the below:

$$\sigma_p = w_D\sigma_D + w_E\sigma_E$$

If D and E are perfectly negatively correlated:

$$\sigma_p = abs(w_D\sigma_D - w_E\sigma_E)$$

A **perfectly hedged position** can be created when D and E are perfectly negatively correlated. The portfolio will have zero risk and will have the corresponding the weights below:

$$w_D = \frac{\sigma_E}{\sigma_D + \sigma_E}$$

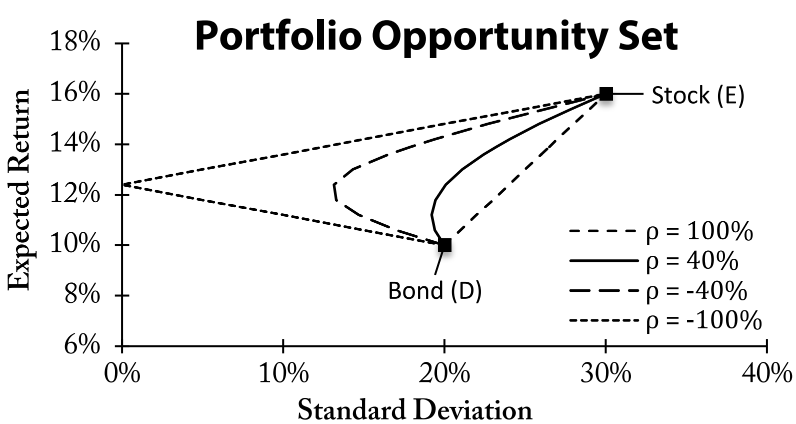
Minimum-Variance Portfolio

Using calculus, BKM derives the weights that would result in the **minimum-variance portfolio**. This portfolio will have a standard deviation less than either asset in isolation:

$$w_{\min,D} = \frac{\sigma_E^2 - Cov(r_D, r_E)}{\sigma_D^2 + \sigma_E^2 - 2Cov(r_D, r_E)} \qquad w_{\min,E} = 1 - w_{\min,D}$$

The weights for each asset will depend on the covariance of D and E, and therefore the correlation between the assets.

A **portfolio opportunity set** shows all combinations of return and risk when combining assets D and E and depends on the correlation between the two assets.



The above shows the advantage of diversification between D and E. If they are perfectly correlated, there is no diversification benefit to add D to E in the portfolio, as the risk always increases. Even when correlation is positive, there are diversification benefits (to a certain point). When D and E are perfectly negatively correlated, a zero-risk, hedged position is available.

Asset Allocation with Stocks, Bonds, and Bills

The goal in any allocation is to be efficient. An investor wants to maximize return while minimizing risk. Though the minimum-variance portfolio gives the lowest achievable risk for two assets, it may not be the optimal portfolio.

Graphically, the **optimal risky portfolio** is where the capital allocation line runs tangent to the portfolio opportunity set.

Using calculus, BKM finds the solution for the weights of the optimal risky portfolio:

$$w_D = \frac{E[R_D]\sigma_E^2 - E[R_E]\sigma_{DE}}{E[R_D]\sigma_E^2 + E[R_E]\sigma_D^2 - (E[R_D]+E[R_E])\sigma_{DE}}$$

$$w_E = 1 - w_D$$

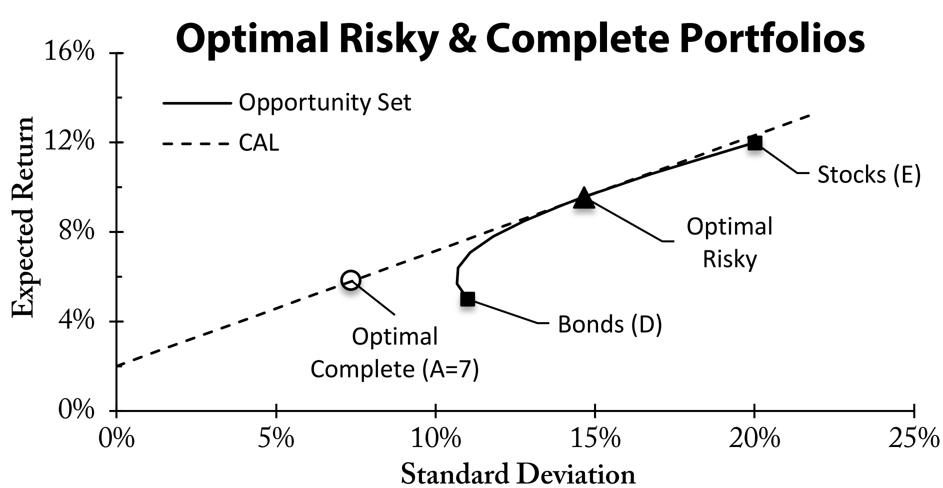
*Note that the above formulas use **risk premium** instead of expected returns.

