Rising Fellow

2024

Exam 5

Cookbook

BASIC RATEMAKING AND RESERVING

60+ **Step-by-Step Recipes**
to Solve CAS Calculation Problems

Exam 5 Cookbook

2024 Sitting

Rising Fellow

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Table of Contents

Introduction

The purpose of the Exam 5 Cookbook is to prepare you to confidently answer calculation-based problems on exam day without wasting time trying to "think through" a problem-solving approach before typing the solution.

Since the 2016 sitting, hundreds of actuaries have used the Exam 7, Exam 8, and Exam 9 Cookbooks to help them pass and get one step closer to their FCAS.

Our goal with Rising Fellow is to help you prepare for the exam with less frustration so that you have your best exam sitting yet!

The Structure

The Exam 5 Cookbook goes through the different calculation-based problem-types that I believe are reasonably testable based on the syllabus. By exam day, you should know how to solve each one.

Inside, you'll find a separate section for each testable problem-type. Each section has the following structure:

Original Practice Problem

Each section has an original practice problem that demonstrates the problem-type. I wrote these based off of the syllabus papers to have a similar difficulty-level and style to what you might see on an exam.

Solution Recipe

The solution recipe solves the practice problem from start to finish and shows the step-by-step approach you should take to answer a similar problem. For each step, you'll see:

- The description for what to do in the step
- The formula(s) necessary for the step
- The formula(s) translated from symbolic notation to plain-English
- Calculations for the step to solve the example problem

Discussion

Each section includes discussion to add clarity and more context. The discussion also covers underlying concepts that might come up on a part b or part c essay question.

For many problems, I point out potential "twists" that could show up on the exam that would make an exam problem more difficult. Since you've taken actuarial exams up to this point, you know that straightforward exam problems are more the exception than the rule.

CBT Spreadsheet Tips

This new section provides Excel formulas and tips for how to solve a problem more efficiently in the computer-based testing (CBT) PearsonVue spreadsheet environment. There are many types of problems where setting up your solution intelligently and taking advantage of the spreadsheet capabilities such as SUMIF( ), COUNTIF( ), and array formulas, will save you valuable time on the exam.

Source

Each section references the pages in the syllabus reading that you can cross-reference for more information and details. Make sure to check the syllabus section for more context if you get stuck on a problem or to see how the author discusses the concepts.

More Practice

Here, you'll see references to past CAS problems. You'll find this helpful especially closer to the exam if there are particular types of problems that you are struggling with. This section includes references to CAS problems from the 2015-2019 exams.

How to Best Use the Exam 5 Cookbook

Below is a suggested guide for how you can incorporate the Exam 5 Cookbook in your own study schedule along with the syllabus material and a typical study manual. This is the general approach that I used when I took my fellowship exams.

For each of those exams I had a main study manual as well as the Exam Cookbook, which I built out while I studied for the exam (but you don't need to waste time doing that part!)

First pass through the syllabus

While you're reading a particular paper in the syllabus and your main study manual to learn the material, use the Exam 5 Cookbook to clearly identify what problem-types you need to know from the paper. Study the steps in the solution recipe to learn how to solve the problem-types. Make sure to do some practice problems as you go through the syllabus. This will help you learn faster.

Second pass through the syllabus

Review the steps for the problem-types and make sure you have an intuitive understanding of how to solve the problems. Start working the past CAS problems.

The first level of understanding is to be able to follow the recipe and understand the steps and calculations.

The next level of understanding is to be able to recall and apply the steps to solve a problem without relying on study material. During your second pass, focus on building this deeper level of understanding.

Review and Practice Problems (around 6 weeks to 2 weeks before the exam)

At this point you should have a good understanding of the syllabus and how to use the recipe steps to systematically solve the different calculation problems. During this period, you should be doing lots of problems across the syllabus and targeting problem-types that you are finding particularly challenging.
By the end of this phase, you might not have all the formulas memorized, but you should know all the steps and how to apply them to solve problems without needing to think too much before beginning to write the solution.

During this phase, make sure to focus on the types of problems and concepts that you're weak at. This may require some struggle, but struggling with some of the challenging problems will help you master these concepts.

You also should continue building your understanding of the concepts and preparing for essay and more complicated integrative questions. I found it helpful to create flashcards from the papers as well as to re-read sections of the syllabus papers that appear to be likely sources of essay problems.

Final Weeks

In the final weeks, focus on taking practice exams to see problems from the entire syllabus. When taking practice exams, work on your exam strategy to make sure you're able to finish the exam and maximize your points.

Prepare for essay problems in the final weeks by using flashcards to make sure that you know all the details necessary. An approach I found helpful is to say flashcards out loud and to explain the flashcard response in my own words as if I were teaching someone. It sounds weird, but it is a much more efficient way to learn and memorize than simply scanning the front and back of the flashcard.

Prepare for calculation problems by reviewing the recipes in the Exam 5 Cookbook in a similar fashion to how you use flashcards for essay problems. Using this approach on my fellowship exams, I was able to rapidly review the steps and formulas for how to solve each problem-type that might show up on the exam. This was a huge benefit and gave me a lot of confidence going into the exam.

Exam Day

I used the original Exam Cookbooks together with a traditional study manual using the approach above to take my fellowship exams. On exam day, for almost every calculation problem I was able to start writing the solution without wasting time trying to think through how to solve the problem. I had an intuitive understanding of how to solve each of the problems following the step-by-step recipes.

If you follow this approach, you should be able to develop a similar level of understanding and confidence going into the exam room.

Excel Version for Computer-Based Testing Preparation

For each recipe, there is an accompanying Excel version. Make sure to review those so that you know how to solve problems in the spreadsheet format. The CBT Spreadsheet Tips sections and the Excel version showing the formulas and setup for the spreadsheet solution will help you understand how to solve exam problems in the PearsonVue spreadsheet environment.

Errata

I always hated seeing errors in study manuals when I studied for exams, so I make every effort to ensure the study materials are accurate. Nevertheless, there may still be some errors in the final version, so I keep an updated errata. Please make sure to check it regularly for any fixes. The link is below:

https://risingfellow.com/errata

If you find any errors, please send me a message using the contact form on the Errata page so that I can make a correction.

Feedback

I am always working to improve the Exam 5 Cookbook and the rest of the Rising Fellow study material. Please send me an email to exam5@RisingFellow.com if you have feedback about any of the following:

- Recipes or sections that are confusing or could be improved
- New recipes I should include in future versions
- Better ways you've found to solve a problem-type in a spreadsheet
- Any comments or other feedback you have

Good luck as you start studying and I hope this will be your best sitting yet.

Aggregating Exposures

Werner Ratemaking Ch. 4

Problem

Given the following automobile policy information, at policy inception, for an insurance company that writes 6-month and 12-month policies:

Policy	Effective Date	Expiration Date	# Autos on policy
A	10/1/20	3/31/21	1
B	4/1/21	3/31/22	3
C	7/1/21	6/30/22	2
D	8/1/21	1/31/22	2
E	1/1/22	6/30/22	1
F	4/1/22	3/31/23	2

The exposure base is car-years and the probability of a claim is evenly distributed through the year.

The following policy changes occurred during the policy effective period:

- Policy B: Canceled on 1/1/22
- Policy C: One of the autos was removed from the policy on 4/1/22
- Policy E: Canceled on 3/31/22

Assume there are no other policies written between 2020 and 2022

a. Calculate the 2021 and 2022 written exposures as of 12/31/22 both aggregated by calendar year and policy year.

b. Calculate the 2021 and 2022 earned exposures as of 12/31/22 both aggregated by calendar year and policy year.

c. Calculate the 2021 and 2022 unearned exposures as of 12/31/22 both aggregated by calendar year and policy year.

d. Calculate the 2021 and 2022 in-force exposures as of 2/15/22 with insured units defined as:
   - Automobile units exposed to loss
   - Written exposures

Solution Recipe

Part a – Written Exposures

Written exposures - The total exposures on policies written during the time period, based on their *effective date*.

1) **For *Calendar Year*, exposures are aggregated by *transaction date*. Policy cancelations will record a negative partial exposure for the portion of the policy canceled for the calendar year of the *transaction date*.**

$$Written\ Exposures = \#Exposures \times Policy\ Length$$

$$Written\ Exposures_{B,CY\ 2021} = 3 \times 1 = 3.0$$

Note: Exposure base is car-years, so the policy length is the term-length in years (0.5 years, 1 year, etc.).

	Written Exposures		
Policy	CY 2021	CY 2022	Comments
A	-	-	Effective date in 2020
B	3.00	-0.75	Canceled in CY 2022 with 3 months remaining
C	2.00	-0.25	1 auto removed with 3 months remaining
D	1.00	-	6-month policy, so each auto is worth 0.5 car-years
E	-	0.25	6-month policy written 2022, canceled halfway through
F	-	2.00	Annual policy written 2022
Total	6.00	1.25	

Note: For policy C, an alternative way to calculate the CY 2022 written exposure is that 3 months of the original policy is canceled (-2 * 3/12) and the newly modified policy has 3 months remaining with 1 autos (+1 * 3/12). The sum is -0.25 car-years.

2) **For *Policy Year*, exposures are aggregated by *policy effective date*. Therefore, policy cancelations will record a negative partial exposure for the portion of the policy canceled for the calendar year of the *transaction date*.**

$$Written\ Exposures_{B,PY\ 2021} = 3 \times 1 - 3 \times \frac{3}{12} = 2.25$$

$$Written\ Exposures_{E,PY\ 2022} = 1 \times 0.5 - 1 \times 0.5 \times \frac{3}{6} = 0.25$$

	Written Exposures		
Policy	PY 2021	PY 2022	Comments
A	-	-	
B	2.25	-	Cancelation contributes to PY 2021 (original effective date)
C	1.75	-	Modification contributes to PY 2021 (original policy effective date)
D	1.00	-	
E	-	0.25	
F	-	2.00	
Total	5.00	2.25	

Part b – Earned exposures

Earned exposures - The portion of written exposures for which coverage has already been provided as of a point of time.

