Rising Fellow

2026

Exam 5

Cookbook

BASIC RATEMAKING AND RESERVING

60+ **Step-by-Step Recipes**
to Solve CAS Calculation Problems

Exam 5 Cookbook

Fall 2025 Sitting

Rising Fellow

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Table of Contents

Introduction

The purpose of the Exam 5 Cookbook is to prepare you to confidently answer calculation-based problems on exam day without wasting time trying to "think through" a problem-solving approach before typing the solution. This is the same approach I used to help pass my upper-level CAS exams on the first sittings to earn my FCAS.

Since the 2016 sitting, 1,000+ actuaries have used the Exam 5- 9 Cookbooks and Online Courses to help them pass their exams and earn their FCAS. We want to see you be one of them.

Our goal with Rising Fellow is to help you prepare for the exam with less frustration so that you have your best exam sitting yet!

The Structure

The Exam 5 Cookbook goes through the different calculation-based problem-types that I believe are reasonably testable based on the syllabus. By exam day, you should know how to solve each one.

Inside, you'll find a separate section for each testable problem-type. Each section has the following structure:

Original Practice Problem

Each section has an original practice problem that demonstrates the problem-type. I wrote these based off of the syllabus papers to have a similar difficulty-level and style to what you might see on an exam.

Solution Recipe

The solution recipe solves the practice problem from start to finish and shows the step-by-step approach you should take to answer a similar problem. For each step, you'll see:

- The description for what to do in the step
- The formula(s) necessary for the step
- The formula(s) translated from symbolic notation to plain-English
- Calculations for the step to solve the example problem

Discussion

Each section includes discussion to add clarity and more context. The discussion also covers underlying concepts that might come up on a part b or part c essay question.

For many problems, I point out potential "twists" that could show up on the exam that would make an exam problem more difficult. Since you've taken actuarial exams up to this point, you know that straightforward exam problems are more the exception than the rule.

Exam Spreadsheet Tips

This new section provides Excel formulas and tips for how to solve a problem more efficiently in the computer-based testing (CBT) PearsonVue spreadsheet environment. There are many types of problems where setting up your solution intelligently and taking advantage of the spreadsheet capabilities such as SUMIF( ), COUNTIF( ), and array formulas, will save you valuable time on the exam.

Source

Each section references the pages in the syllabus reading that you can cross-reference for more information and details. Make sure to check the syllabus section for more context if you get stuck on a problem or to see how the author discusses the concepts.

More Practice

Here, you'll see references to past CAS problems. You'll find this helpful especially closer to the exam if there are particular types of problems that you are struggling with. This section includes references to CAS problems from the 2015-2019 exams.

How to Best Use the Exam 5 Cookbook

Below is a suggested guide for how you can incorporate the Exam 5 Cookbook in your own study schedule along with the syllabus material and a typical study manual. This is the general approach that I used when I took my fellowship exams.

For each of those exams I had a main study manual as well as the Exam Cookbook, which I built out while I studied for the exam (but you don't need to waste time doing that part!)

First pass through the syllabus

While you're reading a particular paper in the syllabus and your main study manual to learn the material, use the Exam 5 Cookbook to clearly identify what problem-types you need to know from the paper. Study the steps in the solution recipe to learn how to solve the problem-types. Make sure to do some practice problems as you go through the syllabus. This will help you learn faster.

Second pass through the syllabus

Review the steps for the problem-types and make sure you have an intuitive understanding of how to solve the problems. Start working the past CAS problems.

The first level of understanding is to be able to follow the recipe and understand the steps and calculations.

The next level of understanding is to be able to recall and apply the steps to solve a problem without relying on study material. During your second pass, focus on building this deeper level of understanding.

Review and Practice Problems (around 6 weeks to 2 weeks before the exam)

At this point you should have a good understanding of the syllabus and how to use the recipe steps to systematically solve the different calculation problems. During this period, you should be doing lots of problems across the syllabus and targeting problem-types that you are finding particularly challenging.
By the end of this phase, you might not have all the formulas memorized, but you should know all the steps and how to apply them to solve problems without needing to think too much before beginning to write the solution.

During this phase, make sure to focus on the types of problems and concepts that you're weak at. This may require some struggle, but struggling with some of the challenging problems will help you master these concepts.

You also should continue building your understanding of the concepts and preparing for essay and more complicated integrative questions. I found it helpful to create flashcards from the papers as well as to re-read sections of the syllabus papers that appear to be likely sources of essay problems.

Final Weeks

In the final weeks, focus on taking practice exams to see problems from the entire syllabus. When taking practice exams, work on your exam strategy to make sure you're able to finish the exam and maximize your points.

Prepare for essay problems in the final weeks by using flashcards to make sure that you know all the details necessary. An approach I found helpful is to say flashcards out loud and to explain the flashcard response in my own words as if I were teaching someone. It sounds weird, but it is a much more efficient way to learn and memorize than simply scanning the front and back of the flashcard.

Prepare for calculation problems by reviewing the recipes in the Exam 5 Cookbook in a similar fashion to how you use flashcards for essay problems. Using this approach on my fellowship exams, I was able to rapidly review the steps and formulas for how to solve each problem-type that might show up on the exam. This was a huge benefit and gave me a lot of confidence going into the exam.

Exam Day

I used the original Exam Cookbooks together with a traditional study manual using the approach above to take my fellowship exams. On exam day, for almost every calculation problem I was able to start writing the solution without wasting time trying to think through how to solve the problem. I had an intuitive understanding of how to solve each of the problems following the step-by-step recipes.

If you follow this approach, you should be able to develop a similar level of understanding and confidence going into the exam room.

Excel Version for Computer-Based Testing Preparation

For each recipe, there is an accompanying Excel version. Make sure to review those so that you know how to solve problems in the spreadsheet format. The Exam Spreadsheet Tips sections and the Excel version showing the formulas and setup for the spreadsheet solution will help you understand how to solve exam problems in the PearsonVue spreadsheet environment.

