

Exam 5

Study Guide

BASIC RATEMAKING AND RESERVING

Comprehensive study guide
with past CAS practice problems

Exam 5 Study Guide

Spring 2024 Sitting

Rising Fellow



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Introduction

How to Use This Guide

This guide is intended to **supplement** the syllabus readings. Although we believe it provides a thorough review of the exam material, the readings provide additional context that is invaluable. Please do NOT skip the syllabus readings.

Past CAS Exam Problems

Past CAS exam problems & solutions from the Spring 2015 – Fall 2018 exams are included for each paper. Note that these questions are solely owned by the CAS. They are included in the online course for student convenience. All past CAS problems are Excel-based and can be downloaded from the online course.

Feedback

We always working to improve the Exam 5 Study Guide and the rest of the Rising Fellow study material. Please send us an email at exam5@risingfellow.com if you have feedback about any of the following:

- Sections that are confusing or could be improved
- Errors (ex. formatting, spelling, calculations, grammar, etc.)

Note that errata will be posted on the Rising Fellow website on an as-needed basis.

Blank Pages

Since many students want a printed copy of the study guide, blank pages have been inserted throughout the guide to ensure that all outlines start on odd pages.

Bookmarks

Bookmarks have been added for easier PDF navigation.

Werner/Modlin

Ch. 1 (Introduction)

Outline

The basic economic relationship for the price of any product is $\text{Price} = \text{Cost} + \text{Profit}$. Unlike standard consumer products, the cost of an insurance “product” is unknown at the time the product is sold. Thus, there is additional complexity when setting prices for insurance products. This textbook outlines the fundamentals of setting insurance prices.

This chapter begins with several definitions and concepts that set the stage for the rest of the textbook.

I. Basic Insurance Terms

- **Rating Manual** – a document that contains the information necessary to classify each risk and calculate the premium associated with that risk
- **Exposure** – basic unit of risk that underlies the insurance premium
 - **Written exposures (WE)** – the total exposures arising from policies issued during a specified period of time
 - **Earned exposures (EE)** – the portion of the written exposures for which coverage has already been provided as of a certain point in time
 - **Unearned exposures** – the portion of the written exposures for which coverage has not yet been provided as of a certain point in time
 - **In-force exposures** – the number of insured units that are exposed to loss at a given point in time
- **Premium** – the amount the insured pays for insurance coverage
 - **Written premium (WP)** – the total premium associated with policies issued during a specified period of time
 - **Earned premium (EP)** – the portion of the written premium for which coverage has already been provided as of a certain point in time

- **Unearned premium** – the portion of the written premium for which coverage has yet to be provided as of a certain point in time
- **In-force premium** – the full-term premium for policies that are in effect at a given point in time
- **Claim** – a demand made by the insured to the insurer for indemnification under an insurance policy
- **Claimant** – the individual making the demand for indemnification
- **Date of loss (i.e., accident date or occurrence date)** – the date of the event that caused the loss
- **Report date** – the date in which the claimant reports the claim to the insurer
- **Incurred but not reported (IBNR) claims** – claims not currently known by the insurer (ex. an insured has an auto accident on January 1 but does not report the claim until January 15. From January 1 to January 15, this is an IBNR claim)
- **Loss** – the amount of compensation paid or payable to the claimant under the terms of the insurance policy (note that Friedland uses the term “Claim” when referring to the dollar amount of “Losses”)
 - **Paid losses** – the amounts that have been paid to claimants
 - **Case reserve** – the amount of additional money required to ultimately settle a claim. When a claim is reported and a payment is expected to be made in the future, the insurer will establish a case reserve. As payments are made on the claim, the case reserve is reduced accordingly since it represents the additional money needed to ultimately settle the claim. Actuaries are not typically involved in the setting of case reserves. Instead, these are usually set by the claims department
 - **Reported losses (i.e., case incurred losses)** – the sum of the paid losses and the current case reserve for a claim. Mathematically, “**Reported Losses = Paid Losses + Case Reserve**”
 - **Incurred but not reported (IBNR) reserve** – the amount estimated to ultimately settle unreported claims
 - **Incurred but not enough reported (IBNER) reserve** – the additional amount above the aggregate reported losses needed to settle reported claims

- **Ultimate losses** – the amount of money required to close and settle all claims for a defined group of policies. Mathematically, “**Ultimate Losses = Reported Losses + IBNR + IBNER = Paid Losses + Case Reserves + IBNR + IBNER**”
- At first glance, it may be difficult to distinguish ultimate losses and reported losses. However, they are different for two reasons: 1) ultimate losses include an IBNR reserve for unreported claims whereas reported losses only consider reported claims and 2) ultimate losses include an IBNER reserve to cover loss development on reported claims
- If you are confused by the various loss definitions above, do not fret! We will look at in-depth examples in later chapters that clarify these differences. In other words, “just roll with it” for now!
- **Loss adjustment expense (LAE)** – the expense dollars incurred by the insurer in the process of settling claims
 - **Allocated loss adjustment expense (ALAE)** – claims-related expenses that are directly attributable to a specific claim (ex. fees associated with outside legal counsel to defend a claim). These may also be referred to as Defense and Cost Containment (DCC)
 - **Unallocated loss adjustment expense (ULAE)** – claim-related expenses that cannot be directly attributed to a specific claim (ex. salaries of claims department personnel that are not readily assignable to a specific claim). These may also be referred to as Adjusting and Other (A&O)
 - $LAE = ALAE + ULAE$
- **Underwriting expenses** – the expense dollars incurred by the insurer in the acquisition and servicing of policies
 - Commissions and brokerage – amounts paid to insurance agents or brokers as compensation for generating business
 - Other acquisition – expenses other than commissions and brokerage paid to acquire business (ex. insurer media advertisements)
 - Taxes, licenses, and fees – all taxes and miscellaneous fees paid by the insurer excluding federal income taxes (ex. premium taxes)