3) **For *Calendar Year*, earned exposure is based on the portion of the policy term used up aggregated by calendar year. This assumes the probability of a claim is evenly distributed throughout the year (an even earning pattern).**

$$Earned\ Exposures = \#Exposures \times \%Earned$$

$$Earned\ Exposures_{B,CY\ 2021} = 3 \times \frac{9}{12} = 2.25$$

Earned Exposures

Policy	CY 2021	CY 2022	Comments
A	0.25	-	Written in 2020, but in effect for 3 months of 2021
B	2.25	-	Canceled on 1/1/22
C	1.00	0.75	1 auto removed with 3 months remaining
D	0.83	0.17	2 autos earned (5 months in 2021 and 1 month in 2022)
E	-	0.25	Canceled on 3/31/22, so only 3-months earned
F	-	1.50	9 months earned in 2022
Total	4.33	2.67	

4) **For *Policy Year*, earned exposure is based on the portion of the policy term used up *by the as-of date* and contributes to the policy year corresponding to the policy effective date.**

$$Earned\ Exposures_{F,PY\ 2022} = 2 \times \frac{9}{12} = 1.50$$

Earned Exposures

Policy	PY 2021	PY 2022	Comments
A	-	-	Policy effective date is in PY 2020
B	2.25	-	Policies B, C and D contribute to PY 2021
C	1.75	-	
D	1.00	-	
E	-	0.25	Policies E and F contribute to PY 2021
F	-	1.50	9 months earned by the as of date of 12/31/22
Total	5.00	1.75	

Part c – Unearned exposures

Unearned exposures - The portion of written exposures for which coverage has ***not yet*** been provided. For a single policy, it's the difference between written and earned exposure at a point in time.

5) **For *Calendar Year*, unearned exposure is the difference between written exposure and earned exposure for a given calendar year *plus* unearned exposures as of the beginning of the calendar year.**

$$CY\ Unearned_{\ End\ CY} = CY\ Written - CY\ Earned + CY\ Unearned_{Beg.\ CY}$$

$$CY\ Unearned_{\ CY\ 2021} = 6.0 - 4.33 + 0.25$$
$$= 1.92$$

	Unearned Exposures		
	CY 2020	CY 2021	CY 2022
Total	0.25	1.92	0.50

Note: To calculate CY 2021 Unearned Exposures, we need the Unearned Exposures at the beginning of the year, the CY 2020 Unearned Exposures. By the end of CY 2020, the only unearned exposure is from the 3 months remaining of Policy A which is earned in CY 2021.

6) **For *Policy Year*, unearned exposure is the difference between written exposure and earned exposure for a given policy year.**

$$Written\ Exposures = Earned\ Exposures + Unearned\ Exposures$$

$$Unearned_{\ PY\ 2022} = 2.25 - 1.75$$
$$= 0.50$$

	Unearned Exposures		
	PY 2021	PY 2022	Comments
Total	-	0.50	*As of 12/31/22*, PY 2021 is fully earned (written = earned)

Note: Policy A is effective in 2020, so it contributes to PY 2020 and can be ignored. *As of 12/31/22*, PY 2021 is fully earned (written exposures = earned exposures), so unearned exposure is zero. For PY 2022, the only unearned exposure *as of 12/31/22* is 3 months remaining of policy F with two exposures.

Part d – In-force exposures

In-force exposures - The number of insured units exposed to claims at a given point in time for policies that are in force.

7) **Sum up the number of exposed units on policies that are in-force as of the given date. Include policies where the "as of" date is after the policy effective date and before expiration or cancelation date.**

Policy	In-Force 2/15/22	Comments
A	-	Expired
B	-	Canceled before 2/15/22
C	2.00	1 auto was removed on 4/1/22, after 2/15/22
D	-	Expired
E	1.00	
F	-	Written *after* 2/15/22
Total	3.00	

8) **For in-force "written exposures", sum up the written exposures on policies that are in-force *as of the given date.***

Policy	In-Force 2/15/22	Comments
A	-	Expired
B	-	
C	2.00	At 2/15/22 policy C has 2 written *car-years*
D	-	Expired
E	0.50	6-month policy, so each auto is 0.5 written car-years
F	-	Written *after* 2/15/22
Total	2.50	

Discussion

Questions about aggregating exposures are straight-forward but it's easy to overlook some of the details and make a mistake.

There are two main ways to aggregate exposures, by policy year and by calendar (accident) year.

- Policy year exposures are aggregated according to the original policy effective date
- Calendar year exposures are aggregated based on the transaction date.

A key thing to remember is that at the end of a year, calendar year exposures are fixed. In contrast, policy year exposures aren't fixed until all policies written during the year expire (24 months after the start of the policy year for annual policies).

Possible Problem Modifications

- Uneven earning pattern (e.g. for watercraft where most claims occur in summer)

    - Calculate earned exposures based on the % of the exposure earning pattern used
- Semi-annual policies
    - Each policy counts as 50% of a written exposure (0.5 years)
- Policy cancelations and modifications
    - **Calendar Year:** The written exposure for the cancelation/modification contributes to the calendar year of the *transaction date*
    - **Policy Year:** The written exposure for the cancelation/modification contributes to the policy year of the *original policy effective date*

Aggregating Premium for Individual Policies

Aggregating Premium for Individual policies is done almost entirely the same. The only real difference is that premium is used instead of exposures. Also, for in-force premium, it's important to use the full-term premium for the policy that is in-force at the as of date.

Criteria for Exposure Bases

Proportional to Expected Loss - The exposure base should be directly proportional to loss.

Practical - The exposure base should be well-defined, objective and relatively easy/inexpensive to measure and verify.

Historical Precedence - Because of the difficulties of implementing a new exposure base (large premium swings for insureds, changing the rating algorithm, and requiring data adjustments for future analyses), a good exposure base should have historical precedence.

Source

Werner Ratemaking Ch. 4 – pg. 51-61

More Practice

CAS Fall 2019 – 2
CAS Spring 2019 – 1
CAS Spring 2018 – 1
CAS Spring 2018 Makeup – 1
CAS Fall 2018 – 2
CAS Fall 2017 – 3
CAS Spring 2017 – 1
CAS Spring 2016 – 2
CAS Fall 2015 – 1
CAS Fall 2014 – 2

Aggregating Blocks of Exposure

Werner Ratemaking Ch. 4

Problem

Given the following written exposures for a company summarized by month as of 3/31/24:

Month	Written Exposures
Jan-23	192
Feb-23	192
Mar-23	184
Apr-23	190
May-23	191
Jun-23	185
Jul-23	191
Aug-23	195
Sep-23	193
Oct-23	191
Nov-23	190
Dec-23	195
Jan-24	204
Feb-24	203
Mar-24	200

- Exposures are written uniformly during each month
- All policies have an annual policy term
- Assume no policies were written prior to 1/1/23.

Calculate the earned exposures for calendar year 2023 and policy year 2023 as of 3/31/24.

Solution Recipe

1) **Calculate earned exposures for a block of policies *as if* the entire block was written on the mid-point of the time period. This is the assumed effective date. Determine the % earned as the portion of the year between the assumed effective date and the end of the calendar year.**

$$\boxed{Assumed\ Effective\ Date = Time\ Period\ Mid\text{-}Point}$$

$$CY\ Earned\ Exposures = \#Exposures \times \%Earned$$

$$CY\ Earned\ Premium_{Jan\text{-}23} = 192 \times 96\%$$
$$= 184$$

Calendar Year and Month	Assumed Effective Date	Written Exposures	Calendar Year 2023 % Earned	Earned Exposures
Jan-23	1/15/23	192	96%	184.00
Feb-23	2/15/23	192	88%	168.00
Mar-23	3/15/23	184	79%	145.67
Apr-23	4/15/23	190	71%	134.58
May-23	5/15/23	191	63%	119.38
Jun-23	6/15/23	185	54%	100.21
Jul-23	7/15/23	191	46%	87.54
Aug-23	8/15/23	195	38%	73.13
Sep-23	9/15/23	193	29%	56.29
Oct-23	10/15/23	191	21%	39.79
Nov-23	11/15/23	190	13%	23.75
Dec-23	12/15/23	195	4%	8.12
Total				**1,140**

Note: Because the problem assumes no policies are written before 1/1/23, we don't need to calculate the amount of calendar year 2023 earned premium from policies written in 2022.

2) For policy year, calculate earned exposures for a block of policies between the mid-point of the time period and the as-of date.

$$\boxed{PY\ Earned\ Exposures = \#Exposures \times \%Earned\ by\ as\text{-}of\text{-}date}$$

$$PY\ Earned\ Premium_{Jan\text{-}23} = 192 \times 100\%$$
$$= 192$$

Policy Year and Month	Assumed Effective Date	Written Exposures	Policy Year 2023 % Earned	Earned Exposures
Jan-23	1/15/23	192	100%	192.00
Feb-23	2/15/23	192	100%	192.00
Mar-23	3/15/23	184	100%	184.00
Apr-23	4/15/23	190	96%	182.08
May-23	5/15/23	191	88%	167.13
Jun-23	6/15/23	185	79%	146.46
Jul-23	7/15/23	191	71%	135.29
Aug-23	8/15/23	195	63%	121.88
Sep-23	9/15/23	193	54%	104.54
Oct-23	10/15/23	191	46%	87.54
Nov-23	11/15/23	190	38%	71.25
Dec-23	12/15/23	195	29%	56.88
Total				**1,641**

Note:
Mar-23 policies and prior are 100% earned *by 3/31/24.*
Apr-23 policies have an assumed effective date of 4/15/23 and have a % earned of (= 23 / 24) by 3/31/24.

Discussion

As long as policies are uniformly written during each time period, this method is a good approximation. The longer the time periods (quarters or years), the less likely that this assumption is appropriate.

This approach is necessary if data is summarized. With greater computational power these days, it's more realistic to aggregate exposures looking at the individual policy data. That would be more accurate than using blocks of exposures.

Note that aggregating premium using blocks of policies is done the same way as above, just with premium numbers.

CBT Spreadsheet Tips

For the %Earned, start with the first %Earned (=23/24 for January 2023) and subtract 1/12 for each subsequent value. This is much quicker than manually typing in 23/24, 21/24, 19/24… for the whole series.