It is also worth noting that the job may not be done once you have found the optimal weights. Remember that we have only found the weights of **each asset in the risky portfolio** (asset allocation). We still need to complete capital allocation and determine the proportion of the **optimal risky** portfolio in the **optimal complete** portfolio.

To arrive at the optimal complete portfolio:

1. Determine the weights, w_D and w_E, of each asset in the risky portfolio
2. Calculate the resulting expected return, $E[r_p]$ and standard deviation, σ, of the risky portfolio
3. Determine the proportion, y*, on the risky portfolio using the formula from BKM 6:

$$y^* = \frac{E[r_p] - r_f}{A\sigma^2}$$



The Markowitz Portfolio Optimization Model

The **minimum-variance frontier** of risky assets shows the lowest possible risk attainable for a given return in a portfolio. Individual assets would be graphed inside this frontier. The **global minimum-variance portfolio** results in the lowest possible risk for any return in the portfolio. If we envision a horizontal line through the global minimum-variance portfolio, the **efficient frontier** is the resulting top half of the minimum-variance frontier.

The idea behind Markowitz optimization is the same as all optimization theory: for a given risk level, an investor should only be interested in the portfolio generating the highest expected return.

BKM notes that any combination of portfolios on the minimum-variance frontier will also be on that frontier.

The Separation Property

A portfolio selection will come down to two tasks:

1. **Determine the optimal risky portfolio.** This is purely technical, and will be the same for all investors using the same input lists. BKM notes that different managers may have differing input lists, which would result in different "optimal" portfolios (quotes used to denote that they are not true optimal portfolios, likely due to bad security analysis).
2. **Capital allocation.** This will differ based on the risk aversion of the investor.

The Power of Diversification

BKM discusses a portfolio of n equally-weighted securities. The conclusion is the following regarding the variance of the portfolio:

$$\sigma_p^2 = \frac{1}{n}\bar{\sigma}^2 + \frac{n-1}{n}\overline{COV}$$

The systematic risk is calculated using the following:

$$systematic\,risk = \overline{COV}$$

$$systematic\,risk = \sigma_i \sigma_j \rho$$

The firm-specific risk is the difference between the total portfolio risk and the systematic risk.

It can be seen that, as n approaches infinity, (1/n) approaches 0, while (n – 1)/n approaches 1. Therefore, as we add more assets, we are only left with the systematic risk. This reconciles with the definition of systematic, or *undiversifiable* risk.

Risk Pooling

BKM defines **risk pooling** as "merging uncorrelated, risky projects as a means to reduce risk." However, it warns that adding risks (though independent) increases one's exposure to risk. An insurer would place equal weight on two (or more) separate risky assets. The remaining weight would still be on the risk-free asset.

The resulting risk premium and standard deviation would be as follows:

$$\mathrm{E}\left[R_Z\right] = \sum w_i \cdot \mathrm{E}\left[R_i\right] = 2yR$$

$$\sigma_Z^2 = y^2\sigma^2 + y^2\sigma^2 = 2y^2\sigma^2$$

$$\rightarrow \boxed{\sigma_Z = y\sigma\sqrt{2}}$$

The Sharpe ratio is calculated as such:

$$S_Z = \frac{R_Z}{\sigma_Z} = \frac{2yR}{y\sigma\sqrt{2}} = \frac{\sqrt{2}R}{\sigma} = \boxed{S_P\sqrt{2}}$$

While the Sharpe ratio has increased by a factor of 1.41 (good), the standard deviation has also increased by a factor of 1.41 (bad). BKM concludes that risk pooling itself does not reduce risk.