- General expenses – any remaining expenses associated with the insurance operations (ex. electricity costs for the home office)
- **Underwriting profit** – the sum of the profits generated from individual policies. In the simplest terms, this is equal to premiums – losses – expenses

II. Fundamental Insurance Equation

As mentioned earlier, basic economics tells us that $\text{Price} = \text{Cost} + \text{Profit}$. We can translate this to the insurance industry in a form known as the **fundamental insurance equation**:

$$\text{Premium} = (\text{Losses} + \text{LAE} + \text{UW Expenses}) + \text{UW Profit}$$

When we set rates for an insurance product, the goal is to determine the premium required to cover all costs and achieve the target underwriting profit.

There are **two key points** to consider in regard to achieving the appropriate balance in the fundamental insurance equation:

- 1) Ratemaking is prospective
- 2) Balance should be attained at the aggregate and individual levels

Ratemaking is Prospective

As mentioned earlier, we do not know the cost of an insurance product upon sale. Thus, we must estimate the cost. The ratemaking process involves estimating the components of the fundamental equation to set a rate that is expected to achieve the target profit **during the period the rates will be in effect**. For example, if the rates go into effect six months from now, then we want to set a rate that covers what costs will look like six months from now (i.e., the expected cost) and leaves enough premium left over to achieve the target underwriting profit.

Overall and Individual Balance

During the ratemaking process, we want to ensure that the fundamental equation is in balance at both an overall level, as well as at an individual or segment level.

Equilibrium at the aggregate level ensures that the total premium for all policies written is sufficient to cover the total expected losses and expenses and achieve the target profit. **Equilibrium at the individual level** ensures that individual policies with higher risks of loss have higher premiums than policies with lower risks of loss (i.e., minimal levels of subsidization).

III. Basic Insurance Ratios

This section introduces several key insurance ratios:

- **Frequency** = $\frac{\text{Number of Claims}}{\text{Number of Exposures}}$
 - **Changes in claims frequency** can be used to identify general industry trends associated with the incidence of claims or the utilization of the insurance coverage
- **Severity** = $\frac{\text{Losses}}{\text{Number of Claims}}$
 - In general, **paid severity** is calculated as paid losses on closed claims divided by closed claims
 - In general, **reported severity** is calculated as reported losses on reported claims divided by reported claims
 - ALAE may or may not be included in the numerator
 - **Changes in severity** can provide information about loss trends and highlights the impact of any changes in claims handling procedures
- **Pure Premium** = $\frac{\text{Losses}}{\text{Number of Exposures}} = \text{Frequency} \times \text{Severity}$
 - Pure premium is also known as the **loss cost**
 - Pure premium describes the portion of the risk's expected costs that is purely attributable to loss
 - ALAE may or may not be included in the numerator
 - **Changes in pure premium** can highlight industry trends in overall loss costs due to change in both frequency and severity
- **Average Premium** = $\frac{\text{Premium}}{\text{Number of Exposures}}$
 - The numerator and denominator need to be on the same basis. For example, to calculate average written premium, written premium and written exposures should be used

- **Changes in average premium**, if adjusted for rate change activity, can highlight changes in the mix of business written (ex. shift towards higher or lower risk characteristics reflected in rate). We will look at how to adjust premiums for rate change activity in a later chapter
- **Loss Ratio** = $\frac{\text{Losses}}{\text{Premium}} = \frac{\text{Pure Premium}}{\text{Average Premium}}$
 - There are many common variations of a loss ratio. One common variation divides total reported losses by total earned premium
 - Total LAE may or may not be included in the numerator. When included, the ratio is known as the **loss and LAE ratio**
 - **The loss and LAE ratio** is a primary measure of the adequacy of the rates overall and for key segments of the portfolio
- **LAE Ratio** = $\frac{\text{LAE}}{\text{Losses}}$
 - First and foremost, notice that this definition of LAE ratio has losses in the denominator instead of premium. Thus, the loss and LAE ratio we covered above is equal to (Loss Ratio) x (1 + LAE Ratio)
 - Both paid and reported figures are used by companies when defining the LAE ratio
 - **The LAE ratio** is used to determine if costs associated with claim settlement procedures are stable or not. An LAE ratio that changes drastically from year to year is not stable
- **Underwriting (UW) Expense Ratio** = $\frac{\text{UW Expenses}}{\text{Premium}}$
 - This ratio is typically split into two ratios: 1) expenses that are generally incurred at the onset of the policy (commissions and brokerage, other acquisition, and taxes, licenses, and fees) and 2) expenses that are incurred throughout the policy (general expense)
 - The expenses incurred at the onset are divided by written premium and the expected incurred throughout the policy are divided by earned premium. This is done to **better match the expense payments to the premium associated with the expense**