Source

Werner Ratemaking - pg. 60-61

More Practice

CAS Fall 2016 – 1
CAS Spring 2015 – 3

Uneven Earning Pattern

Werner Ratemaking Ch. 5

Problem

Given the following for an insurer writing Recreational Vehicle (RV) insurance as of 3/31/2024:

Calendar Year and Quarter	Written Premium
2022 Q1	182
2022 Q2	765
2022 Q3	707
2022 Q4	155
2023 Q1	208
2023 Q2	842
2023 Q3	735
2023 Q4	98

Earning Pattern

Quarter	%Earned
Q1	10%
Q2	40%
Q3	40%
Q4	10%

- Policies are written uniformly during each quarter
- There are no policy cancellations or modifications
- All policies are annual
- The company began writing the RV line of business on 1/1/22

a. Calculate the 2022 and 2023 **earned premium** as of 3/31/2024 both aggregated by:
   i. Calendar year and quarter
   ii. Policy year and quarter

b. Calculate the 2022 and 2023 **unearned premium** as of 3/31/2024 both aggregated by:
   i. Calendar year and quarter
   ii. Policy year and quarter

Solution Recipe

Part a – Earned premium with uneven earning pattern

1) **For *calendar quarter*, calculate the earned premium as the exposed written premium that is earned during the quarter based on the earning pattern. With blocks of premium (or exposures), treat the policies as if they were written at the mid-point of the time period.**

$$CY\ Earned\ Premium = \sum_{\substack{Exposed \\ Policies}} Written\ Premium \times \%Earned\ in\ time\ period$$

Calendar Quarter Earned Premium

Calendar Year and Quarter	Average Written Date	Written Premium	Earned Premium
2022 Q1	2/15/22	182	9.10
2022 Q2	5/15/22	765	225.80
2022 Q3	8/15/22	707	520.20
2022 Q4	11/15/22	155	173.15
2023 Q1	2/15/23	208	182.20
2023 Q2	5/15/23	842	749.40
2023 Q3	8/15/23	735	770.40
2023 Q4	11/15/23	98	191.15

Notes:

- **2022 Q1** - 2022 Q1 written premium is exposed for 50% of Q1.
- **2022 Q2** - 2022 Q1 written premium is exposed for 100% of Q2 and 2022 Q2 written premium is exposed for 50% of Q2.
- **2022 Q3** - 2022 Q1 & Q2 written premium is exposed for 100% of Q3 and 2022 Q3 written premium is exposed for 50% of Q3.

2) **For *policy quarter*, calculate earned premium as the written premium that is earned *by the as of* date based on the earning pattern *for policies effective during the quarter*. With blocks of premium (or exposures), treat the policies as if they were written at the mid-point of the time period.**

$$PY\ Earned\ Premium = PY\ Written\ Premium \times \%Earned\ by\ as\ of\ date$$

Policy Quarter Earned Premium as of 3/31/24

Policy Year and Quarter	Average Written Date	Written Premium	Earned Premium
2022 Q1	2/15/22	182	182
2022 Q2	5/15/22	765	765
2022 Q3	8/15/22	707	707
2022 Q4	11/15/22	155	155
2023 Q1	2/15/23	208	208
2023 Q2	5/15/23	842	673.60
2023 Q3	8/15/23	735	294.00
2023 Q4	11/15/23	98	14.70

Notes:

- By 3/31/24, all policies written on or before 3/31/23 have been fully earned and expired.
- **2023 Q2** - By 3/31/24, this policy block has been exposed for 50% of 2023 Q2 and 100% of 2023 Q3, Q4 and 2024 Q1.
- **2023 Q3** - By 3/31/24, this policy block has been exposed for 50% of 2023 Q3 and 100% of 2023 Q4 and 2024 Q1.

Part b – Unearned premium with uneven earning pattern

3) **For *Calendar Year*, unearned premium is the difference between written premium and earned premium for a given calendar year plus unearned premium as of the beginning of the calendar year.**

$$CY\ Unearned\ Prem_{End\ CY} = CY\ Written\ Prem - CY\ Earned\ Prem + CY\ Unearned\ Prem_{Beg.\ CY}$$

Calendar Year	Unearned Premium
2022	881
2023	871

Note:
The company started writing RV policies on 1/1/22, so there is no unearned premium at the beginning of CY 2022.

4) **For Policy Year, unearned premium is the difference between written premium and earned premium for a given policy year evaluated at the as of date.**

$$Written\ Exposures = Earned\ Exposures + Unearned\ Exposures$$

Policy Year	Unearned Premium	
2022	0	← By 3/31/24, all 2022 policies have been fully earned
2023	693	

Discussion

The text doesn't show any numerical examples, but does discuss this concept and it has been asked on past exams. The approach is the same for both earned exposures and earned premium using an uneven earning pattern.

Lines of business like recreational vehicle, boat owners, and warranty don't have an equal probability of loss over the whole policy term. Exposures and premiums for lines of business with seasonal fluctuations (like RV and boat owners) will have an uneven earning pattern. In the problem here, recreational vehicles are most susceptible to loss over Q2 and Q3.

With an uneven earning pattern, you must calculate the earned exposures and premium using the earning pattern, not using the percent of the policy term length exposed.

Source

Werner Ratemaking Ch. 5 - pg. 55-57, 68-70

More Practice

CAS Fall 2019 – 1

Extension of Exposures

Werner Ratemaking Ch. 5

Problem

An insurance company writes annual policies and has the following historical policies in the experience period grouped by rating characteristics for an indicated rate review as of 3/31/23:

- Policies are written uniformly throughout each year

Policy Effective Dates	Number of Policies	Number of Exposures	Territory	Class
Jan 1, 2021 - Dec 31, 2021	129	774	A	X
	74	296	A	Y
	68	476	B	X
	31	155	B	Y
Jan 1, 2022 - Dec 31, 2022	133	798	A	X
	78	390	A	Y
	76	456	B	X
	35	210	B	Y

The current rating algorithm as of 3/31/23 is:

$$Premium = Exposure \times Rate\ per\ Exposure \times Territory\ Factor \times Class\ Factor + Policy\ Fee$$

- Base rate per exposure $350
- Current policy fee $200 per policy

Territory	Rating Factor
A	1.00
B	1.43

Class	Rating Factor
X	1.00
Y	0.78

Calculate the on-level earned premium for Calendar Year 2022 using the extension of exposures method.

Solution Recipe

Part a – Written premium at current rate level with extension of exposures

1) **Rerate all policies in the experience period *using the current rating algorithm* to restate the historical premium at current rate levels. Make sure to apply the correct class factors to each policy.**

$$Premium = Exposure \times Rate\ per\ Exposure \times \prod_{i=1}^{n} Class\ Factors_i + Policy\ Fee$$

$$Premium_{2021,Terr\ A,Class\ Y} = 296 \times \$350 \times 1.00 \times 0.78 + \$200 \times 74$$
$$= 95{,}608$$

In Excel
Use VLOOKUP( ) to look up the correct class rating factors for a policy group:
Class Factor$_i$ = VLOOKUP(Class$_i$, rating factor table, rating factor column #, FALSE)

Effective Year	Territory	Class	Premium at CRL
2021	A	X	296,700
2021	A	Y	95,608
2021	B	X	251,838
2021	B	Y	66,710
2022	A	X	305,900
2022	A	Y	122,070
2022	B	X	243,428
2022	B	Y	88,982

2) Aggregate the on-level written premium by calendar year based on the policy effective date.

$$Premium_{2021} = 296{,}700 + 95{,}608 + 251{,}838 + 66{,}710$$
$$= 710{,}856$$

In Excel
Written Premium = SUMIF(effective year, calendar year criteria, premium at CRL)

Calendar Year	Written Premium at CRL
2021	710,856
2022	760,380

3) Determine the percentage of each policy year that is earned in the calendar year and calculate the aggregate on-level earned premium.

$$CY\ Earned\ Premium = \sum Written\ Premium \times \%Earned\ in\ time\ period\ Avg\ Factor$$

$$CY\ Earned\ Prem_{2022} = 710{,}856 \times 50\% + 760{,}380 \times 50\%$$
$$= \boxed{735{,}618}$$

Effective Year	% in CY 2022
2021	50%
2022	50%

Note:
The average written date for the block of policies effective in 2021 is 7/1/21. Since these are annual policies, 50% is earned in calendar year 2021 and 50% is earned in calendar year 2022.

Discussion

The key idea of the extension of exposures method is to rerate all policies written in the historical experience period with the *current rating algorithm*. This restates the historical premium to current rates.

The extension of exposures method has the following advantages and disadvantages:

Advantages	Disadvantages
• Most accurate current rate level method	• Requires detailed data - rating characteristics for all policies in the historical period • Difficult to determine what subjective debits/credits would be applied under current rating guidelines

Possible Problem Modifications

- Different rating algorithm used
  - The problem should clearly state the rating algorithm
- Given rating factors at different effective dates
  - Make sure to use the latest rating factors
- 6-month policies used
  - Make sure to apply the rating algorithm and aggregate earned premium correctly

CBT Spreadsheet Tips

For this type of problem, using VLOOKUP( ) to look up rating factors for a given rating characteristic and SUMIF( ) to aggregate premium by year may be helpful on the exam. Of course, you can always reference rating factors and aggregate premium by year manually if it's a very simple problem.

SUMPRODUCT(array 1, array 2, ...) makes it easy to help calculate weighted averages or the sum of a product of multiple arrays in one step.

Source

Werner Ratemaking Ch. 5 – pg. 72-73

More Practice

CAS Fall 2018 – 3
CAS Fall 2015 – 1
CAS Fall 2014 – 3
CAS Fall 2013 – 2

Parallelogram Method

Werner Ratemaking Ch. 5

Problem

Given the following historical premium and rate changes for an insurer:

Historical Premium

Calendar Year	Earned Premium
2020	870,000
2021	935,000
2022	980,000

Rate Change History

Effective Date	Overall Average Rate Change
4/1/20	3%
7/1/21	12%
7/1/22	-2%

- All policies are annual-term policies
- Policies are written uniformly over the year

a. Calculate the earned premium at current rate level for Calendar Years 2020-2022.

b. Calculate the appropriate on-level factors for Policy Years 2020-2022.