Exam-Related Questions

If you purchased the Exam 5 Online Course, **please post your question in the Exam 5 course forum**. We answer exam-related questions through the forum for people in the online course.

Errata

I always hated seeing errors in study manuals when I studied for exams, so I make every effort to ensure the study materials are accurate. Nevertheless, there may still be some errors in the final version, so I keep an updated errata. Please make sure to check it regularly for any fixes. The link is below:

https://risingfellow.com/errata

If you find any errors, please send me a message using the contact form on the Errata page so that I can make a correction.

Feedback

I am always working to improve the Exam 5 Cookbook and the rest of the Rising Fellow study material. Please send me an email to exam5@RisingFellow.com if you have feedback about any of the following:

- Recipes or sections that are confusing or could be improved
- New recipes I should include in future versions
- Better ways you've found to solve a problem-type in a spreadsheet
- Any comments or other feedback you have

Reviews

If you find the Exam 5 Cookbook helpful this sitting, please leave us a review and let us know how it helped you prepare for the exam. Other actuaries look at reviews to help decide what study material to buy and it's helpful for us to hear feedback from actuaries like you so that we can better understand what's working and what can be improved.

You can leave us a review by sending us an email to info@RisingFellow.com. Thank you!

Good luck as you start studying and I hope this will be your best sitting yet.

Aggregating Exposures

Werner Ratemaking Ch. 4

Problem

Given the following automobile policy information, at policy inception, for an insurance company that writes 6-month and 12-month policies:

Policy	Effective Date	Expiration Date	# Autos on policy
A	10/1/20	3/31/21	1
B	4/1/21	3/31/22	3
C	7/1/21	6/30/22	2
D	8/1/21	1/31/22	2
E	1/1/22	6/30/22	1
F	4/1/22	3/31/23	2

The exposure base is car-years and the probability of a claim is evenly distributed through the year.

The following policy changes occurred during the policy effective period:

- Policy B: Canceled on 1/1/22
- Policy C: One of the autos was removed from the policy on 4/1/22
- Policy E: Canceled on 3/31/22

Assume there are no other policies written between 2020 and 2022

a. Calculate the 2021 and 2022 written exposures as of 12/31/22 both aggregated by calendar year and policy year.

b. Calculate the 2021 and 2022 earned exposures as of 12/31/22 both aggregated by calendar year and policy year.

c. Calculate the 2021 and 2022 unearned exposures as of 12/31/22 both aggregated by calendar year and policy year.

d. Calculate the in-force exposures as of 2/15/22 with insured units defined as:
   - Automobile units exposed to loss
   - Written exposures

Solution Recipe

Part a – Written Exposures

Written exposures - The total exposures on policies written during the time period, based on their *effective date*.

1) **For *Calendar Year*, exposures are aggregated by *transaction date*. When a policy is canceled, a negative partial exposure for the portion of the policy canceled is recorded in the calendar year of the *transaction date.***

$$Written\ Exposures = \#Exposures \times Policy\ Length$$

$$Written\ Exposures_{B,CY\ 2021} = 3 \times 1 = 3.0$$

Note: Exposure base is car-years, so the policy length is the term-length in years (0.5 years, 1 year, etc.).

	Written Exposures		
Policy	CY 2021	CY 2022	Comments
A	-	-	Effective date in 2020
B	3.00	-0.75	Canceled in CY 2022 with 3 months remaining
C	2.00	-0.25	1 auto removed with 3 months remaining
D	1.00	-	6-month policy, so each auto is worth 0.5 car-years
E	-	0.25	6-month policy written 2022, canceled halfway through
F	-	2.00	Annual policy written 2022
Total	6.00	1.25	

Note: For policy C, an alternative way to calculate the CY 2022 written exposure is that 3 months of the original policy is canceled (-2 * 3/12) and the newly modified policy has 3 months remaining with 1 autos (+1 * 3/12). The sum is -0.25 car-years.

2) **For *Policy Year*, exposures are aggregated by *policy effective date*. When a policy is canceled, a negative partial exposure for the portion of the policy canceled is recorded in the policy year of the *policy's effective date.***

$$Written\ Exposures_{B,PY\ 2021} = 3 \times 1 - 3 \times \frac{3}{12} = 2.25$$

$$Written\ Exposures_{E,PY\ 2022} = 1 \times 0.5 - 1 \times 0.5 \times \frac{3}{6} = 0.25$$

	Written Exposures		
Policy	PY 2021	PY 2022	Comments
A	-	-	
B	2.25	-	Cancelation contributes to PY 2021 (original effective date)
C	1.75	-	Modification contributes to PY 2021 (original policy effective date)
D	1.00	-	
E	-	0.25	
F	-	2.00	
Total	5.00	2.25	

Part b – Earned exposures

Earned exposures - The portion of written exposures for which coverage has already been provided as of a point of time.

3) **For *Calendar Year*, earned exposure is based on the portion of the policy term used up aggregated by calendar year. This assumes the probability of a claim is evenly distributed throughout the year (an even earning pattern).**

$$Earned\ Exposures = \#Exposures \times \%Earned$$

$$Earned\ Exposures_{B,CY\ 2021} = 3 \times \frac{9}{12} = 2.25$$

Earned Exposures

Policy	CY 2021	CY 2022	Comments
A	0.25	-	Written in 2020, but in effect for 3 months of 2021
B	2.25	-	Canceled on 1/1/22
C	1.00	0.75	1 auto removed with 3 months remaining
D	0.83	0.17	2 autos earned (5 months in 2021 and 1 month in 2022)
E	-	0.25	Canceled on 3/31/22, so only 3-months earned
F	-	1.50	9 months earned in 2022
Total	4.33	2.67	

4) **For *Policy Year*, earned exposure is based on the portion of the policy term used up *by the as-of date* and contributes to the policy year corresponding to the policy effective date.**

$$Earned\ Exposures_{F,PY\ 2022} = 2 \times \frac{9}{12} = 1.50$$

Earned Exposures

Policy	PY 2021	PY 2022	Comments
A	-	-	Policy effective date is in PY 2020
B	2.25	-	Policies B, C and D contribute to PY 2021
C	1.75	-	
D	1.00	-	
E	-	0.25	Policies E and F contribute to PY 2021
F	-	1.50	9 months earned by the as of date of 12/31/22
Total	5.00	1.75	

Part c – Unearned exposures

Unearned exposures - The portion of written exposures for which coverage has ***not yet*** been provided. For a single policy, it's the difference between written and earned exposure at a point in time.