- Based on the two bullet above, we can re-write the UW Expense Ratio as

$$UW \text{ Expense Ratio} = \frac{C\&B + OA + TLF}{Written \text{ Premium}} + \frac{GE}{Earned \text{ Premium}}$$
- **Analysis of the UW ratio** involves comparing actual changes in the ratio to expected changes based on general inflation
- **Operating Expense Ratio (OER) = UW Expense Ratio + $\frac{LAE}{Earned \text{ Premium}}$**
 - The OER represents the portion of each premium dollar used to pay for LAE and UW expenses
 - **The OER** is used to monitor operational expenditures and is key to understanding the overall profitability of the insurer
- **Combined Ratio = Loss Ratio + $\frac{LAE}{Earned \text{ Premium}}$ + $\frac{UW \text{ Expenses}}{Written \text{ Premium}}$**
 - Notice that LAE is considered separately in the formula above. If the insurer uses a loss and LAE ratio instead of a loss ratio, then the second component above would be excluded. The point is just to make sure that LAE is not double counted
 - Notice that the total UW expenses are being divided by written premium. If we wanted to use the “more accurate” version where UW expenses incurred through the policy are divided by earned premium, then the formula would be re-written as

$$Combined \text{ Ratio} = Loss \text{ Ratio} + OER$$
 - **The combined ratio** is the primary measure of the profitability of the book of business since $UW \text{ Profit} = 1.00 - Combined \text{ Ratio}$
- **Retention Ratio = $\frac{Number \text{ of Policies Renewed}}{Number \text{ of Potential Renewal Policies}}$**
 - There are many variations of the retention ratio. For example, some companies exclude policies that cancel due to death
 - **Retention rates and changes in retention ratios** are used to gauge the competitiveness of rates. Retention rates are also central to lifetime customer value and/or projecting future premium values
- **Close Ratio = $\frac{Number \text{ of Accepted Quotes}}{Number \text{ of Quotes}}$**
 - There are variations of the close ratio. For example, suppose a prospective insured receives multiple quotes. Some companies may count that as one quote, while others may count each quote separately

- **Close rates and changes in retention ratios** are used to gauge the competitiveness of rate for new business

Werner/Modlin

Ch. 3 (Ratemaking Data)

Outline

This chapter discusses various types of ratemaking data, along with data aggregation methods.

I. Internal Data

There are **two types** of internal data involved in a ratemaking analysis:

- 1) Risk information – includes exposures, premiums, claim counts, losses, and explanatory characteristics about the policy or the claim
- 2) Accounting information – includes underwriting expenses and ULAE (both are typically only available at the aggregate level)

Risk Information

When analyzing risk information, we want to link exposure and premium data with the corresponding claim and loss data. In general, companies record this information in **two separate databases**: 1) a policy database and 2) a claim database.

Policy Database

The policy database is defined according to records (i.e., individual policies or some further subdivision of the policy) and fields (i.e., explanatory information about the record). Records are defined differently based on the line of business. For example:

- Homeowners – a record may be a home for an annual policy period
- Workers' Compensation – a record may be the relevant industry classification

In addition to the record definitions above, records are also subdivided according to any changes in the risk(s) during the policy period. For example, if a policy is amended during the policy term, then

separate records are created for the partial policy periods before and after the change. We will explore an example later to illustrate this.

Fields for each record on the policy database may include the following:

- Policy identifier – a key that identifies each policy
- Risk identifier – a key that identifies each risk if there are multiple risks on a single policy (ex. multiple operators on a single auto policy)
- Relevant dates – policy start date, policy amendment date, etc.
- Premium – typically written premium (by coverage, if applicable)
- Exposure – typically written exposure (by coverage, if applicable)
- Characteristics – rating variables, underwriting variables, etc.; the characteristics should reflect what was present on the policy during that specific period

Example: Given the following homeowners policies:

- Policy A
 - Written date: January 1, 2010
 - Annual premium: \$1,100
 - Territory: 1
 - Deductible: \$250
 - Policy remains unchanged for the full term of the policy
- Policy B
 - Written date: April 1, 2010
 - Annual premium: \$600 at policy issue
 - Territory: 2
 - Deductible: \$250 at policy issue
 - Policy is canceled on December 31, 2010
- Policy C
 - Written date: July 1, 2010
 - Annual premium: \$1,000 at policy issue; increased to \$1,200 after deductible change described below
 - Territory: 3

- Deductible: \$500 at policy issue; decreases to \$250 on January 1, 2011

The policy database is shown below, where each row represents a “record:”

Policy	Orig. Effect. Date	Orig. Term. Date	Trans. Effect. Date	Ded.	Terr.	Other Chars.	Written Exposure	Written Premium
A	01/01/10	12/31/10	01/01/10	\$250	1	...	1.00	\$1,100
B	04/01/10	03/31/11	04/01/10	\$250	2	...	1.00	\$600
B	04/01/10	03/31/11	12/31/10	\$250	2	...	-0.25	-\$150
C	07/01/10	06/30/11	07/01/10	\$500	3	...	1.00	\$1,000
C	07/01/10	06/30/11	01/01/11	\$500	3	...	-0.50	-\$500
C	07/01/10	06/30/11	01/01/11	\$250	3	...	0.50	\$600

How did we build the table above?