Solution Recipe

Part a – Calendar Year Parallelogram Method

1) **Determine the rate level groups based on the rate change history and calculate the cumulative rate level index for each group.**

$$CRL\ Index_1 = 1.00$$
$$CRL\ Index_i = CRL\ Index_{i-1} \times (1 + rate\ change_i)$$

$$CRL\ Index_2 = 1.00 \times (1 + 0.03)$$
$$= 1.030$$

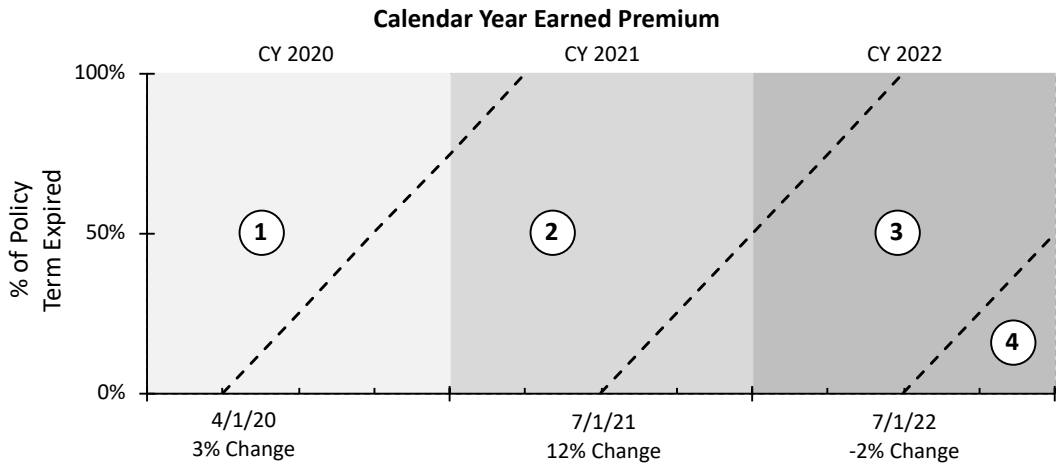
Rate Level Group	Effective Date	Cum. Rate Level Index
1	Initial	1.000
2	4/1/20	1.030
3	7/1/21	1.154
4	7/1/22	1.131

2) **Calculate the portion of each calendar year's earned premium that was earned during each rate level group. Use a calendar year earned premium diagram with the rate changes to help with this.**

$$Area\ of\ a\ triangle = {}^{1}/_{2} \times base \times height$$

$$Area\ of\ a\ parallelogram = base \times height$$

$$Area\ of\ a\ trapezoid = {}^{1}/_{2} \times (base_1 + base_2) \times height$$



3) **Calculate the weighted average cumulative rate level index for each year.**

$$\boxed{Average\ CRL\ Index = \sum \%EP\ in\ Rate\ Level\ Group_i \times CRL\ Index_i}$$

$$\%\ EP\ in\ Rate\ Level\ Group_{CY\ 2020,Group\ 1} = 1 - 0.5 \times 0.75^2$$
$$= 71.88\%$$

$$Average\ CRL\ Index_{CY\ 2020} = 71.88\% \times 1.000 + 28.13\% \times 1.030$$
$$= 1.0084$$

In Excel

Average CRL Index = SUMPRODUCT(%EP in rate level groups , CRL Indices of rate level groups)

Calendar Year	Portion of EP in Each Rate Level Group 1	2	3	4	Average CRL Index
2020	71.88%	28.13%			1.0084
2021	3.13%	84.38%	12.50%		1.0445
2022		12.50%	75.00%	12.50%	1.1353
CRL Index	1.000	1.030	1.154	1.131	

Note: As a quick reasonability check, the sum of the portion of EP in *each row* should be 100%.

4) **Calculate the on-level factor for each year as the cumulative rate level index for the current group divided by the weighted average rate level index for the year.**

$$On\text{-}Level\ Factor = \frac{Current\ CRL\ Index}{Average\ CRL\ Index}$$

$$On\text{-}Level\ Factor_{2020} = \frac{1.131}{1.0084} = 1.1211$$

Calendar Year	On-Level Factor
2020	1.1211
2021	1.0823
2022	0.9958

Note: Make sure to use the CRL Index for the current *rate level group* (1.131 for group 4 here), not the weighted average CRL index for the latest historical year.

5) **Apply the on-level factor to the earned premium for the appropriate year to calculate the on-level earned premium.**

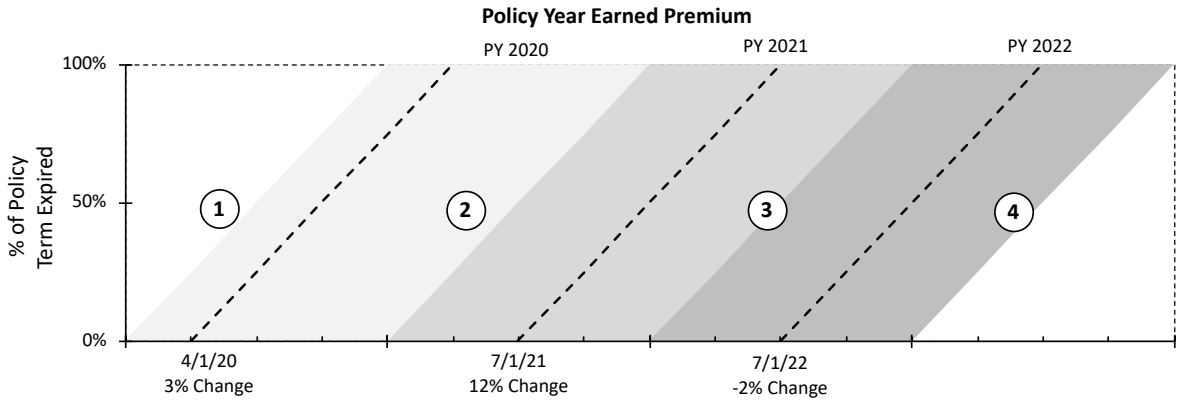
$$On\text{-}Level\ EP = On\text{-}Level\ Factor \times Earned\ Premium$$

$$On\text{-}Level\ EP_{2020} = 1.1211 \times 870{,}000 = 975{,}330$$

Calendar Year	On-Level Earned Prem
2020	975,330
2021	1,011,997
2022	975,910

Part b – Policy Year Parallelogram Method

6) **Redo steps 2-4 to get the on-level factors for policy years. The key difference is that the portion of each policy year's earned premium in the rate level groups will be different than for calendar year. Use a policy year earned premium diagram to help.**



$\%\ EP\ in\ Rate\ Level\ Group_{PY\ 2020,Group\ 1} = 1 \times 0.25 = 25\%$

$\text{Average CRL Index}_{\text{CY 2020}} = 25\% \times 1.000 + 75\% \times 1.030 = 1.0225$

$On\text{-}Level\ Factor_{2020} = \frac{1.131}{1.0225} = \boxed{1.1057}$

Policy Year	Portion of EP in Each Rate Level Group 1	2	3	4	Average CRL Index	On-Level Factor
2020	25%	75%			1.0225	**1.1057**
2021		50%	50%		1.0918	**1.0355**
2022			50%	50%	1.1421	**0.9899**
CRL Index	1.000	1.030	1.154	1.131		

Discussion

The parallelogram method works at the aggregate level to adjust historical premium to current rate level. This is a significant difference compared to the Extension of Exposures method which rerates all policies to current rates at the policy level.

The tricky part in a parallelogram method problem is calculating the portion of earned premium in each rate level group. For policy years, this is easy for normal rate changes, but it requires some geometry for calendar year on-leveling. I would sketch a quick earned premium diagram to help and avoid mistakes.

Problem Variations

There are three main ways a parallelogram method can be modified:

- Calendar year vs. policy year (or months/quarters)
- 12-Month vs. 6-Month policy terms
- Rate changes affect: policies written after the change vs. policies midterm

For each variation, the same general approach as above is used with some modification.

Two Problems with the Parallelogram Method

Problem 1 - *Assumes policies are written evenly throughout the year*

- This assumption isn't always valid, especially for seasonal lines of business like boat owners insurance.
    - The parallelogram method can still be used with more refined time periods like months or quarters.

Another approach is to use the actual distribution of writings during the year to more accurately determine the portion of earned premium in each rate level group (step 2). The paper doesn't go into this approach.

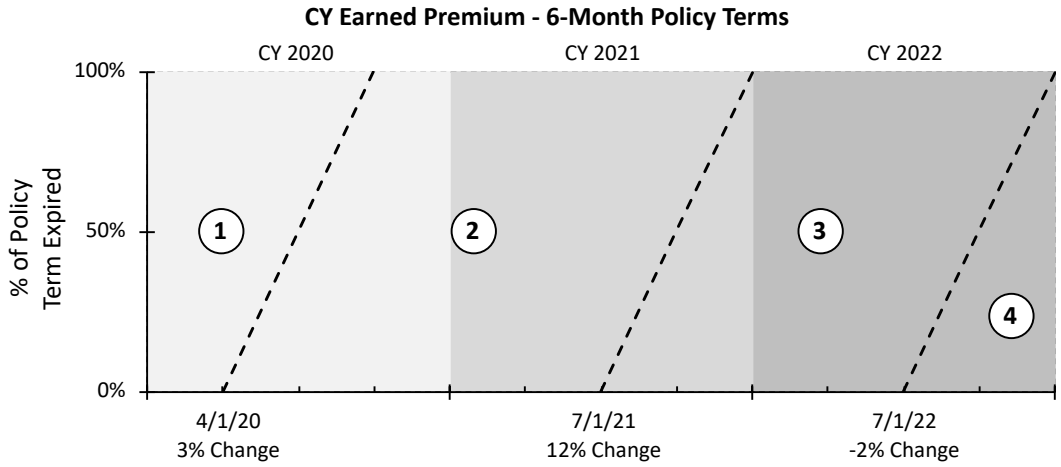
Problem 2 - *Applies at the aggregate level with overall average rate changes*

- If rate changes varied by class (class factors were modified), then premium at the class level won't be properly on-leveled. This means the adjusted premium won't be acceptable for classification ratemaking analysis.