Risk Sharing Risk of Long-Term Investing

When risk pooling is combined with **risk sharing**, the increase in risk can be overcome. By selling shares of a risky portfolio, insurance companies can limit risk while maintaining the increased Sharpe ratio.

To illustrate, BKM assumes the investor will sell off half of his position in each risky asset when a second asset is added. This results in a position of y/2 in each risky asset and (1 – y) in the risk-free asset. For this portfolio V, the expected risk premium and standard deviation:

$$\mathrm{E}\left[R_V\right] = \sum w_i \cdot \mathrm{E}\left[R_i\right] = yR$$

$$\sigma_V^2 = \left(\frac{y}{2}\right)^2\sigma^2 + \left(\frac{y}{2}\right)^2\sigma^2 = \frac{y^2\sigma^2}{2}$$

$$\rightarrow \boxed{\sigma_V = \frac{y\sigma}{\sqrt{2}}}$$

The Sharpe ratio is calculated as such:

$$S_V = \frac{R_V}{\sigma_V} = \frac{yR}{y\sigma/\sqrt{2}} = \frac{\sqrt{2}R}{\sigma} = \boxed{S_P\sqrt{2}}$$

In this example, risk sharing results in the same increased Sharpe ratio but the standard deviation is *reduced* by a factor of 1.41. The combination of risk pooling and risk sharing is the key to the insurance industry.

This example can be extended to n risks and the conclusion holds (substituting n for 2 in the above results). However, there are realistic limits to this proposition, as growing the portfolio to an unwieldy size will be a significant burden. Also, the inputs are estimates, and a slight error could have compounding impacts on imprecision.

Risk of Long-Term Investing

Since risk does not fade over time, extending an investment horizon for another period is analogous to risk pooling.

Rather, using the example of a two-period horizon, the soundest move would be to place half the budget in each of the time periods (similar to risk sharing).

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Index Models

Single-Factor Security Market

The Markowitz model is unwieldy for the following reasons:

- Large number of estimates required for input
- Correlation estimate errors can lead to nonsensical results
- It is difficult to judge consistency of correlation matrices quickly

The single-factor model simplifies these requirements.

Formulas

For a single-factor model, where beta is the sensitivity coefficient of a security to macro forces:

$$r_i = E(r_i) + \beta_i m + e_i$$

$$risk_{systematic} = \beta_i^2 \sigma_m^2$$

$$risk_{total} = \beta_i^2 \sigma_m^2 + \sigma^2(e_i)$$

$$\text{Cov}\left(r_i, r_j\right) = \beta_i \beta_j \sigma_m^2$$

where e_i is the unexpected return of a stock. Since there are market forces impacting the return, we call that market factor m.

Single-Index Model

So called because it uses the market index as a proxy for the common (macro) factor. Using regression:

$$R_i(t) = \alpha_i + \beta_i R_M(t) + e_i(t)$$

Since the expected value of the error term is zero:

$$E(R_i) = \alpha_i + \beta_i E(R_M)$$

$E(R_M)$ is called systematic risk premium because it derives from the risk premium of the entire market. Alpha in these equations is called **nonmarket premium.** It is return in excess of market premium. (Note: alpha can be negative if a security is deemed to be overpriced).

The table below shows the input list for the single-index model:

Estimate	Symbol
Non-market premium	α_i
Return due to movements in overall market	$\beta_i\,(r_M - r_f)$
Firm-specific risk	e_i
Variance due to the uncertainty of the macro factor	$\beta_i^2\sigma_m^2$
Firm-specific variance	$\sigma^2(e_i)$

For *n* stocks, the following parameter estimates are needed:

- *n* estimates of α_i
- *n* estimates of β_i
- *n* estimates of σ_i
- 1 estimate for $E[R_M]$
- 1 estimate for σ_M^2

In total, 3*n* + 2 estimates are required, a significant decrease from the Markowitz model.