- Policy A – This is straight-forward. Since there were no changes during the policy period, we have a single record and a written exposure of 1.00
- Policy B – The only policy change was the cancellation. Thus, we have two records. The first record shows the information known at policy issue. The second record shows the cancellation adjustment. Since the policy was cancelled 75% of the way through the policy period, the second record shows a written exposure of -0.25 and a written premium of -\$150. In doing so, the “net” result after aggregating the two records is a net written exposure of $0.75 = 1.00 - 0.25$ and a net written premium of $\$450 = \$600 - \$150$
- Policy C – The only policy change is a mid-term adjustment to decrease the deductible. Since the policy was not cancelled, we have three records. The first record shows the information at policy issue. The second record negates the portion of the original policy that was unearned at the time of the amendment. Since half of the record was unearned, the written exposure is -0.50 and the written premium is -\$500. The third record shows the information for the amended portion of the policy. Since half of the policy remains, the written exposure is 0.50. Since the annual premium associated with the amendment is \$1,200, the written premium associated with the amendment is $\$600 = (0.50)(\$1,200)$

Claims Database

In a claims database, each record represents a transaction tied to a specific claim. Examples of transactions include claim payments and changes in case reserves. Each record also includes dates or other explanatory information relevant to each claim.

Fields included for each record on the claims database include the following:

- Policy identifier
- Risk identifier
- Claim identifier – a key that identifies each claim. If a claim has multiple claimants, each claimant will have the same claim identifier
- Claimant identifier – a key that identifies each claimant on a particular claim
- Relevant loss dates – loss date, report date, etc.
- Claim status – shows whether the claim is open, closed, re-opened, etc.
- Claim count – identifies the number of claims by coverage associated with the loss occurrence
- Paid loss – payments made for each claim record
- Case reserve – the case reserve or change in the case reserve at the time the transaction is recorded
- ALAE – the ALAE for each claim record
- Salvage/subrogation – companies can often recoup some claim payments in the form of salvage or subrogation. When damaged property is sold by the insurer, this is known as salvage and can be used to offset claim payments. When a company pays for an insured's loss, the company reserves the right to recover any damages from a third party who was at fault. This is known as subrogation and can be used to offset claim payments
- Claim characteristics – type of injury, loss location, age of injured party, etc.

Example: Given the following claim information for three policies:

- Policy A
 - Loss date: January 10, 2010
 - Report date: January 15, 2010

- Initial case reserve: \$10,000
- Payment #1: \$1,000 made on March 1, 2010, with a corresponding \$1,000 reduction in the case reserve
- Payment #2: \$9,000 made on May 1, 2010, and the claim is closed
- Policy B: No claims
- Policy C
 - Loss date: October 1, 2010
 - Report date: October 15, 2010
 - Initial case reserve: \$18,000
 - Payment #1: \$2,000 made on December 15, 2010, with a \$1,000 reduction in the case reserve. Notice that the change in the case reserve is not always a direct offset to the claim payment. This happens when the claims adjuster feels that additional payments beyond the initial case reserve will be required. Remember that we referred to this as development on a known claim in Chapter 1 (IBNER)
 - Payment #2: \$7,000 made on March 1, 2011, with a \$2,000 reduction in the case reserve
 - Payment #3: \$15,000 made on March 1, 2012, and the claim is closed. The insurer receives a \$1,000 salvage recovery by selling damaged property

The claims database is shown below, where each row represents a “record:”

Policy	Claim Number	Loss Date	Report Date	Trans. Date	Claim Status	Claim Chars.	Loss Pmt.	Case Res.	Sal/Sub
A	1	01/10/10	01/15/10	01/15/10	Open	...	\$ -	\$10,000	\$ -
A	1	01/10/10	01/15/10	03/01/10	Open	...	\$1,000	\$9,000	\$ -
A	1	01/10/10	01/15/10	05/01/10	Closed	...	\$9,000	\$ -	\$ -
C	2	10/01/10	10/15/10	10/15/10	Open	...	\$ -	\$18,000	\$ -
C	2	10/01/10	10/15/10	12/15/10	Open	...	\$2,000	\$17,000	\$ -
C	2	10/01/10	10/15/10	03/01/11	Open	...	\$7,000	\$15,000	\$ -
C	2	10/01/10	10/15/10	03/01/12	Closed	...	\$15,000	\$ -	\$1,000

How did we build the table above?

- Policy A – There are three records: one when the claim is reported; one when the first payment is made; and one when the last payment is made and the claim is closed
- Policy B – There are no claims and thus, no records
- Policy C – There are four records: one when the claim is reported; one when the first payment is made; one when the second payment is made; and one when the last payment is made, salvage/subrogation is received, and the claim is closed
- At any given point in time for a single claim, we can sum up all payments and add on the latest case reserve to obtain the reported loss gross of sal/sub. If we subtract off all sal/sub received from the reported loss gross of sal/sub, we obtain the reported loss net of sal/sub. We can do the same thing across all claims to obtain the aggregate reported loss net of sal/sub

Accounting Information

Accounting information refers to financial information not specific to any one claim (ex. UW expenses and ULAE). Thus, accounting information is usually tracked at the aggregate level. Companies typically track accounting information by calendar year.

II. Data Aggregation

When aggregating data for ratemaking purposes, three general objectives apply:

- 1) Accurately match losses and premium for the policy
- 2) Use the most recent data available
- 3) Minimize the cost of data collection and retrieval

We will explore **four methods of data aggregations**: calendar year (CY), accident year (AY), policy year (PY), and report year (RY).

CY Aggregation

CY aggregation **considers all premium and loss transactions** that occur during the twelve-month calendar **without regard to the date** of policy issuance, the accident date, or the report date of the claim. Regarding earned premium, earned exposures, paid losses, and reported losses:

- CY Earned Premium – all premium earned during the twelve-month period
- CY Earned Exposures – all exposures earned during the twelve-month period
- CY Paid Losses – all loss payments made during the twelve-month period
- CY Reported Losses – equal to CY Paid Losses + Change in Case Reserves during the twelve-month period

An **advantage** of CY aggregation is that data is available quickly. Once a CY ends, all premium, exposure, and loss information is fixed and never changes.