Calendar Year Parallelogram Method for 6-Month Policies

For 6-month policies, rate changes are fully realized over 6 months instead of 12 months. To on-level calendar year earned premium, make sure to calculate the portion of earned premium in each rate level group correctly (step 2) since the geometry is different.

Example:
For the same problem above, assuming all policies have 6-month terms, this is how to solve for the on-level factors:



Calendar Year	Portion of EP in Each Rate Level Group 1	2	3	4	Average CRL Index	**On-Level Factor**
2020	50%	50%			1.0150	**1.1138**
2021		75%	25%		1.0609	**1.0656**
2022			75%	25%	1.1478	**0.9849**
CRL Index	1.000	1.030	1.154	1.131		

For policy years with 6-month terms, you get the same answer as in part b of the problem above. The diagram will look a different since policy years are fully earned after only 18 months, but the portion of earned premium in each rate level group is the same. You can set up an earned premium diagram to see why.

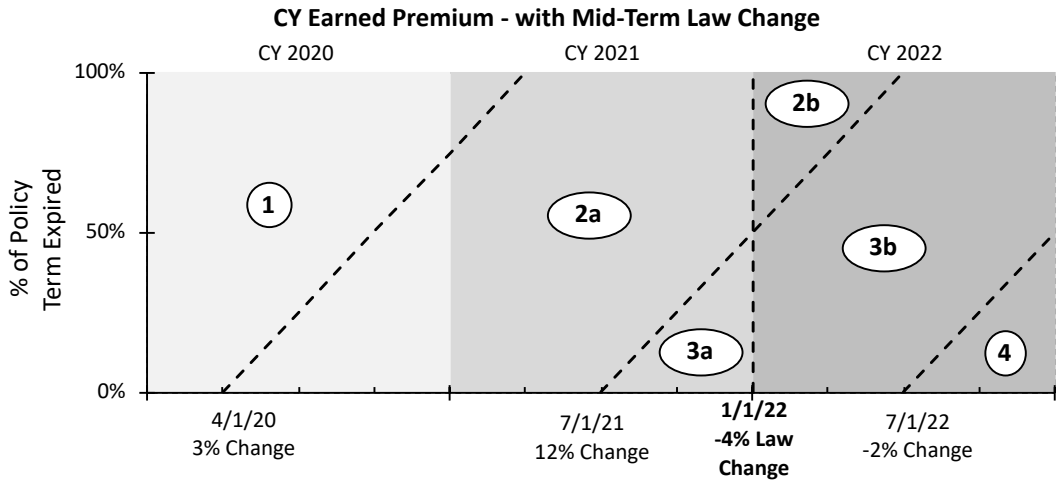
Rate Changes Affecting Policies Midterm (Mandated by Law)

A rate change mandated by law that affects ALL policies on or after the a specific date, including in-force policies, must be handled differently. It's represented as a vertical line on an earned premium diagram and will split one or more rate level groups.

Be careful when calculating the cumulative rate level indexes and the portion of earned premium in each group and sub-group.

Example:
Calculate the on-level factors for the same problem as part a, but with a mandated rate change of -4% on 1/1/22 affecting all policies midterm.

Rate Level Group	Effective Date	Cum. Rate Level Index	
1	Initial	1.000	
2a	4/1/20	1.030	***Before*** midterm rate change
3a	7/1/21	1.154	
2b	1/1/22	0.989	***After*** midterm rate change
3b	1/1/22	1.107	
4	7/1/22	1.085	



Calendar Year	Portion of EP in Each Rate Level Group						Average CRL Index	**On-Level Factor**
	1	2a	3a	2b	3b	4		
2020	71.88%	28.13%					1.0084	**1.0762**
2021	3.13%	84.38%	12.50%				1.0445	**1.0391**
2022				12.50%	75.00%	12.50%	1.0899	**0.9958**
CRL Index	1.000	1.030	1.154	0.989	1.107	1.085		

CBT Spreadsheet Tips

Use SUMPRODUCT( ) to help calculate the weighted average rate level index for each year:

Average CRL Index = SUMPRODUCT(%EP in rate level groups , CRL Indices of rate level groups)

It is possible to set up formulas to directly calculate the portion of earned premium in each rate level group using YEARFRAC(start date, end date) formulas and the rate level effective dates. However, it requires complicated Excel formulas that are tricky to set up. On an exam, I think it's faster to sketch a quick diagram on scratch paper and use geometry as shown above.

Source

Werner Ratemaking Ch. 5 - pg. 73-80

More Practice

CAS Spring 2019 – 2
CAS Fall 2018 – 3
CAS Spring 2017 – 2
CAS Fall 2016 – 2
CAS Spring 2015 – 5
CAS Spring 2015 – 4
CAS Spring 2014 – 1

Premium Development

Werner Ratemaking Ch. 5

Problem

Given the following for a workers compensation insurer as of 12/31/22:

Policy Year	Earned Premium Evaluated as of 12 months	24 months	36 months	Ultimate 48 months
2018	3,348	6,663	6,958	6,958
2019	3,481	6,725	7,315	7,315
2020	3,603	6,718	6,940	
2021	3,711	7,076		
2022	3,823			

- Premium audits are performed between 3 and 6 months after policies expire.
- All policies have an annual policy term.

Calculate the estimated ultimate earned premium for by policy year for years 2018 – 2022.

Solution Recipe

1) **Calculate the age-to-age development factors (link ratios) for the premium development triangle.**

$$Age\text{-}to\text{-}Age\ Factor_t = \frac{Premium_{t+1}}{Premium_t}$$

2) **Select a suitable link ratio for each development period based on how development is expected to occur in the future. Reviewing various averages can help with selection. There's no single "correct" selection, but the selection should be reasonable.**

Policy Year	Age-to-Age Factors 12-24	24-36	36-48
2018	1.990	1.044	1.000
2019	1.932	1.088	1.000
2020	1.865	1.033	
2021	1.907		
All-Year Avg	1.923	1.055	1.000
Weighted Avg	1.922	1.055	1.000
Selected	**1.922**	**1.055**	**1.000**

$$All\text{-}Year\ Avg_{12-24} = \frac{1.990 + \cdots + 1.907}{4} = 1.923$$

$$Weighted\ Avg_{12-24} = \frac{6{,}663 + \cdots + 7{,}076}{3{,}348 + \cdots + 3{,}711} = 1.922$$

3) **Calculate the age-to-ultimate development factors for each maturity period as the product of the age-to-age factors between the maturity period and ultimate.**

$$Age\text{-}to\text{-}Ult\ Factor_t = \prod_{i=t}^{Ult} Age\text{-}to\text{-}Age\ Factor_i$$

$$Age\text{-}to\text{-}Ult\ Factor_{12-Ult} = 1.922 \times 1.055$$
$$= 2.028$$

	12-Ult	24-Ult	36-Ult
Age-to-Ultimate Factor	2.028	1.055	1.000

4) **Apply the appropriate age-to-ultimate development factor to the premium for each policy year to estimate ultimate premium.**

$$Ultimate\ Premium = Premium_{at\ latest\ valuation} \times Age\text{-}to\text{-}Ult\ Factor_t$$

$$Ultimate\ Premium_{2022} = 3{,}823 \times 2.028$$
$$= 7{,}753$$

Policy Year	Earned Prem as of 12/31/23	Age-to-Ult Factor	**Ultimate Earned Prem**
2018	6,958	1.000	**6,958**
2019	7,315	1.000	**7,315**
2020	6,940	1.000	**6,940**
2021	7,076	1.055	**7,465**
2022	3,823	2.028	**7,752**

Note: 2018 and 2019 are already at ultimate (48 months here), so their age-to-ultimate factor is set to 1.

Discussion

There are two main scenarios where premium development is used:

- When an incomplete year of data is used
- When the line of business uses premium audits

In this problem, we see both of these scenarios:

Incomplete Policy Year - Because policies have annual terms and the premium is aggregated by policy year, policies for 2022 won't fully expire until 12/31/23 after 24 months. This is why the 12-24 age-to-age factor is so high. It's reflecting the fact that a policy year evaluated at 12 months maturity is an incomplete year.

Premium Audits - At 24 months, all policies are expired, but only some have had a premium audit. Premium audits aren't fully complete for a year until 6 months after the policy year expires (30 months).

Premium development is generally *only done for policy year* premium because calendar year premium is fixed at the end of each year, so there is no development. Some actuaries may adjust calendar year premium if audit patterns are changing, but this isn't elaborated on.

Source

Werner Ratemaking Ch. 5 - pg. 80-81

More Practice

One-Step Premium Trend

Werner Ratemaking Ch. 5

Problem

An actuary is performing a rate review for a Home insurance book of business using the following premium information as of 3/31/2022:

Year Ending Quarter - X	Written Premium at CRL	Written Exposures
2019 - 1Q	134,300	128
2019 - 2Q	137,511	129
2019 - 3Q	139,447	130
2019 - 4Q	141,394	132
2020 - 1Q	144,364	133
2020 - 2Q	147,201	135
2020 - 3Q	150,063	136
2020 - 4Q	152,949	138
2021 - 1Q	155,859	139
2021 - 2Q	158,794	141
2021 - 3Q	161,753	142
2021 - 4Q	164,736	144

Calendar Year	Earned Premium at CRL
2019	139,994
2020	151,434
2021	163,104

- All policies have an annual policy term
- The proposed effective date for the rate change is: 1/1/2023
- The proposed rates are expected to be in effect for one year

Calculate the projected earned premium at current rate level for years 2019-2021 using the one-step premium trend method.