5) **For *Calendar Year*, unearned exposure is the difference between written exposure and earned exposure for a given calendar year *plus* unearned exposures as of the beginning of the calendar year.**

$$CY\ Unearned_{\ End\ CY} = CY\ Written - CY\ Earned + CY\ Unearned_{Beg.\ CY}$$

$$CY\ Unearned_{\ CY\ 2021} = 6.0 - 4.33 + 0.25$$
$$= 1.92$$

	Unearned Exposures		
	CY 2020	CY 2021	CY 2022
Total	0.25	1.92	0.50

Note: To calculate CY 2021 Unearned Exposures, we need the Unearned Exposures at the beginning of the year, the CY 2020 Unearned Exposures. By the end of CY 2020, the only unearned exposure is from the 3 months remaining of Policy A which is earned in CY 2021.

6) **For *Policy Year*, unearned exposure is the difference between written exposure and earned exposure for a given policy year.**

$$Written\ Exposures = Earned\ Exposures + Unearned\ Exposures$$

$$Unearned_{\ PY\ 2022} = 2.25 - 1.75$$
$$= 0.50$$

	Unearned Exposures		
	PY 2021	PY 2022	Comments
Total	-	0.50	*As of 12/31/22*, PY 2021 is fully earned (written = earned)

Note: Policy A is effective in 2020, so it contributes to PY 2020 and can be ignored. *As of 12/31/22*, PY 2021 is fully earned (written exposures = earned exposures), so unearned exposure is zero. For PY 2022, the only unearned exposure *as of 12/31/22* is 3 months remaining of policy F with two exposures.

Part d – In-force exposures

In-force exposures - The number of insured units exposed to claims at a given point in time for policies that are in force.

7) **Sum up the number of exposed units on policies that are in-force as of the given date. Include policies where the "as of" date is after the policy effective date and before expiration or cancelation date.**

Policy	In-Force 2/15/22	Comments
A	-	Expired
B	-	Canceled before 2/15/22
C	2.00	1 auto was removed on 4/1/22, after 2/15/22
D	-	Expired
E	1.00	
F	-	Written *after* 2/15/22
Total	3.00	

8) **For in-force "written exposures", sum up the written exposures on policies that are in-force *as of the given date.***

Policy	In-Force 2/15/22	Comments
A	-	Expired
B	-	
C	2.00	At 2/15/22 policy C has 2 written *car-years*
D	-	Expired
E	0.50	6-month policy, so each auto is 0.5 written car-years
F	-	Written *after* 2/15/22
Total	2.50	

Discussion

Questions about aggregating exposures are straight-forward but it's easy to overlook some of the details and make a mistake.

There are two main ways to aggregate exposures, by policy year and by calendar (accident) year.

- Policy year exposures are aggregated according to the original policy effective date
- Calendar year exposures are aggregated based on the transaction date.

A key thing to remember is that at the end of a year, calendar year exposures are fixed. In contrast, policy year exposures aren't fixed until all policies written during the year expire (24 months after the start of the policy year for annual policies).

Possible Problem Modifications

- Uneven earning pattern (e.g. for watercraft where most claims occur in summer)

    - Calculate earned exposures based on the % of the exposure earning pattern used
- Semi-annual policies
    - Each policy counts as 50% of a written exposure (0.5 years)
- Policy cancellations and modifications
    - **Calendar Year:** The written exposure for the cancellation/modification contributes to the calendar year of the *transaction date*
    - **Policy Year:** The written exposure for the cancellation/modification contributes to the policy year of the *original policy effective date*

Aggregating Premium for Individual Policies

Aggregating Premium for Individual policies is done almost entirely the same. The only real difference is that premium is used instead of exposures. Also, for in-force premium, it's important to use the full-term premium for the policy that is in-force at the as of date.

Criteria for Exposure Bases

Proportional to Expected Loss - The exposure base should be directly proportional to loss.

Practical - The exposure base should be well-defined, objective and relatively easy/inexpensive to measure and verify.

Historical Precedence - Because of the difficulties of implementing a new exposure base (large premium swings for insureds, changing the rating algorithm, and requiring data adjustments for future analyses), a good exposure base should have historical precedence.

Source

Werner Ratemaking Ch. 4 – pg. 51-61

More Practice

CAS Fall 2019 – 2
CAS Spring 2019 – 1
CAS Spring 2018 – 1
CAS Spring 2018 Makeup – 1
CAS Fall 2018 – 2
CAS Fall 2017 – 3
CAS Spring 2017 – 1
CAS Spring 2016 – 2
CAS Fall 2014 – 2

Aggregating Blocks of Exposure

Werner Ratemaking Ch. 4

Problem

Given the following written exposures for a company summarized by month as of 3/31/24:

Month	Written Exposures
Jan-23	192
Feb-23	192
Mar-23	184
Apr-23	190
May-23	191
Jun-23	185
Jul-23	191
Aug-23	195
Sep-23	193
Oct-23	191
Nov-23	190
Dec-23	195
Jan-24	204
Feb-24	203
Mar-24	200

- Exposures are written uniformly during each month
- All policies have an annual policy term
- Assume no policies were written prior to 1/1/23.

Calculate the earned exposures for calendar year 2023 and policy year 2023 as of 3/31/24.