Pros & Cons of Single-Index Model vs. Markowitz

Advantages

- The number of parameters is significantly lower than Markowitz.
- 3*n* + 2 estimates are needed*
- It allows for specialization in security analysis since covariance is based on the relationships to the market index.

Disadvantages

- It oversimplifies uncertainty between macro and micro risk and doesn't allow for industry-specific effects.
- The optimal portfolio from the single-index model can be inferior to the optimal portfolio from the Markowitz model.

Single-Index Model and Diversificaiton

A portfolio of n equally-weighted stocks has a total beta equal to the average of the sum of the individual betas in the single-index model. The same is true for the portfolio alpha and firm-specific risk:

$$\beta_P = \frac{1}{n}\sum_{i=1}^{n}\beta_i \qquad \alpha_P = \frac{1}{n}\sum_{i=1}^{n}\alpha_i \qquad e_P = \frac{1}{n}\sum_{i=1}^{n}e_i$$

Since the portfolio's variance is still a function of the variance of the market, there will still be systematic risk remaining after diversification.

Estimating the Single-Index Model

Using regression on the excess returns of a specific stock and the market index, a **security characteristic line** (SCL) is found with y-intercept equal to alpha and slope equal to beta.

We would then test the following hypotheses:

- Alpha = 0
- Beta = 0

It is important to remember that we reject these hypotheses only if the resulting p-values are deemed statistically significant.

Portfolio Construction and the Single-Index Model

Below is the preparation process for an input list for the single-index model:

1. Macroeconomic analysis: to estimate R_M and the risk of the market index
2. Statistical analysis: to estimate betas and firm-specific (residual) variances
3. Portfolio manager calculates expected market-driven return of security
4. Security analysis: to estimate security alphas

BKM notes that estimating betas tends to be standardized across managers.

The following rules hold for alphas:

- If an alpha is positive, it should be overweighted in the portfolio compared to the market index.
- If alpha is negative, it should be underweighted, possibly to a weight < 0. This would mean taking a short position (if able) would be preferable.

Optimal Risky Portfolio in the Single-Index Model

Using the n equally-weighted stock example and, instead, allowing weights to vary, we get:

$$\alpha_P = \sum_{i=1}^{n+1} w_i \alpha_i \qquad \beta_P = \sum_{i=1}^{n+1} w_i \beta_i \qquad \sigma^2(e_P) = \sum_{i=1}^{n+1} w^2 \sigma^2(e_i)$$

The (n+1) term is for the market portfolio. Alpha for the market portfolio is zero, beta is one, and the firm-specific risk is zero.

The objective is to maximize the Sharpe ratio by using the portfolio weights, which must sum to one.

Summary of Optimization Procedure

To find the weights in the optimal risky portfolio of a single-index model:

1. Calculate initial position of each security in the active portfolio. $w_i^0 = \dfrac{\alpha_i}{\sigma^2(e_i)}$
2. Force the weights to sum to 1. $w_i = \dfrac{w_i^0}{\sum_{i=1}^{n} w_i^0}$
3. Calculate the resulting alpha of the active portfolio. $\alpha_A = \sum_{i=1}^{n} w_i \alpha_i$
4. Calculate the residual variance of the active portfolio. $\sigma^2(e_A) = \sum_{i=1}^{n} w^2 \sigma^2(e_i)$
5. Calculate the resulting beta of the active portfolio. $\beta_A = \sum_{i=1}^{n} w_i \beta_i$
6. Calculate the initial position in the active portfolio. $w_A^0 = \dfrac{\alpha_A / \sigma^2(e_A)}{E(R_M) / \sigma_M^2}$
7. Adjust the weight in the active portfolio. $w_A^* = \dfrac{w_A^0}{1 + (1 - \beta_A) w_A^0}$
8. Calculate the weights of the optimal risky portfolio, including the passive portfolio and each security in the active portfolio. $w_M^* = 1 - w_A^*$ $\quad w_i^* = w_A^* \cdot w_i$
9. Calculate the expected risk premium for the portfolio. This calculation uses a weighted alpha and beta based on the active and passive portfolio. $\mathrm{E}[R_P] = w_A^* \alpha_A + (w_M^* + w_A^* \beta_A)\mathrm{E}[R_M]$
10. Calculate the variance of the optimal risky portfolio. $\sigma_p^2 = (w_M^* + w_A^* \beta_A)^2 \sigma_M^2 + w_A^2 \sigma(e_A)^2$