Solution Recipe

1) **Calculate the average written premium per exposure at *current rate level* for the quarterly premium data.**

$$Avg\ WP\ at\ CRL = \frac{WP\ at\ CRL}{Exposures}$$

$$Avg\ WP\ at\ CRL_{2020-1Q} = \frac{144{,}364}{133} = 1{,}085$$

2) **Calculate the annual changes in average written premium and select an appropriate trend factor to adjust historical premium to projected levels. Fitting an exponential or linear trend may help, but isn't necessary if the annual changes are consistent.**

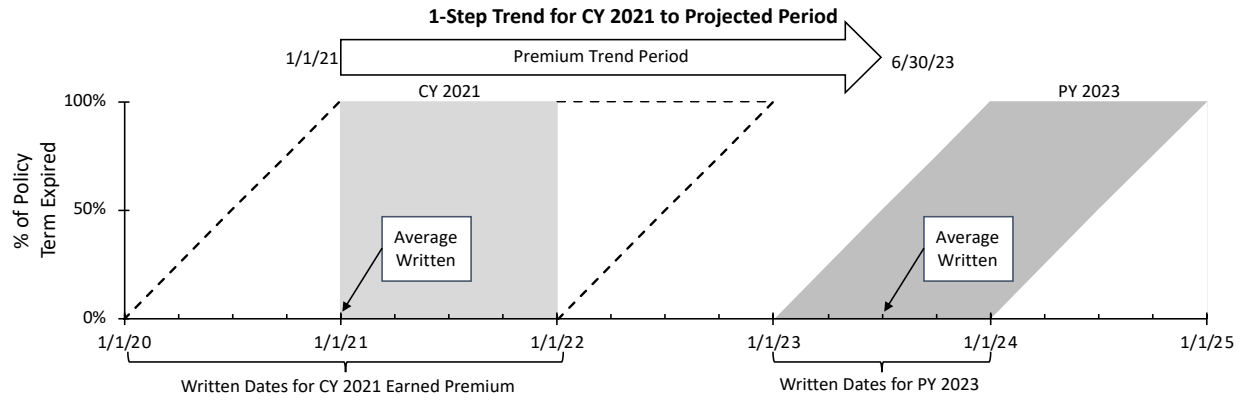
$$Annual\ Change_t = \frac{Avg\ WP\ at\ CRL_t}{Avg\ WP\ at\ CRL_{t-1}} - 1 \qquad Annual\ Change_{2020-1Q} = \frac{1{,}085}{1{,}049} - 1 = 3.5\%$$

Year Ending Quarter - X	Written Premium at CRL	Written Exposures	Average Written Prem at CRL	Annual Change
2019 - 1Q	134,300	128	1,049	-
2019 - 2Q	137,511	129	1,066	-
2019 - 3Q	139,447	130	1,073	-
2019 - 4Q	141,394	132	1,071	-
2020 - 1Q	144,364	133	1,085	3.5%
2020 - 2Q	147,201	135	1,090	2.3%
2020 - 3Q	150,063	136	1,103	2.9%
2020 - 4Q	152,949	138	1,108	3.5%
2021 - 1Q	155,859	139	1,121	3.3%
2021 - 2Q	158,794	141	1,126	3.3%
2021 - 3Q	161,753	142	1,139	3.2%
2021 - 4Q	164,736	144	1,144	3.2%
			Average	3.1%
			Selected	**3.1%**

3) **Determine the trend period for each calendar year of premium. The trend period is the time from the average written date of policies earned during the experience period to the average written date for the effective period of the proposed rates.**

$$1\text{-}Step\ Trend\ Period = Historical\ Average\ Written\ Date \rightarrow Effective\ Average\ Written\ Date$$

$$1\text{-}Step\ Trend\ Period_{2021} = 1/1/21 \text{ to } 6/30/23 = 2.5\ years$$



4) **Calculate the premium trend factor and apply it to the calendar year earned premium to calculate the projected earned premium at current rate level. Make sure to use *earned premium*.**

$$Projected\ Earned\ Prem\ at\ CRL = Earned\ Prem\ at\ CRL \times (1 + trend)^t$$

$$Projected\ Earned\ Prem\ at\ CRL_{2021} = 163{,}104 \times (1.031)^{2.5}$$
$$= 176{,}040$$

Calendar Year	Earned Premium at CRL	Trend Period	Premium Trend Factor	**Projected Earned Prem at CRL**
2019	139,994	4.5	1.147	**160,610**
2020	151,434	3.5	1.113	**168,511**
2021	163,104	2.5	1.079	**176,040**

Discussion

In addition to adjusting premium to current rate level, it's important to adjust the historical on-level premium to expected levels during the *future time period* due to premium trend.

Premium trend reflects how average written premium levels change over time due to distributional changes in a book's mix of business.

One-Step Premium Trend

The basic one-step trending approach involves selecting a premium trend based on historical changes in average written premium per exposure and applying it to the experience period premium.

Written premium *at current rate level* should be used. It's best to use *quarterly written premium* for responsiveness and 12-month rolling periods are often used. For example, written premium for the year ending 2019 Q2 includes written premium between 7/1/18 and 6/30/19. See the exhibits in the appendix.

Determining the correct trend period length is critical. The trend period is the length of time from the *average written date of policies underlying the historical earned premium to the average written date of policies effective with the new rates.*

The one-step approach may not be appropriate when changes in average premium vary significantly year-by-year or when historical average premium changes differ from expected future changes. In such cases, a two-step trending approach is more appropriate.

Factors That Impact Trend Period Length

- **Different Policy Term Length**

  For example, if all policies above were 6-month policies, the average written date for policies underlying CY 2021 earned premium would be 4/1/21 (policies written between 7/1/20 and 12/31/21). The trend-to date is the same and the trend period would be 2.25 years.

- Historical Policy Year Premium

  For example, if policy year earned premium was used, the average written date for policies underlying PY 2021 earned premium would be the mid-point of the policy year, 7/1/21. The trend-to date is the same and the trend period would be 2 years for PY 2021 earned premium.

- Different Effective Period for Proposed Rates

  For example, if rates are expected to be in effect for two years (1/1/23 to 12/31/24), then the trend-to date would be 12/31/23. For CY 2021 earned premium, the trend period would be 3 years (1/1/21 to 12/31/23).

Exposure Trend

If the exposure base is inflation-sensitive (like payroll for workers compensation), then average premium can change over time due to inflation in the exposure base. For lines of business with inflation-sensitive exposure bases, an exposure trend is used to project future exposures (and therefore premium) to future levels.

This isn't discussed further in the text, but you can see an example of it in appendix D. The adjustment is very similar to premium trend.

Source

Werner Ratemaking Ch. 5 - pg. 81-86

More Practice

CAS Spring 2019 – 2
CAS Spring 2015 – 4
CAS Spring 2013 – 2

Two-Step Premium Trend

Werner Ratemaking Ch. 5

Problem

An actuary is performing a rate review for an Auto insurance book of business using the following premium information as of 3/31/2022:

Quarter - X	Written Premium at CRL	Written Exposures
2020 - 1Q	309,960	492
2020 - 2Q	340,896	536
2020 - 3Q	347,964	542
2020 - 4Q	326,592	504
2021 - 1Q	331,578	507
2021 - 2Q	364,872	552
2021 - 3Q	372,744	558
2021 - 4Q	350,325	519

Calendar Year	Earned Premium at CRL	Earned Exposures
2017	1,128,805	2,009
2018	1,182,755	2,001
2019	1,237,389	2,046
2020	1,298,903	2,067
2021	1,398,917	2,130

The company acquired a small insurer on 1/1/2022 with a sub-prime book of auto business. After consolidating the books, the actuary expects the average premium level to increase. The actuary selects a **3% annual premium trend** for the prospective period.

- All policies have a six-month policy term
- The proposed effective date for the rate change is: 4/1/2023
- The proposed rates are expected to be in effect for 12 months

Calculate the projected earned premium at current rate level for years 2017-2021 using the two-step premium trend method.

Solution Recipe

1) **Calculate the average written premium per exposure *at current rate level* for the quarterly premium data for the latest point in the written premium trend data.**

$$Latest\ Average\ WP\ at\ CRL = \frac{Latest\ WP\ at\ CRL}{Latest\ \#Exposures}$$

$$Latest\ Average\ WP\ at\ CRL = \frac{350{,}325}{519}$$

$$= 675$$

2) **[Step 1 - Current Trend Factor] Calculate the current premium trend factor to adjust each year's historical earned premium *at current rate level* to the average premium level of the latest point in the written premium trend data.**

$$Current\ Trend\ Factor = \frac{Latest\ Average\ WP\ at\ CRL}{Historical\ Average\ EP\ at\ CRL}$$

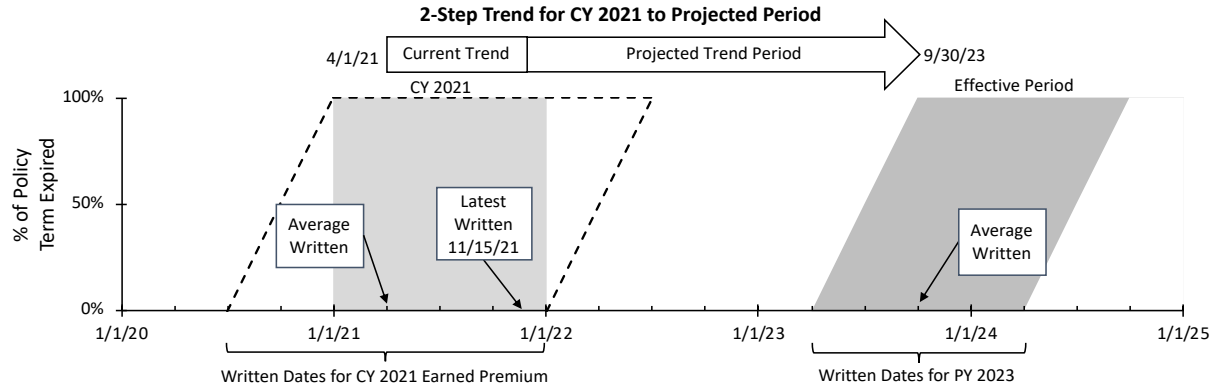
$$Current\ Trend\ Factor_{2021} = \frac{675}{657}$$
$$= 1.028$$

Calendar Year	Earned Premium at CRL	Earned Exposures	Average Earned Prem at CRL	Current Trend Factor
2017	1,128,805	2,009	562	1.201
2018	1,182,755	2,001	591	1.142
2019	1,237,389	2,046	605	1.116
2020	1,298,903	2,067	628	1.074
2021	1,398,917	2,130	657	1.028

3) **Determine the trend period for the projected trend factor. The projected trend period is the time from the *average written date of the latest point in the trend data* to the *average written date for the effective period* of the proposed rates.**

$$Projected\ Trend\ Period = Latest\ Average\ Written\ Date \rightarrow Effective\ Average\ Written\ Date$$

Latest Average Written Date	11/15/2021
Avg Written Date for Effective Period	9/30/2023
Projected Trend Period	**1.875**



Note: This problem gives written premium by quarter, not 12-month rolling periods like the 1-step premium trend problem or the exhibit in Appendix A of Werner. You should be prepared for either.