Solution Recipe

1) **Calculate earned exposures for a block of policies *as if* the entire block was written on the mid-point of the time period. This is the assumed effective date. Determine the % earned as the portion of the year between the assumed effective date and the end of the calendar year.**

$$\boxed{Assumed\ Effective\ Date = Time\ Period\ Mid\text{-}Point}$$

$$CY\ Earned\ Exposures = \#Exposures \times \%Earned$$

$$CY\ Earned\ Premium_{Jan\text{-}23} = 192 \times 96\%$$
$$= 184$$

Calendar Year and Month	Assumed Effective Date	Written Exposures	Calendar Year 2023 % Earned	Earned Exposures
Jan-23	1/15/23	192	96%	184.00
Feb-23	2/15/23	192	88%	168.00
Mar-23	3/15/23	184	79%	145.67
Apr-23	4/15/23	190	71%	134.58
May-23	5/15/23	191	63%	119.38
Jun-23	6/15/23	185	54%	100.21
Jul-23	7/15/23	191	46%	87.54
Aug-23	8/15/23	195	38%	73.13
Sep-23	9/15/23	193	29%	56.29
Oct-23	10/15/23	191	21%	39.79
Nov-23	11/15/23	190	13%	23.75
Dec-23	12/15/23	195	4%	8.12
Total				**1,140**

Note: Because the problem assumes no policies are written before 1/1/23, we don't need to calculate the amount of calendar year 2023 earned premium from policies written in 2022.

2) For policy year, calculate earned exposures for a block of policies between the mid-point of the time period and the as-of date.

$$\boxed{PY\ Earned\ Exposures = \#Exposures \times \%Earned\ by\ as\text{-}of\text{-}date}$$

$$PY\ Earned\ Premium_{Jan\text{-}23} = 192 \times 100\%$$
$$= 192$$

Policy Year and Month	Assumed Effective Date	Written Exposures	Policy Year 2023 % Earned	Earned Exposures
Jan-23	1/15/23	192	100%	192.00
Feb-23	2/15/23	192	100%	192.00
Mar-23	3/15/23	184	100%	184.00
Apr-23	4/15/23	190	96%	182.08
May-23	5/15/23	191	88%	167.13
Jun-23	6/15/23	185	79%	146.46
Jul-23	7/15/23	191	71%	135.29
Aug-23	8/15/23	195	63%	121.88
Sep-23	9/15/23	193	54%	104.54
Oct-23	10/15/23	191	46%	87.54
Nov-23	11/15/23	190	38%	71.25
Dec-23	12/15/23	195	29%	56.88
Total				**1,641**

Note:
Mar-23 policies and prior are 100% earned *by 3/31/24.*
Apr-23 policies have an assumed effective date of 4/15/23 and have a % earned of (= 23 / 24) by 3/31/24.

Discussion

As long as policies are uniformly written during each time period, this method is a good approximation. The longer the time periods (quarters or years), the less likely that this assumption is appropriate.

This approach is necessary if data is summarized. With greater computational power these days, it's more realistic to aggregate exposures looking at the individual policy data. That would be more accurate than using blocks of exposures.

Note that aggregating premium using blocks of policies is done the same way as above, just with premium numbers.

Exam Spreadsheet Tips

For the %Earned, start with the first %Earned (=23/24 for January 2023) and subtract 1/12 for each subsequent value. This is much quicker than manually typing in 23/24, 21/24, 19/24… for the whole series.

Source

Werner Ratemaking - pg. 60-61

More Practice

CAS Fall 2016 – 1
CAS Spring 2015 – 3

Uneven Earning Pattern

Werner Ratemaking Ch. 5

Problem

Given the following for an insurer writing Recreational Vehicle (RV) insurance as of 3/31/2024:

Calendar Year and Quarter	Written Premium
2022 Q1	182
2022 Q2	765
2022 Q3	707
2022 Q4	155
2023 Q1	208
2023 Q2	842
2023 Q3	735
2023 Q4	98

Earning Pattern

Quarter	%Earned
Q1	10%
Q2	40%
Q3	40%
Q4	10%

- Policies are written uniformly during each quarter
- There are no policy cancellations or modifications
- All policies are annual
- The company began writing the RV line of business on 1/1/22

a. Calculate the 2022 and 2023 **earned premium** as of 3/31/2024 both aggregated by:
   i. Calendar year and quarter
   ii. Policy year and quarter

b. Calculate the 2022 and 2023 **unearned premium** as of 3/31/2024 both aggregated by:
   i. Calendar year and quarter
   ii. Policy year and quarter

Solution Recipe

Part a – Earned premium with uneven earning pattern

1) For ***calendar quarter***, **calculate the earned premium as the exposed written premium that is earned during the quarter based on the earning pattern. With blocks of premium (or exposures), treat the policies as if they were written at the mid-point of the time period.**

$$CY\ Earned\ Premium = \sum_{\substack{Exposed \\ Policies}} Written\ Premium \times \%Earned\ in\ time\ period$$

Calendar Quarter Earned Premium

Calendar Year and Quarter	Average Written Date	Written Premium	Earned Premium
2022 Q1	2/15/22	182	9.10
2022 Q2	5/15/22	765	225.80
2022 Q3	8/15/22	707	520.20
2022 Q4	11/15/22	155	173.15
2023 Q1	2/15/23	208	182.20
2023 Q2	5/15/23	842	749.40
2023 Q3	8/15/23	735	770.40
2023 Q4	11/15/23	98	191.15

Notes:

- **2022 Q1** - 2022 Q1 written premium is exposed for 50% of Q1.
- **2022 Q2** - 2022 Q1 written premium is exposed for 100% of Q2 and 2022 Q2 written premium is exposed for 50% of Q2.
- **2022 Q3** - 2022 Q1 & Q2 written premium is exposed for 100% of Q3 and 2022 Q3 written premium is exposed for 50% of Q3.