The Sharpe ratio of the optimal risky portfolio will be greater than that of the index portfolio (assuming nonzero alphas). The relationship is:

$$S_p^2 = S_M^2 + \left[\frac{\alpha_A}{\sigma(e_A)}\right]^2 \qquad \qquad \textit{Information Ratio} = \frac{\alpha_A}{\sigma(e_A)}$$

The information ratio measures the extra return obtained from security analysis.

It is worth noting that alphas should be dampened due to the fact that these forecasts are subject to estimation errors. Also, alphas may not hold for very long, as estimates from past data may not be the best predictor of future alphas.

Betas have similar behaviors, and as a firm becomes more "conventional", it is likely its beta will converge towards 1. We can use this fact to calculate an **adjusted beta**:

$$\beta_{adjusted} = \frac{2}{3}\beta_{sample} + \frac{1}{3}(1)$$

Betas can also be predicted using firm-specific variables, such as:

- Variance of earnings
- Variance of cash flow
- Growth in earnings per share
- Market capitalization (firm size)
- Dividend yield
- Debt-to-asset ratios

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The Capital Asset Pricing Model (CAPM)

Overview

CAPM relates the risk of an asset to its expected return. It can provide benchmark rates of return and assist in hypothesizing returns of assets not yet traded. BKM notes that CAPM struggles with some empirical tests, but is still used widely (within acceptable accuracy).

The Capital Asset Pricing Model

The core idea of CAPM is that if all investors shared the same investable universe, they would:

- Have the same input lists
- Conclude the same efficient frontiers
- Have an identical tangent CAL
- Have risky portfolios with identical weights on each risky asset

Therefore, CAPM concludes that the optimal risky portfolio in this universe must be the **market portfolio**, in which each security is held in proportion to its market value.

This also leads to the conclusion that a passive strategy is efficient (**mutual fund theorem**). This implies a proportion of 1 on the market portfolio. Any active strategy will incur costs and will perform no better, creating net inferior results.

Since y = 1 in the optimal risky portfolio, BKM determines that since

$$\boxed{y^* = \frac{\mathrm{E}\left[r_M\right] - r_f}{A\sigma_M^2}}$$

it follows that

$$\boxed{\mathrm{E}\left[R_M\right] = \bar{A}\sigma_M^2}$$

concluding the risk premium on the market portfolio is proportional to the average risk aversion of investors.

The reward-to-risk ratio for investing in the market portfolio is known as the **market price of risk:**

$$\boxed{\frac{\mathrm{E}\left[R_M\right]}{\sigma_M^2}}$$

Note that this is <u>not</u> the same as the Sharpe ratio, which uses standard deviation in the denominator.

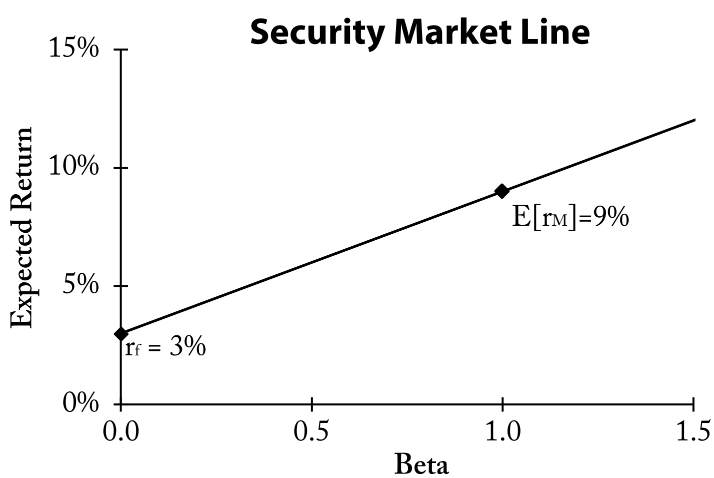
The CAPM Equation

The CAPM equation (the second uses the simplified view of risk premium):

$$E(r_i) = r_f + \beta_i(\mathrm{E}[r_M] - r_f) \qquad E(r_i) = r_f + \beta_i \mathrm{E}[R_M]$$

This is also called the **expected return-beta** (or **mean-beta**) relationship. Betas greater than 1 are considered aggressive and sensitive to market swings. Betas below 1 are considered defensive.