4) **[Step 2 - Projected Trend Factor] Calculate the projected trend factor using the projected annual premium trend and the projected trend period.**

$$\boxed{Projected\ Trend\ Factor = (1 + projected\ trend)^{t}}$$

$$Projected\ Trend\ Factor = (1 + 3\%)^{1.875}$$
$$= 1.057$$

5) **Calculate the total premium trend factor as the product of the current trend factor and the projected trend factor. Apply it to the calendar year earned premium to calculate the projected earned premium at current rate level.**

$$\boxed{Projected\ EP\ at\ CRL = EP\ at\ CRL \times Current\ Trend\ Factor \times Projected\ Trend\ Factor}$$

$$Projected\ EP\ at\ CRL_{2021} = 1{,}398{,}917 \times 1.028 \times 1.057$$
$$= 1{,}519{,}684$$

Calendar Year	Earned Premium at CRL	(Step 1) Current Trend Factor	(Step 2) Projected Trend Factor	**Projected Earned Prem at CRL**
2017	1,128,805	1.201	1.057	**1,433,354**
2018	1,182,755	1.142	1.057	**1,427,646**
2019	1,237,389	1.116	1.057	**1,459,752**
2020	1,298,903	1.074	1.057	**1,474,735**
2021	1,398,917	1.028	1.057	**1,519,684**

Discussion

The two-step trending method for premium trend should be used instead of the one-step method in situations where the historical premium trend has been significantly different from what is expected to occur in the future. This allows us to separately adjust the historical earned premium data to current average premium levels (step 1) and then apply a different trend into the projected period to reflect anticipated future changes to average premiums.

Alternative Approach for the Current Trend Factor

When the historical average premium is volatile, an alternative approach is to select an annual premium trend for the current trend period instead of the ratio shown in the Current Trend Factor formula.

Example:
Assume we selected an annual premium trend of 4% for the current period. Below is how to calculate the current trend factor for CY 2021 Earned Premium:

Avg Written Date of Policies Earned$_{CY\ 2021}$	4/1/2021	← All policies are 6-month
Latest Average Written Date	11/15/2021	
Current Trend Period$_{CY\ 2021}$	**0.625**	← 4/1/21 to 11/15/21

$$Current\ Trend\ Factor_{CY\ 2021} = (1 + 4\%)^{0.625}$$
$$= 1.0248$$

The projected trend factor would be the same as before.

Source

Werner Ratemaking Ch. 5 - pg. 86-88

More Practice

CAS Spring 2019 – 2 ← One-step trending is easier for this problem
CAS Spring 2017 – 1

Loss Aggregation

Werner Ratemaking Ch. 6 & Friedland Reserving Ch. 4

Problem

Given the following claims organized by accident date as of 12/31/2023:

Claim	Policy Effective Date	Accident Date	Report Date	Transaction Date	Claim Status	Incremental Payment	Case Reserve
1	4/18/2019	2/22/2020	2/4/2022	2/4/2022	Open	-	10,000
				5/14/2022	Closed	5,000	-
				8/2/2022	Reopened	-	20,000
				2/1/2023	Closed	25,000	-
2	12/15/2019	6/8/2020	6/15/20	6/15/2020	Open	-	3,000
				7/28/2020	Open	2,000	12,000
				9/3/2020	Open	8,000	4,000
				1/12/2021	Closed	3,000	-
3	7/5/2020	11/24/2020	12/8/2020	12/8/2020	Open	-	4,000
				3/15/2022	Open	-	20,000
				5/2/2023	Closed	-	-
4	12/19/2020	5/12/2021	5/29/2021	5/29/2021	Open	-	8,000
				7/30/2021	Open	5,000	10,000
				3/1/2022	Open	2,000	15,000
				2/15/2023	Open	10,000	5,000

- The case reserve amounts are the case reserves as of the transaction date.

a. Calculate the 2020 reported loss aggregated by:
   i. Calendar year
   ii. Accident year evaluated as of 12/31/2022
   iii. Policy year evaluated as of 12/31/2022
   iv. Report year evaluated as of 12/31/2022

b. Build an accident year paid loss cumulative development triangle for accident years 2019-2021 with annual December 31 valuations.

Solution Recipe

Part a – Loss Data Aggregation

1) For ***Calendar Year***, losses are aggregated by ***transaction date***, regardless of the policy effective date, accident date, or report date. Calendar year reported loss includes all loss payments in the year plus the change in case reserves over the year. Calendar year losses are fixed at year-end.

For **transactions in the Calendar Year**:

$$\boxed{CY\ Reported\ Loss = \sum Paid\ Losses + \Delta Case\ Reserve}$$

$$\boxed{CY\ Paid\ Loss = \sum Paid\ Losses}$$

$$CY\ 2020\ Reported\ Loss = (2{,}000 + 8{,}000 + 0) + [(4{,}000 + 4{,}000) - 0]$$
$$= \boxed{18{,}000}$$

Note: Only include transactions with a transaction date in 2020. For case reserves, use the difference between the ending case reserve (as of 12/31/2020) and the beginning case reserve (as of 1/1/2020) for the year.

2) **For *Accident Year*, losses are aggregated by *accident date*. Accident year reported loss incudes all loss payments up to the evaluation date for claims with a loss occurrence date during the year plus the case reserve *as of the evaluation date*.**

For losses occurring during the Accident Year:

$$\boxed{AY\ Reported\ Loss = \sum Paid\ Losses + Case\ Reserves_{eval\ date}}$$

$$\boxed{AY\ Paid\ Loss = \sum Paid\ Losses}$$

$$AY\ 2020\ Reported\ Loss = (5{,}000 + 2{,}000 + 8{,}000 + 3{,}000 + 0) + (20{,}000 + 0 + 20{,}000)$$
$$= \boxed{58{,}000}$$

Note: The evaluation date is 12/31/2022, so only include transactions that occurred by then and for claims with a loss occurrence in 2020.

3) **For *Policy Year*, losses are aggregated by *policy effective date*. Policy year reported loss incudes all loss payments up to the evaluation date for claims from policies with an inception date during the year plus the case reserve *as of the evaluation date*.**

For **policies incepted during the Policy Year**:

$$\boxed{PY\ Reported\ Loss = \sum Paid\ Losses + Case\ Reserves_{eval\ date}}$$

$$\boxed{PY\ Paid\ Loss = \sum Paid\ Losses}$$

$$PY\ 2020\ Reported\ Loss = (0 + 5{,}000 + 2{,}000) + 20{,}000 + 15{,}000$$
$$= \boxed{42{,}000}$$

4) **For *Report Year*, losses are aggregated by *report date*. Report year is similar to accident year except that losses are aggregated by report date instead of accident date.**

For **losses reported during the Report Year**:

$$RY\ Reported\ Loss = \sum Paid\ Losses + Case\ Reserves_{eval\ date}$$

$$RY\ Paid\ Loss = \sum Paid\ Losses$$

$$RY\ 2020\ Reported\ Loss = (2{,}000 + 8{,}000 + 3{,}000) + 0 + 20{,}000$$
$$= \boxed{33{,}000}$$

Part b – Cumulative Loss Development Triangle

5) **Set up a loss triangle with rows by accident year (or policy/report year) and columns at valuation month relative to the start of the year (e.g. 12 months, 24 months, …).**

Accident Year	12 Months	24 Months	36 Months	48 Months
2019	0	0	0	0
2020	18,000	17,000	**58,000**	43,000
2021	15,000	22,000	22,000	

Note: The 2020 Accident Year loss evaluated as of 12/31/2022 from part a is the 2020 Accident Year loss at 36 months shown in the triangle here. Claim #1 isn't reported until *after* 24 months development.

Discussion

Data aggregation questions (loss, premium or exposure) are simple in theory, but *attention to detail is critical*. The calculation is defined by the following elements:

- **Relevant statistic measured** - Paid or reported loss, earned premium, written exposure, etc.
- **Data aggregation method** - Calendar, accident, policy, or report year/month/quarter
- **Time period** - E.g. 2020, Fiscal Year 2022, or 2021 Q1
- **Valuation Date** - When the data is evaluated (part of the time period)

Calendar Year Aggregation

Calendar year aggregation considers all premium and loss transactions that occur during the calendar year regardless of the policy effective date, accident date, or report date. It's most appropriate for lines of business where losses are reported and settled quickly.

Advantages	Disadvantages
• Data is available quickly and is fixed at year-end	• Mismatch in timing between premium and losses

• Requires no additional expense to aggregate	• May not be appropriate for lines of business where losses are reported and settled relatively slowly.

Accident Year Aggregation

Accident year aggregation includes losses for accidents that occurred during the accident year, regardless of when the policy was issued or the claim was reported. Premium and exposures are the same as calendar year.

Advantages	Disadvantages
• Better match of premium and losses than calendar year aggregation	• Future development on known losses needs to be estimated

Policy Year Aggregation

Policy year aggregation (underwriting year) includes all premium and loss transactions on policies that were written during the year, regardless of when the claim occurred or when it was reported, reserved, or paid. Premium and exposures aren't fixed until after all policies written during the policy year have expired.

Advantages	Disadvantages
• The best match between losses and premium	• Data takes longer to develop than both calendar year and accident year

Report Year Aggregation

Report year aggregation is similar to calendar-accident year, except that the losses are aggregated according to when the claim was reported, as opposed to loss occurrence date. It is typically used for claims-made policies.