2) **For *policy quarter*, calculate earned premium as the written premium that is earned *by the as of* date based on the earning pattern *for policies effective during the quarter*. With blocks of premium (or exposures), treat the policies as if they were written at the mid-point of the time period.**

$$PY\ Earned\ Premium = PY\ Written\ Premium \times \%Earned\ by\ as\ of\ date$$

Policy Quarter Earned Premium as of 3/31/24

Policy Year and Quarter	Average Written Date	Written Premium	Earned Premium
2022 Q1	2/15/22	182	182
2022 Q2	5/15/22	765	765
2022 Q3	8/15/22	707	707
2022 Q4	11/15/22	155	155
2023 Q1	2/15/23	208	208
2023 Q2	5/15/23	842	673.60
2023 Q3	8/15/23	735	294.00
2023 Q4	11/15/23	98	14.70

Notes:

- By 3/31/24, all policies written on or before 3/31/23 have been fully earned and expired.
- **2023 Q2** - By 3/31/24, this policy block has been exposed for 50% of 2023 Q2 and 100% of 2023 Q3, Q4 and 2024 Q1.
- **2023 Q3** - By 3/31/24, this policy block has been exposed for 50% of 2023 Q3 and 100% of 2023 Q4 and 2024 Q1.

Part b – Unearned premium with uneven earning pattern

3) **For *Calendar Year*, unearned premium is the difference between written premium and earned premium for a given calendar year plus unearned premium as of the beginning of the calendar year.**

$$CY\ Unearned\ Prem_{End\ CY} = CY\ Written\ Prem - CY\ Earned\ Prem + CY\ Unearned\ Prem_{Beg.\ CY}$$

Calendar Year	Unearned Premium
2022	881
2023	871

Note:
The company started writing RV policies on 1/1/22, so there is no unearned premium at the beginning of CY 2022.

4) **For Policy Year, unearned premium is the difference between written premium and earned premium for a given policy year evaluated at the as of date.**

$$Written\ Exposures = Earned\ Exposures + Unearned\ Exposures$$

Policy Year	Unearned Premium	
2022	0	← By 3/31/24, all 2022 policies have been fully earned
2023	693	

Discussion

The text doesn't show any numerical examples, but does discuss this concept and it has been asked on past exams. The approach is the same for both earned exposures and earned premium using an uneven earning pattern.

Lines of business like recreational vehicle, boat owners, and warranty don't have an equal probability of loss over the whole policy term. Exposures and premiums for lines of business with seasonal fluctuations (like RV and boat owners) will have an uneven earning pattern. In the problem here, recreational vehicles are most susceptible to loss over Q2 and Q3.

With an uneven earning pattern, you must calculate the earned exposures and premium using the earning pattern, not using the percent of the policy term length exposed.

Source

Werner Ratemaking Ch. 5 - pg. 55-57, 68-70

More Practice

CAS Fall 2019 – 1

Extension of Exposures

Werner Ratemaking Ch. 5

Problem

An insurance company writes annual policies and has the following historical policies in the experience period grouped by rating characteristics for an indicated rate review as of 3/31/23:

- Policies are written uniformly throughout each year

Policy Effective Dates	Number of Policies	Number of Exposures	Territory	Class
Jan 1, 2021 - Dec 31, 2021	129	774	A	X
	74	296	A	Y
	68	476	B	X
	31	155	B	Y
Jan 1, 2022 - Dec 31, 2022	133	798	A	X
	78	390	A	Y
	76	456	B	X
	35	210	B	Y

The current rating algorithm as of 3/31/23 is:

$$Premium = Exposure \times Rate\ per\ Exposure \times Territory\ Factor \times Class\ Factor + Policy\ Fee$$

- Base rate per exposure $350
- Current policy fee $200 per policy

Territory	Rating Factor
A	1.00
B	1.43

Class	Rating Factor
X	1.00
Y	0.78

Calculate the on-level earned premium for Calendar Year 2022 using the extension of exposures method.

Solution Recipe

1) **Rerate all policies in the experience period *using the current rating algorithm* to restate the historical premium at current rate levels. Make sure to apply the correct class factors to each policy.**

$$Premium = Exposure \times Rate\ per\ Exposure \times \prod_{i=1}^{n} Class\ Factors_i + Policy\ Fee$$

$$Premium_{2021,Terr\ A,Class\ Y} = 296 \times \$350 \times 1.00 \times 0.78 + \$200 \times 74$$
$$= 95{,}608$$

In Excel
Use VLOOKUP( ) to look up the correct class rating factors for a policy group:
Class Factor$_i$ = VLOOKUP(Class$_i$, rating factor table, rating factor column #, FALSE)

Effective Year	Territory	Class	Premium at CRL
2021	A	X	296,700
2021	A	Y	95,608
2021	B	X	251,838
2021	B	Y	66,710
2022	A	X	305,900
2022	A	Y	122,070
2022	B	X	243,428
2022	B	Y	88,982

2) Aggregate the on-level written premium by calendar year based on the policy effective date.

$$Premium_{2021} = 296{,}700 + 95{,}608 + 251{,}838 + 66{,}710$$
$$= 710{,}856$$

In Excel
Written Premium = SUMIF(effective year, calendar year criteria, premium at CRL)

Calendar Year	Written Premium at CRL
2021	710,856
2022	760,380

3) Determine the percentage of each policy year that is earned in the calendar year and calculate the aggregate on-level earned premium.

$$CY\ Earned\ Premium = \sum Written\ Premium \times \%Earned\ in\ time\ period\ Avg\ Factor$$

$$CY\ Earned\ Prem_{2022} = 710{,}856 \times 50\% + 760{,}380 \times 50\%$$
$$= \boxed{735{,}618}$$

Effective Year	% in CY 2022
2021	50%
2022	50%

Note:
The average written date for the block of policies effective in 2021 is 7/1/21. Since these are annual policies, 50% is earned in calendar year 2021 and 50% is earned in calendar year 2022.

Discussion

The key idea of the extension of exposures method is to rerate all policies written in the historical experience period with the *current rating algorithm*. This restates the historical premium to current rates.