A **disadvantage** of CY aggregation is the mismatch between premiums and losses. The premiums earned during the CY come from policies in-force during the year. For annual policies, those premiums come from policies written in the previous CY or the current CY. The losses could come from any open claims. For example, CY 2010 can include loss payments made on claims that occurred in 1985. Complicated claims can remain open for decades. If a loss payment on a 1985 claim occurs in CY 2010, then the CY 2010 paid losses will include that payment.

CY aggregation **is most appropriate** for lines of business in which **losses are reported quickly and settled quickly** (ex. homeowners).

AY Aggregation

Under AY aggregation, premiums and exposures are aggregated in the same manner as CY aggregation. However, AY aggregation of losses only **considers losses for accidents that have occurred during that twelve-month period**, regardless of when the policy was issued or when the claim was reported. Given that premiums and exposures follow CY aggregation whereas losses follow AY aggregation, this is sometimes known as CY/AY aggregation. Regarding earned premium, earned exposures, paid losses, and reported losses:

- CY/AY Earned Premium – all premium earned during the twelve-month period
- CY/AY Earned Exposures – all exposures earned during the twelve-month period
- AY Paid Losses – all loss payments made on claims that occurred during the twelve-month period
- AY Reported Losses – all loss payments and the current case reserve on claims that occurred during the twelve-month period

An **advantage** of AY aggregation is that it provides a better match of premium and losses than CY aggregation. Since it only considers claims that occurred during the twelve-month period, any losses will come from policies issued in the previous CY or the current CY.

A **disadvantage** of AY aggregation is that the AY is not fixed at the end of the twelve-month period. As mentioned in Chapter 1, we often have development on known claims (IBNER). We also have to consider pure IBNR claims that occurred in the AY but have yet to be reported. It can take several years for all claims that occurred in a specific AY to be settled. Thus, AY losses change (i.e., develop) over time.

PY Aggregation

Also known as underwriting year (UY) aggregation, PY aggregation considers **all premiums and loss transaction on policies that were written during a twelve-month period**, regardless of when the claim occurred or when it was reported. Regarding earned premium, earned exposures, paid losses, and reported losses:

- PY Earned Premium – all premium earned on policies written during the twelve-month period
- PY Earned Exposures – all exposures earned on policies written during the twelve-month period
- PY Paid Losses – all loss payments made on claims from policies written during the twelve-month period
- PY Reported Losses – all loss payments and the current case reserve on claims from policies written during the twelve-month period

An **advantage** of PY aggregation is that it represents an exact match of premiums and losses. The premium comes from policies written during the twelve-month period. The losses come from those exact same policies that were written during the twelve-month period.

A **disadvantage** of PY aggregation is that data takes longer to develop than both calendar year and accident year. For example, PY 2010 includes premium and losses from policies written between January 1, 2010 and December 31, 2010. An annual policy written on December 31, 2010 will earn nearly all of its premium in 2011. In addition, a loss on this policy could occur as late as December

30, 2011. Thus, when dealing with policies with annual policy terms, it takes 24 months for all exposures to fully earn out and for all losses on those policies to occur. Similar to AY aggregation, PY losses develop over time as more information is gathered on known claims and as unreported claims are gradually reported.

R_Y Aggregation

This method is like CY/AY except that the losses are aggregated according to when the claim was reported, as opposed to when the claim occurred. This method **is typically used for claims-made policies (ex. medical malpractice policies)**.

We will show detailed examples of each of the aggregation methods above in a later chapter.

Overall vs. Classification Analysis

When the goal of the ratemaking analysis is to review the adequacy of the overall rate level, the data can be highly summarized (i.e., aggregated at a broad level like year and location). When the goal is classification analysis, more granular data is needed. For example, if the aim is to revise a rating plan's territory relativities, then detailed data is needed by territory. If the aim is to perform multivariate analysis via a predictive model, then detailed data is needed by policy or risk.

III. External Data

External data is useful when internal data is limited (ex. entering a new line of business). External data includes the following:

- Statistical plans
- Other aggregated industry data
- Competitor rate filings or competitor manuals
- Other third-party data

Statistical Plans

Statistical plan data is summarized data required by state regulators. To comply with state requirements for aggregated industry data, industry service organizations such as NCCI and ISO

have been formed to collect and aggregate data from participating insurers. This data can be used to supplement internal data when performing actuarial analyses.

Other Aggregated Industry Data

Many insurers voluntarily report data to other various industry organizations so that it can be aggregated and used by the insurance industry. Examples include the following:

- Fast Track Reports used by insurers and state regulator to analyze loss trends
- Loss data from the Highway Loss Data Institute (HLDI) that provides detailed loss information by type of car for use by insurers and consumers in the form of safety recommendations

Competitor Rate Filings or Competitor Manuals

In certain states, rate filings are publicly available. Since rate filings typically include actuarial justification, they often include detailed premium, loss, and expense information. Insurers can use this to compare themselves to competitors.

Companies may be required to include manual pages as well. Rating manuals include the rules, rating structures, and rating algorithms in use by the company. Insurers can review competitor rating manuals to see how they stack up in terms of rating features. When reviewing rating manuals, insurers should **keep in mind that underwriting tiers** are often not included. Since underwriting tiers can be a large component of the final premium charged, insurers should be cautious when comparing their premiums to competitor premiums.