This relationship can be shown graphically using the **security market line** (SML). An example below:



The risk-free rate will always cross the y-axis (it is risk-free, and therefore beta equals zero). The expected market return relates to beta = 1. Since betas can (and are often) greater than 1.0, the line continues ad infinitum.

The SML graphs individual asset risk – the contribution of the asset to the portfolio variance (beta). In market equilibrium all securities <u>must lie</u> on the SML. **Underpriced** (positive-alpha) stocks will lie above the SML, while negative-alpha stocks will lie below the SML.

CAPM is reduced to five simple ideas:

- Investors can eliminate some risks (ex: management changes in a firm).
- Some risks cannot be eliminated through diversification (ex: global recessions).
- People must be rewarded for investing in risky assets by earning returns higher than safer assets.
- Rewards are independently based on the extent to which it affects the market risk basket.
- The contribution to the market basket's risk can be captured by beta.

CAPM implies that the market portfolio is efficient, and the risk premium on any risky asset is proportional to its beta.

CAPM Assumptions

The assumptions of CAPM can be split into two categories: individual behavior assumptions and market structure assumptions.

1. Individual behavior
   a. Investors are rational, mean-variance optimizers
   b. Planning horizons of a single period
   c. Homogenous expectations (identical input lists)
2. Market structure
   a. All assets are publicly held and traded, short positions allowed, investors can borrow or lend at risk-free rate
   b. All information is publicly available
   c. No taxes
   d. No transaction costs

Some of these assumptions are challenging to accept. The first one noted is assumption (2a). Specifically, the short position assumption has some issues with it:

- Liability of those holding short positions is potentially unlimited and requires large collateral
- Limited supply of shares to be borrowed limits short sales
- Many companies are prohibited from short sales

Extensions of CAPM

Many extensions of CAPM exist to reconcile issues with the assumptions.

Zero-Beta CAPM

The **zero-beta** extension assumes that investors may not borrow at risk-free rates. We replace the risk-free return with the zero-beta companion portfolio return:

$$E(r_i) = E[r_Z] + \beta_i[\mathrm{E}(r_M) - E(r_Z)]$$

Labor Income and Nontraded Assets

Two important assets not traded on the public market are human capital and privately held businesses. This extension of CAPM follows:

$$E(R_i) = E(R_M)\frac{Cov(R_i, R_M) + \frac{P_H}{P_M}Cov(R_i, R_H)}{\sigma_M^2 + \frac{P_H}{P_M}Cov(R_H, R_M)}$$

where

P_H = value of aggregate human capital
P_M = market value of traded assets
R_H = excess rate of return on aggregate human capital

ICAPM

The **intertemporal CAPM (ICAPM)** assumes we can identify K sources of extramarket risk and find K associated hedge portfolios:

$$E(R_i) = \beta_{iM} E[R_M] + \sum_{k=1}^{K} \beta_{ik} \, \mathrm{E}(R_k)$$

Consumption-Based CAPM

The consumption-based CAPM recognizes that any asset with positive covariance with consumption growth is assumed to be riskier (utility drops during difficult economic times):

$$E(R_M) = \alpha_M + \beta_{MC} E(R_C) + \varepsilon_M$$

BKM notes consumption growth figures are infrequent and fraught with errors.

Liquidity and CAPM

Liquidity is the speed and ease with which an asset can be sold at fair market value. **Illiquidity** runs opposite, and is an expected discount from fair market value if an asset must be sold quickly. This can be quite large in many circumstances.

As such, illiquid securities are more subject to risk, and will offer higher rates of return. This **liquidity premium** will increase with trading costs at a decreasing rate. Liquidity across stocks shows significant correlation.