The extension of exposures method has the following advantages and disadvantages:

Advantages	Disadvantages
• Most accurate current rate level method	• Requires detailed data - rating characteristics for all policies in the historical period • Difficult to determine what subjective debits/credits would be applied under current rating guidelines

Possible Problem Modifications

- Different rating algorithm used
  - The problem should clearly state the rating algorithm
- Given rating factors at different effective dates
  - Make sure to use the latest rating factors
- 6-month policies used
  - Make sure to apply the rating algorithm and aggregate earned premium correctly

Exam Spreadsheet Tips

For this type of problem, using VLOOKUP( ) to look up rating factors for a given rating characteristic and SUMIF( ) to aggregate premium by year may be helpful on the exam. Of course, you can always reference rating factors and aggregate premium by year manually if it's a very simple problem.

SUMPRODUCT(array 1, array 2, ...) makes it easy to help calculate weighted averages or the sum of a product of multiple arrays in one step.

Source

Werner Ratemaking Ch. 5 – pg. 72-73

More Practice

CAS Fall 2018 – 3
CAS Fall 2015 – 1
CAS Fall 2014 – 3
CAS Fall 2013 – 2

Parallelogram Method

Werner Ratemaking Ch. 5

Problem

Given the following historical premium and rate changes for an insurer:

Historical Premium

Calendar Year	Earned Premium
2020	870,000
2021	935,000
2022	980,000

Rate Change History

Effective Date	Overall Average Rate Change
4/1/20	3%
7/1/21	12%
7/1/22	-2%

- All policies are annual-term policies
- Policies are written uniformly over the year

a. Calculate the earned premium at current rate level for Calendar Years 2020-2022.

b. Calculate the appropriate on-level factors for Policy Years 2020-2022.

Solution Recipe

Part a – Calendar Year Parallelogram Method

1) Determine the rate level groups based on the rate change history and calculate the cumulative rate level index for each group.

$$CRL\ Index_1 = 1.00$$

$$CRL\ Index_i = CRL\ Index_{i-1} \times (1 + rate\ change_i)$$

$$CRL\ Index_2 = 1.00 \times (1 + 0.03)$$
$$= 1.030$$

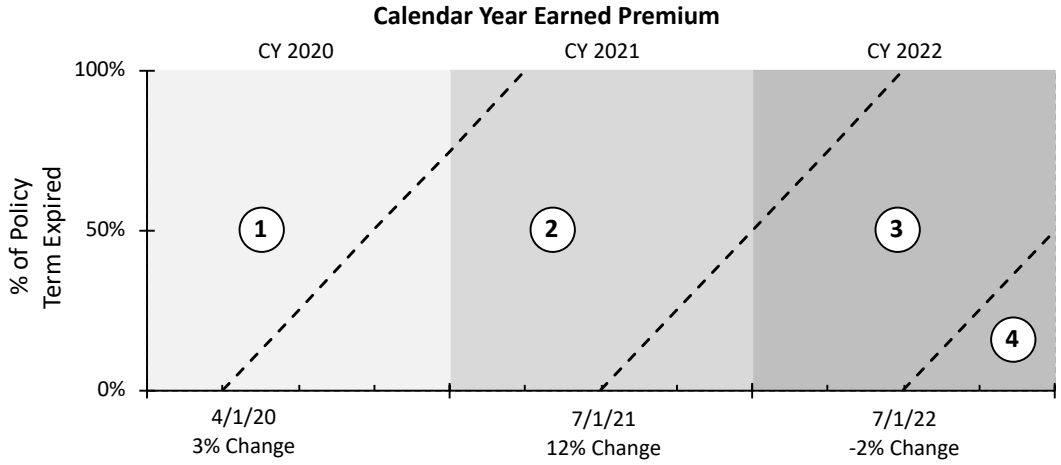
Rate Level Group	Effective Date	Cum. Rate Level Index
1	Initial	1.000
2	4/1/20	1.030
3	7/1/21	1.154
4	7/1/22	1.131

2) **Calculate the portion of each calendar year's earned premium that was earned during each rate level group. Use a calendar year earned premium diagram with the rate changes to help with this.**

$$Area\ of\ a\ triangle = {}^{1}/_{2} \times base \times height$$

$$Area\ of\ a\ parallelogram = base \times height$$

$$Area\ of\ a\ trapezoid = {}^{1}/_{2} \times (base_1 + base_2) \times height$$



3) **Calculate the weighted average cumulative rate level index for each year.**

$$\boxed{Average\ CRL\ Index = \sum \%EP\ in\ Rate\ Level\ Group_i \times CRL\ Index_i}$$

$$\%\ EP\ in\ Rate\ Level\ Group_{CY\ 2020,Group\ 1} = 1 - 0.5 \times 0.75^2$$
$$= 71.88\%$$

$$Average\ CRL\ Index_{CY\ 2020} = 71.88\% \times 1.000 + 28.13\% \times 1.030$$
$$= 1.0084$$

In Excel

Average CRL Index = SUMPRODUCT(%EP in rate level groups , CRL Indices of rate level groups)

Calendar Year	Portion of EP in Each Rate Level Group 1	2	3	4	Average CRL Index
2020	71.88%	28.13%			1.0084
2021	3.13%	84.38%	12.50%		1.0445
2022		12.50%	75.00%	12.50%	1.1353
CRL Index	1.000	1.030	1.154	1.131	

Note: As a quick reasonability check, the sum of the portion of EP in *each row* should be 100%.

4) **Calculate the on-level factor for each year as the cumulative rate level index for the current group divided by the weighted average rate level index for the year.**

$$On\text{-}Level\ Factor = \frac{Current\ CRL\ Index}{Average\ CRL\ Index}$$

$$On\text{-}Level\ Factor_{2020} = \frac{1.131}{1.0084} = 1.1211$$

Calendar Year	On-Level Factor
2020	1.1211
2021	1.0823
2022	0.9958

Note: Make sure to use the CRL Index for the current *rate level group* (1.131 for group 4 here), not the weighted average CRL index for the latest historical year.