Other Third-Party Data

Insurers also make use of non-insurance information sourced from third parties. This includes economic data (ex. Consumer Price Index), geo-demographic data (ex. average characteristics of a particular location) and credit data. Note that there are many other types of third-party data as well.

The Consumer Price Index can assist companies in projecting trends in expenses, premium, or losses. Geo-demographic data and credit data can be included in predictive models used for predicting frequency and severity.

Werner/Modlin

Ch. 4 (Exposures)

Outline

I. Criteria for Exposure Bases

An **exposure** is the basic unit that measures a policy's exposure to loss. A good exposure base should meet the following **three criteria**:

- 1) It should be directly proportional to expected loss
- 2) It should be practical
- 3) It should consider any pre-existing exposure base established within the industry (i.e., historical precedence)

Directly Proportional to Expected Loss

All else being equal, the expected loss of a policy with two exposures should be twice the expected loss of a policy with one exposure. Notice that words "all else being equal." In addition to the number of exposures, the expected loss of a policy varies by several other factors, including rating or underwriting variables.

As an example of this "directly proportional" concept, consider workers' compensation (WC) insurance. The most used exposure base for WC insurance is payroll. As the number of workers increases (or decreases) or the average number of hours worked increases (or decreases), both payroll and risk of loss increases (or decreases). Thus, payroll moves in proportion to expected losses.

Practical

By practical, we mean that the exposure base should be objective and relatively easy and inexpensive to obtain and verify. The exposure base should also be selected so that policyholders (or agents) cannot manipulate exposure information for their own gain (i.e., moral hazard).

As an example, on the surface, annual mileage appears to be a reasonable exposure base for auto policies. However, this information is typically provided by the policyholder and not verified by the insurer. In this case, it's very easy for the policyholder to provide a lower annual mileage figure resulting in a lower premium. Thus, car-years is often used instead to measure exposure.

Another **example** concerns product liability. An intuitive exposure base is the number of products currently in use. Since it is difficult for insurers to track how many of their products are actually being used during the coverage period, gross sales is used instead.

Historical Precedence

Insurer should be cautious before changing an exposure base for a line of business for the following reasons:

- Any change in exposure base can lead to large premium swings for individual insureds
- Any change in exposure base will require a change in the rating algorithm, which could require a lot of time and expense
- Any change in exposure may require significant data adjustments since ratemaking analyses require several years of historical data

Here is a table of exposure bases by LOB:

LOB	Exposure Bases
Personal Auto	Earned Car Year
Homeowners	Earned House Year
WC	Payroll
Commercial General Liability	Sales Revenue, Square Footage
Commercial Business Property	Amount of Insurance Coverage
Physician's Professional Liability	Number of Physician Years
Professional Liability	Number of Professionals
Personal Articles Floater	Value of Item

II. Exposures for Large Commercial Risks

Given the unique nature of large commercial risks, they do not follow traditional exposure base conventions. Instead, **composite rating and loss-rated composite rating** is often used for these risks.

Under **composite rating**, different exposure measures are used for each “sub-coverage.” For example, a commercial multi-peril policy might use sales revenue for general liability and amount of insurance value for commercial business property. These exposure bases are combined with the rating algorithms for each “sub-coverage” to calculate the initial premium. Obviously, these exposure bases can change over time. Instead of regularly auditing each exposure base and re-doing the premium calculation, a **proxy measure** is used to measure the overall change in exposure to loss. Using the same commercial multi-peril policy above, the insurer might choose the property value as the proxy measure. If the property value increases by 20%, then the total policy premium will also increase by 20%.

Under **loss-rated composite rating**, there is no rating algorithm. Instead, the premium is calculated solely using the individual risk’s historical loss experience.

III. Aggregation of Exposures

In Chapter 3, we covered four methods of aggregation: CY, AY, PY, and RY. However, for exposures, there are only two aggregation methods: CY and PY. As mentioned in Chapter 3, AY aggregation is identical to CY aggregation for exposures and premiums. RY aggregation is a loss concept and doesn’t apply to exposures and premiums. In this section, we will apply these concepts to a set of homeowners policies.

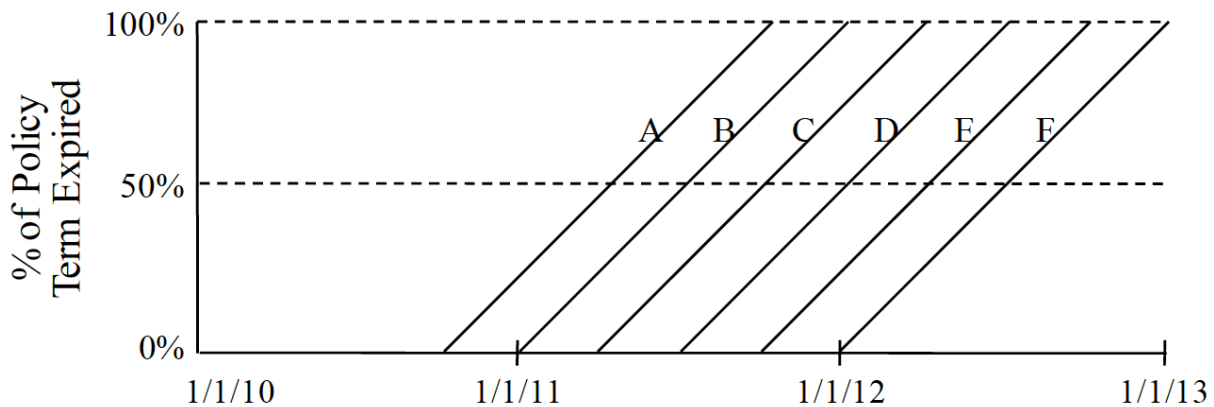
Example 1

Given the following homeowners policies with annual terms:

Policy	Effective Date	Expiration Date	Exposure
A	10/01/10	09/30/11	1.00
B	01/01/11	12/31/11	1.00
C	04/01/11	03/31/12	1.00

D	07/01/11	06/30/12	1.00
E	10/01/11	09/30/12	1.00
F	01/01/12	12/31/12	1.00

We can also represent the policies pictorially:

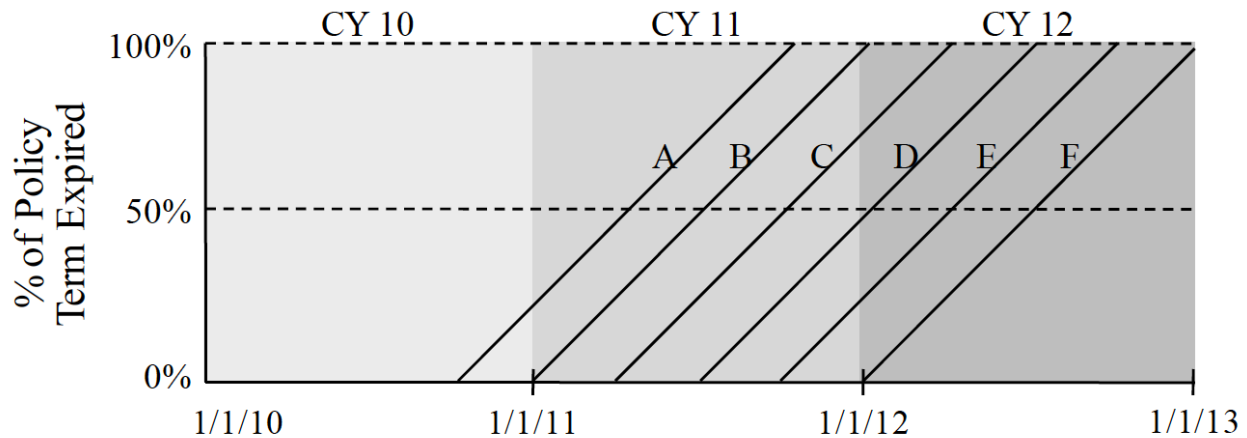


How do we interpret the picture above?

- Each diagonal line represents a single policy (A through F)
- The x-axis represents the effective date or the expiration date. The y-axis represents the % of the policy term that has expired. Thus, each policy starts at 0% (at the policy effective date) and ends at 100% (at the policy expiration date)
- A single point on each diagonal line represents the “current date during the policy term” and the associated “expiration percentage.” Using Policy A as example, the policy term is half-way complete on 03/31/11. Thus, if we were to draw a vertical line at 03/31/11, it would intersect policy A’s diagonal line at the 50% mark

CY (and AY) Aggregation

Recall that a CY is fixed at the end of the CY. In addition, a CY considers **any transactions** that occurred during the CY. Thus, we represent CYs as squares:



As shown above, CY 10 is the lightest gray square that runs from 1/1/10 – 12/31/10. Once we understand the picture above, we can aggregate written exposures (WE) and earned exposures (EE) by CY.

Aggregating written exposures is simple. If a policy is written in CY 10, then it contributes a written exposure of 1.00 to that CY, regardless of whether it was written at the beginning of CY 10 or the end of CY 10. **Aggregating earned exposures** is slightly more complicated. **Assuming that the probability of a claim is evenly distributed throughout the year**, we calculate the percentage of each diagonal line that falls in each year. For example, since policy A is written on 10/01/10, 25% (i.e., 3 months out of 12 months) of the total exposure for Policy A is earned in CY 10. The remaining 75% of the total exposure for Policy A is written in CY 11. Thus, Policy A contributes 0.25 (25% x 1.00) earned exposure to CY 10 and 0.75 (75% x 1.00) to CY 11. Let's look at the results:

Policy	Eff. Date	Exp. Date	Total Exposure	Written Exposures			Earned Exposures		
				CY 10	CY 11	CY 12	CY 10	CY 11	CY 12
A	10/01/10	09/30/11	1.00	1.00	0.00	0.00	0.25	0.75	0.00
B	01/01/11	12/31/11	1.00	0.00	1.00	0.00	0.00	1.00	0.00
C	04/01/11	03/31/12	1.00	0.00	1.00	0.00	0.00	0.75	0.25
D	07/01/11	06/30/12	1.00	0.00	1.00	0.00	0.00	0.50	0.50
E	10/01/11	09/30/12	1.00	0.00	1.00	0.00	0.00	0.25	0.75
F	01/01/12	12/31/12	1.00	0.00	0.00	1.00	0.00	0.00	1.00
Total			6.00	1.00	4.00	1.00	0.25	3.25	2.50

In the table above, we see that the total written exposures from the six policies equals the total earned exposures for each policy. The difference is simply in how they are aggregated by CY.