Source

Werner Ratemaking Ch. 6 - pg. 90-93, 42-44 (advantages/disadvantages)
Friedland Reserving Ch. 4 - pg. 51-60

More Practice

CAS Fall 2018 – 15
CAS Spring 2018 Makeup – 2
CAS Spring 2018 Makeup – 15
CAS Spring 2018 – 16
CAS Fall 2017 – 16
CAS Spring 2017 – 3
CAS Fall 2016 – 16
CAS Spring 2016 – 4
CAS Spring 2016 – 15
CAS Fall 2015 – 16
CAS Spring 2015 – 15

Extraordinary Losses

Werner Ratemaking Ch. 6

Problem

Given the following loss information for a Homeowners book of business undergoing a ratemaking analysis:

Accident Year	Reported Losses	5 Largest Claims				
		1	2	3	4	5
2007	76,875,714	2,022,062	1,787,954	1,242,313	1,131,477	973,805
2008	60,916,907	2,005,466	1,862,951	1,543,141	1,455,848	1,185,086
2009	41,712,714	1,061,301	1,037,846	969,623	951,167	890,444
2010	63,515,325	1,716,360	1,633,779	1,204,630	1,124,469	1,028,317
2011	53,000,933	2,314,636	1,457,525	1,316,407	1,080,797	1,063,648
2012	44,869,468	1,674,273	999,249	915,594	908,755	870,177
2013	43,260,337	2,002,786	1,388,327	1,231,978	1,154,774	1,007,637
2014	61,967,802	1,422,431	1,340,170	1,264,830	1,161,002	1,120,878
2015	47,051,055	2,536,648	1,832,912	1,471,365	1,302,771	1,141,176
2016	43,054,299	1,614,409	1,035,201	1,020,831	961,403	928,812
2017	45,806,432	1,725,250	1,337,870	1,219,562	1,197,121	1,195,792
2018	46,417,329	1,496,189	1,190,257	932,890	850,882	844,945
2019	45,572,683	1,128,725	931,334	848,336	815,734	812,016
2020	46,415,447	1,298,035	1,255,713	1,175,925	1,161,828	1,151,466
2021	40,037,115	1,543,742	1,313,928	1,114,386	1,103,745	986,073

- Large Loss Threshold $1,250,000
- All losses are trended to future levels

a. Calculate the excess loss factor that should be applied to non-excess losses in the historical experience period of the ratemaking analysis.

b. Calculate the non-excess reported losses for accident years 2017-2021 loaded with a provision for shock losses that should be used for the ratemaking analysis.

Solution Recipe

Part a – Excess Loss Factor

1) **Calculate the excess and non-excess losses based on the large loss threshold for all accident years. If given a set of large losses, use SUMIF() and COUNTIF() formulas to easily calculate the excess losses. This should be done on trended losses.**

$$Losses\ Excess\ Threshold = Ground\text{-}Up\ Excess\ Losses - Threshold \times \#Excess\ Claims$$

$$Non\text{-}Excess\ Losses = Reported\ Losses - Losses\ Excess\ Threshold$$

$$Loss\ Excess\ Threshold_{2007} = 2{,}022{,}062 + 1{,}787{,}954 - 1{,}250{,}000 \times 2$$
$$= 1{,}310{,}016$$
$$Non\text{-}Excess\ Losses_{2007} = 76{,}875{,}714 - 1{,}310{,}016$$
$$= 75{,}565{,}698$$

In Excel
Number of Excess Claims = COUNTIF(Large Claims, ">"& Threshold)
Ground-Up Excess Losses = SUMIF(Large Claims, ">"& Threshold)

2) Calculate the excess ratio for each year as the ratio of the excess losses to non-excess losses.

$$Excess\ Ratio = \frac{Losses\ Excess\ Threshold}{Non\text{-}Excess\ Threshold}$$

$$Excess\ Ratio_{2007} = \frac{1{,}310{,}016}{75{,}565{,}698}$$
$$= 1.7\%$$

Accident Year	Reported Losses	Number of Excess Claims	Ground-Up Excess Losses	Losses Excess Threshold	Non-Excess Losses	Excess Ratio
2007	76,875,714	2	3,810,016	1,310,016	75,565,698	1.7%
2008	60,916,907	4	6,867,406	1,867,406	59,049,501	3.2%
2009	41,712,714	0	0	0	41,712,714	0.0%
2010	63,515,325	2	3,350,139	850,139	62,665,186	1.4%
2011	53,000,933	3	5,088,568	1,338,568	51,662,365	2.6%
2012	44,869,468	1	1,674,273	424,273	44,445,195	1.0%
2013	43,260,337	2	3,391,113	891,113	42,369,224	2.1%
2014	61,967,802	3	4,027,431	277,431	61,690,371	0.4%
2015	47,051,055	4	7,143,696	2,143,696	44,907,359	4.8%
2016	43,054,299	1	1,614,409	364,409	42,689,890	0.9%
2017	45,806,432	2	3,063,120	563,120	45,243,312	1.2%
2018	46,417,329	1	1,496,189	246,189	46,171,140	0.5%
2019	45,572,683	0	0	0	45,572,683	0.0%
2020	46,415,447	2	2,553,748	53,748	46,361,699	0.1%
2021	40,037,115	2	2,857,670	357,670	39,679,445	0.9%

3) Calculate the excess loss factor based on the weighted-average excess ratio that should be applied to non-excess losses for the ratemaking analysis.

$$Excess\ Loss\ Factor = 1 + \frac{\sum Losses\ Excess\ Threshold}{\sum Non\text{-}Excess\ Threshold}$$

$$Excess\ Loss\ Factor = 1 + \frac{1{,}310{,}016 + \cdots + 357{,}670}{75{,}565{,}698 + \cdots + 39{,}679{,}445}$$
$$= \boxed{1.014}$$

Part b – Adjust Losses for Large Individual Losses

4) **Apply the excess loss factor to the non-excess losses in the historical experience period to adjust losses for the average expected large loss amount.**

$$Losses\ Adjusted\ for\ Excess\ Losses = Non\text{-}Excess\ Losses \times Excess\ Loss\ Factor$$

$$Losses\ Adjusted\ for\ Excess\ Losses_{AY\ 2017} = 45{,}243{,}312 \times 1.014$$
$$= 45{,}888{,}230$$

Accident Year	Non-Excess Losses	Excess Loss Factor	**Losses Adjusted for Excess Losses**
2017	45,243,312	1.014	**45,888,230**
2018	46,171,140	1.014	**46,829,284**
2019	45,572,683	1.014	**46,222,296**
2020	46,361,699	1.014	**47,022,559**
2021	39,679,445	1.014	**40,245,053**

Discussion

There are two types of extraordinary losses:

- **Large Individual Losses (Shock Losses)** - These infrequent, large *individual* losses may occur with some regularity. Examples include a large multi-claimant liability claim or a total permanent disability of a young worker.
- **Catastrophe Losses** - These large *aggregate* losses result from unusually severe natural or man-made disasters, such as hurricanes and terrorist attacks, that lead to a significant number of claims.

In the ratemaking process, shock losses and catastrophe losses should be removed from the historical data or capped at a large loss threshold, as Werner shows. The actual excess losses are replaced with a longer-term average expected excess loss amount by loading in an **excess loss factor.**

Enough accident years should be used to produce a stable and reasonable estimated excess loss factor, but not so many years such that the historical data is irrelevant.

Note On Trending:
Reported losses should be trended to future levels before removing actual excess losses and loading in an excess loss factor. Alternatively, the large loss threshold can be indexed to older years to reflect the loss trend and then applied to ground-up untrended losses.

Setting the Large Loss Threshold

For shock losses, a large loss threshold is selected that best balances the goals of:

- Including as many losses as possible and
- Minimizing volatility in the ratemaking analysis

One approach is to set the threshold at a given percentile from the severity distribution, such as the 99th percentile. Alternatively, a threshold can be chosen based on a percentage of losses rather than claim counts.

Catastrophe Losses

Catastrophe losses should be removed from the actual historical loss data and replaced with an average expected catastrophe loss amount. This prevents the ratemaking analysis from being distorted by catastrophic losses.

There are two groups of catastrophe losses:

- **Non-Modeled Catastrophes** - These catastrophes affect a large number of claims, but occur with some regularity (e.g. hailstorms impacting automobile policies).
  - → Calculate a ratio of CAT losses to non-CAT losses over a longer experience period (e.g. 10-30 years, balancing stability and responsiveness). Multiply the ratio by non-CAT losses to get an expected CAT loss loading.
  - → Alternatively, a pure premium per exposure can be selected and loaded into the rates.
- **Modeled Catastrophes** - Catastrophic events that are extremely sporadic and cause high severity losses are modeled with sophisticated Catastrophe models (e.g. for hurricanes and earthquakes). These models estimate an expected annual loss for the type of catastrophe based on the insurer's exposure (book of business).
  - → Add the model's catastrophe loss provision to the non-catastrophe losses

Reinsurance

Reinsurance is insurance for primary insurers and helps them manage some of their risk. There are two types of reinsurance:

- **Proportional Reinsurance** - Premium and losses are ceded to the reinsurer with the same proportion (e.g. a 30% quota share reinsurance treaty)
  - → Generally, this doesn't need to be explicitly considered for ratemaking.
- **Non-Proportional Reinsurance** - The insurer pays a portion of premium to the reinsurer (**reinsurance cost**) and the reinsurer assumes a predefined portion of losses (**reinsurance recoveries**) from the insurer (e.g. an excess of loss treaty covering 50% of losses in the $15M to $30M loss layer)
  - → Take into account by reducing projected losses by the expected non-proportional loss recoveries and reducing the total premium by the cost of reinsurance.
  - → Alternatively, add the net cost of non-proportional reinsurance (cost of reinsurance minus expected recoveries) as an expense item in the rate indication.

CBT Spreadsheet Tips

Use the **COUNTIF(range, criteria)** formula to count the number of excess claims and the **SUMIF(range, criteria)** to calculate the ground-up excess losses that exceed the large loss threshold.

Important:
The setup of the COUNTIF( ) and SUMIF( ) formulas is tricky because they use a logical operator. The correct criteria is: **">"& Threshold.**

For example, the full formula is:

COUNTIF(Array of Claim Values , ">"& Large Loss Threshold)

Source

Werner Ratemaking Ch. 6 - pg. 95-99

More Practice

CAS Spring 2019 – 5
CAS Spring 2018 Makeup – 3
CAS Spring 2018 – 5
CAS Spring 2015 – 8
CAS Fall 2013 – 5