5) **Apply the on-level factor to the earned premium for the appropriate year to calculate the on-level earned premium.**

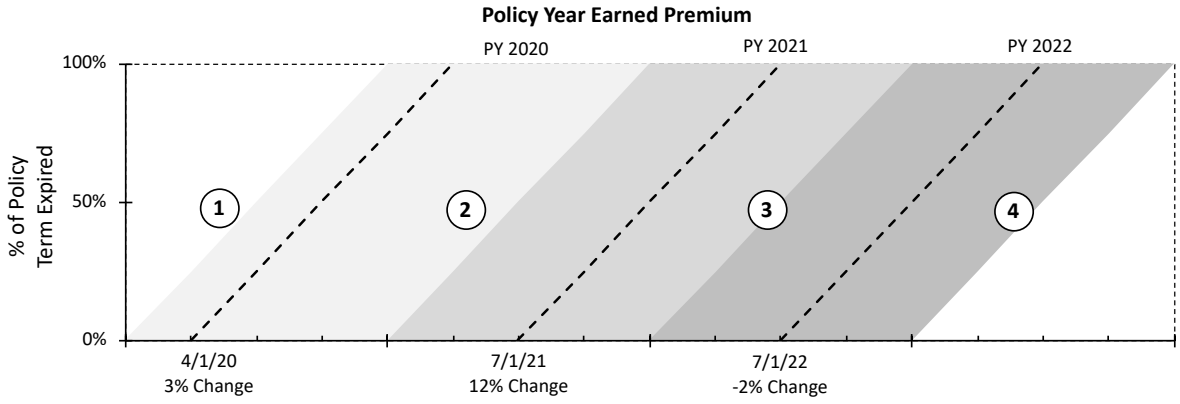
$$On\text{-}Level\ EP = On\text{-}Level\ Factor \times Earned\ Premium$$

$$On\text{-}Level\ EP_{2020} = 1.1211 \times 870{,}000 = 975{,}330$$

Calendar Year	On-Level Earned Prem
2020	975,330
2021	1,011,997
2022	975,910

Part b – Policy Year Parallelogram Method

6) **Redo steps 2-4 to get the on-level factors for policy years. The key difference is that the portion of each policy year's earned premium in the rate level groups will be different than for calendar year. Use a policy year earned premium diagram to help.**



$\%\ EP\ in\ Rate\ Level\ Group_{PY\ 2020,Group\ 1} = 1 \times 0.25 = 25\%$

$\text{Average CRL Index}_{\text{CY 2020}} = 25\% \times 1.000 + 75\% \times 1.030 = 1.0225$

$On\text{-}Level\ Factor_{2020} = \frac{1.131}{1.0225} = \boxed{1.1057}$

Policy Year	Portion of EP in Each Rate Level Group 1	2	3	4	Average CRL Index	**On-Level Factor**
2020	25%	75%			1.0225	**1.1057**
2021		50%	50%		1.0918	**1.0355**
2022			50%	50%	1.1421	**0.9899**
CRL Index	1.000	1.030	1.154	1.131		

Discussion

The parallelogram method works at the aggregate level to adjust historical premium to current rate level. This is a significant difference compared to the Extension of Exposures method which rerates all policies to current rates at the policy level.

The tricky part in a parallelogram method problem is calculating the portion of earned premium in each rate level group. For policy years, this is easy for normal rate changes, but it requires some geometry for calendar year on-leveling. I would sketch a quick earned premium diagram to help and avoid mistakes.

Problem Variations

There are three main ways a parallelogram method can be modified:

- Calendar year vs. policy year (or months/quarters)
- 12-Month vs. 6-Month policy terms
- Rate changes affect: policies written after the change vs. policies midterm

For each variation, the same general approach as above is used with some modification.

Two Problems with the Parallelogram Method

Problem 1 - *Assumes policies are written evenly throughout the year*

- This assumption isn't always valid, especially for seasonal lines of business like boat owners insurance.
    - The parallelogram method can still be used with more refined time periods like months or quarters.

Another approach is to use the actual distribution of writings during the year to more accurately determine the portion of earned premium in each rate level group (step 2). The paper doesn't go into this approach.

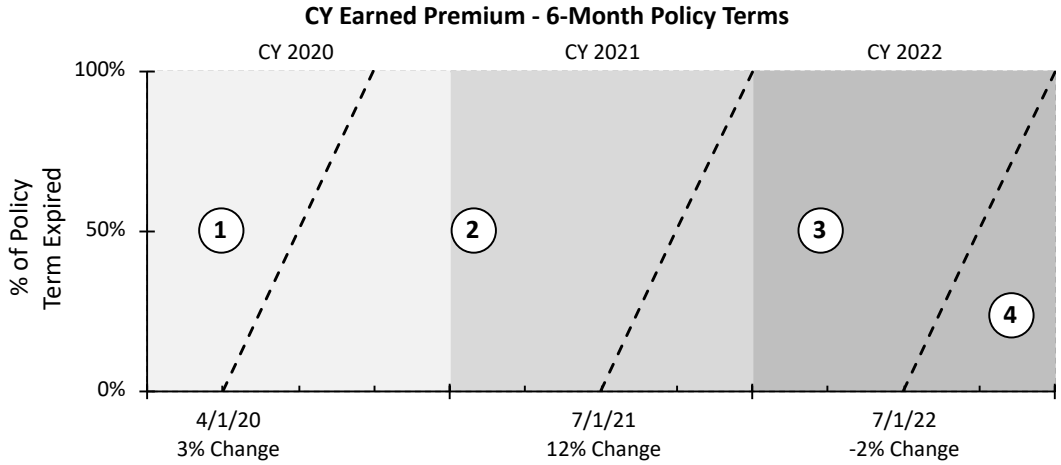
Problem 2 - *Applies at the aggregate level with overall average rate changes*

- If rate changes varied by class (class factors were modified), then premium at the class level won't be properly on-leveled. This means the adjusted premium won't be acceptable for classification ratemaking analysis.

Calendar Year Parallelogram Method for 6-Month Policies

For 6-month policies, rate changes are fully realized over 6 months instead of 12 months. To on-level calendar year earned premium, make sure to calculate the portion of earned premium in each rate level group correctly (step 2) since the geometry is different.

Example:
For the same problem above, assuming all policies have 6-month terms, this is how to solve for the on-level factors:



Calendar Year	Portion of EP in Each Rate Level Group 1	2	3	4	Average CRL Index	**On-Level Factor**
2020	50%	50%			1.0150	**1.1138**
2021		75%	25%		1.0609	**1.0656**
2022			75%	25%	1.1478	**0.9849**
CRL Index	1.000	1.030	1.154	1.131		

For policy years with 6-month terms, you get the same answer as in part b of the problem above. The diagram will look a different since policy years are fully earned after only 18 months, but the portion of earned premium in each rate level group is the same. You can set up an earned premium diagram to see why.

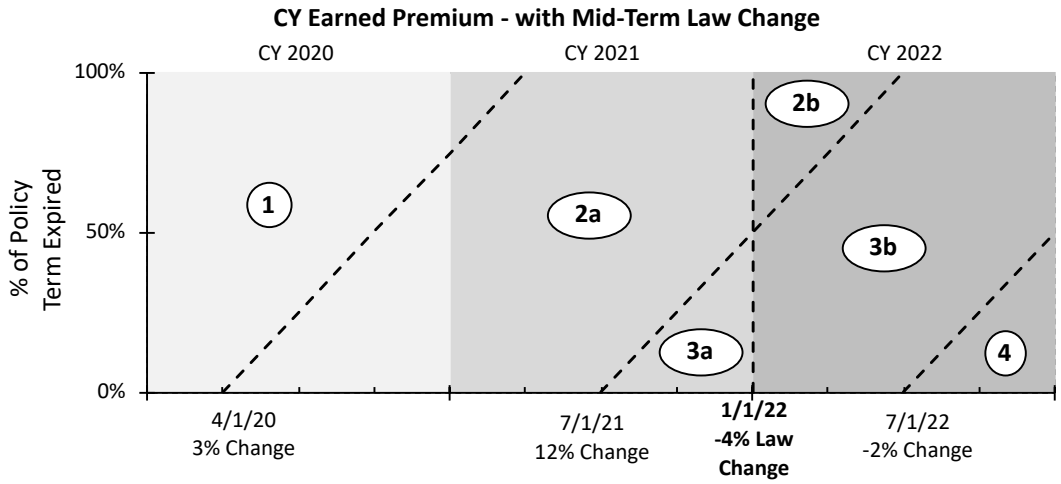
Rate Changes Affecting Policies Midterm (Mandated by Law)

A rate change mandated by law that affects ALL policies on or after the a specific date, including in-force policies, must be handled differently. It's represented as a vertical line on an earned premium diagram and will split one or more rate level groups.

Be careful when calculating the cumulative rate level indexes and the portion of earned premium in each group and sub-group.

Example:
Calculate the on-level factors for the same problem as part a, but with a mandated rate change of -4% on 1/1/22 affecting all policies midterm.

Rate Level Group	Effective Date	Cum. Rate Level Index	
1	Initial	1.000	
2a	4/1/20	1.030	***Before*** midterm rate change
3a	7/1/21	1.154	
2b	1/1/22	0.989	
3b	1/1/22	1.107	***After*** midterm rate change
4	7/1/22	1.085	



Calendar Year	Portion of EP in Each Rate Level Group						Average CRL Index	**On-Level Factor**
	1	2a	3a	2b	3b	4		
2020	71.88%	28.13%					1.0084	**1.0762**
2021	3.13%	84.38%	12.50%				1.0445	**1.0391**
2022				12.50%	75.00%	12.50%	1.0899	**0.9958**
CRL Index	1.000	1.030	1.154	0.989	1.107	1.085		

Exam Spreadsheet Tips

Use SUMPRODUCT( ) to help calculate the weighted average rate level index for each year:

Average CRL Index = SUMPRODUCT(%EP in rate level groups , CRL Indices of rate level groups)

It is possible to set up formulas to directly calculate the portion of earned premium in each rate level group using YEARFRAC(start date, end date) formulas and the rate level effective dates. However, it requires complicated Excel formulas that are tricky to set up. On an exam, I think it's faster to sketch a quick diagram on scratch paper and use geometry as shown above.

Source

Werner Ratemaking Ch. 5 - pg. 73-80

More Practice

CAS Spring 2019 – 2
CAS Fall 2018 – 3
CAS Spring 2017 – 2
CAS Fall 2016 – 2
CAS Spring 2015 – 5
CAS Spring 2015 – 4
CAS Spring 2014 – 1

Premium Development

Werner Ratemaking Ch. 5

Problem

Given the following for a workers compensation insurer as of 12/31/22:

	Earned Premium Evaluated as of			
Policy Year	12 months	24 months	36 months	Ultimate 48 months
2018	3,348	6,663	6,958	6,958
2019	3,481	6,725	7,315	7,315
2020	3,603	6,718	6,940	
2021	3,711	7,076		
2022	3,823			

- Premium audits are performed between 3 and 6 months after policies expire.
- All policies have an annual policy term.

Calculate the estimated ultimate earned premium by policy year for years 2018 – 2022.

Solution Recipe

1) Calculate the age-to-age development factors (link ratios) for the premium development triangle.

$$Age\text{-}to\text{-}Age\ Factor_t = \frac{Premium_{t+1}}{Premium_t}$$

2) Select a suitable link ratio for each development period based on how development is expected to occur in the future. Reviewing various averages can help with selection. There's no single "correct" selection, but the selection should be reasonable.

	Age-to-Age Factors		
Policy Year	12-24	24-36	36-48
2018	1.990	1.044	1.000
2019	1.932	1.088	1.000
2020	1.865	1.033	
2021	1.907		
All-Year Avg	1.923	1.055	1.000
Weighted Avg	1.922	1.055	1.000
Selected	**1.922**	**1.055**	**1.000**

$$All\text{-}Year\ Avg_{12-24} = \frac{1.990 + \cdots + 1.907}{4} = 1.923$$

$$Weighted\ Avg_{12-24} = \frac{6{,}663 + \cdots + 7{,}076}{3{,}348 + \cdots + 3{,}711} = 1.922$$