When considering **unearned exposures**, the intent is to calculate the amount of exposure unearned at a **certain point in time**. For **an individual policy**, unearned exposures = written exposures – earned exposures at the time being considered. For example, at 12/31/10, Policy A has an unearned exposure of $0.75 = 1.00 - 0.25$. At 06/30/11, Policy B has an unearned exposure of $0.50 = 1.00 - 0.50$. For **groups of policies**, the formula for unearned exposures depends on the method of data aggregation:

- CY Aggregation: $\text{CY Unearned Exposures} = \text{CY Written Exposures} - \text{CY Earned Exposures} + \text{Unearned Exposures as of the beginning of the CY}$
- PY Aggregation: $\text{PY Unearned Exposures} = \text{PY Written Exposures} - \text{PY Earned Exposures}$

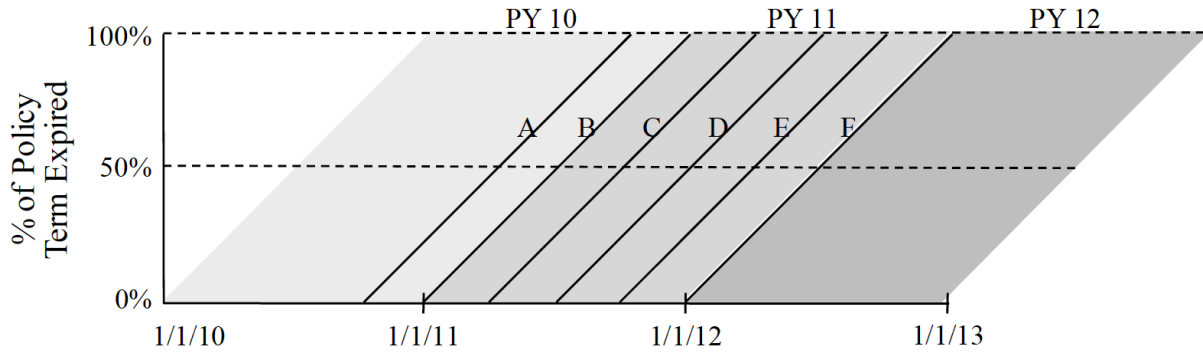
As you can see, the formula for CY Unearned Exposures is a bit strange. Let's apply it to our example above to see it in action:

- $\text{CY 10 Unearned Exposures at 12/31/10} = 1.00 - 0.25 + 0.00 = 0.75$
- $\text{CY 11 Unearned Exposures at 12/31/11} = 4.00 - 3.25 + 0.75 = 1.50$
- $\text{CY 12 Unearned Exposures at 12/31/12} = 1.00 - 2.50 + 1.50 = 0.00$

As expected, there are no unearned exposures at the end of CY 12. Also, if we had applied the “individual policy” formula of $\text{WE} - \text{EE}$ and summed them up for each CY, we would get the same totals.

PY Aggregation

PY aggregation considers all exposures on policies with effective dates during the year. Here’s how it looks pictorially:



As shown in the picture above, PYs are represented as **parallelograms instead of squares**. This is because each PY is defined by policies with effective dates in that PY. Thus, all exposure, premium, and loss from a policy written on 1/1/10 is attributed to PY 10. All exposure, premium, and loss from a policy written on 12/31/10 is attributed to PY 10. In terms of **aggregating written exposures**, it works the same as CY. For PY, **aggregating earned exposures is just as simple**. Since Policy A was written in year 10, 100% of its earned exposure is attributed to PY 10. Thus, we don’t have to worry about partial exposures when performing PY aggregation. Let’s look at the results:

Policy	Eff. Date	Exp. Date	Total Exposure	Written Exposures			Earned Exposures		
				PY 10	PY 11	PY 12	PY 10	PY 11	PY 12
A	10/01/10	09/30/11	1.00	1.00	0.00	0.00	1.00	0.00	0.00
B	01/01/11	12/31/11	1.00	0.00	1.00	0.00	0.00	1.00	0.00
C	04/01/11	03/31/12	1.00	0.00	1.00	0.00	0.00	1.00	0.00
D	07/01/11	06/30/12	1.00	0.00	1.00	0.00	0.00	1.00	0.00
E	10/01/11	09/30/12	1.00	0.00	1.00	0.00	0.00	1.00	0.00
F	01/01/12	12/31/12	1.00	0.00	0.00	1.00	0.00	0.00	1.00
Total			6.00	1.00	4.00	1.00	1.00	4.00	1.00

Cancelled Policies

How do we **handle cancelled policies** under CY and PY aggregation?

- If a policy cancels mid-term, the policy will contribute written exposures to different CYs if the date of the cancellation is in a different CY than the original effective date. For example, if Policy D is cancelled on March 31, 2012, then Policy D will contribute 1.00 WE to CY 11 and -0.25 WE to CY 12
- Unlike CY aggregation, if a policy is cancelled in a different calendar year than the original effective date, the original WE and the WE due to cancellation are still booked in the same PY. For example, if Policy D is cancelled on March 31, 2012, then Policy D will contribute 1.00 WE to PY 11 and -0.25 WE to PY 11

Uneven Earning Patterns

For both CY and PY aggregation, we assumed an even earning pattern. An even earning pattern does NOT hold for certain LOBs (ex. warranty coverage). In these cases, actuaries should specify an earning pattern based on historical loss experience

In-Force Exposures

Before moving on to semi-annual policies, let's briefly discuss **in-force exposures**. Unlike written, earned, and unearned exposures, we don't consider in-force exposures by aggregation year. Instead, we simply count the number of policies in effect at a given point in time. For example, on 06/15/11, there are four policies in effect. Thus, we say that in-force exposures are 4.00 on 06/15/11. It's that simple.

Policy Terms Other Than Annual

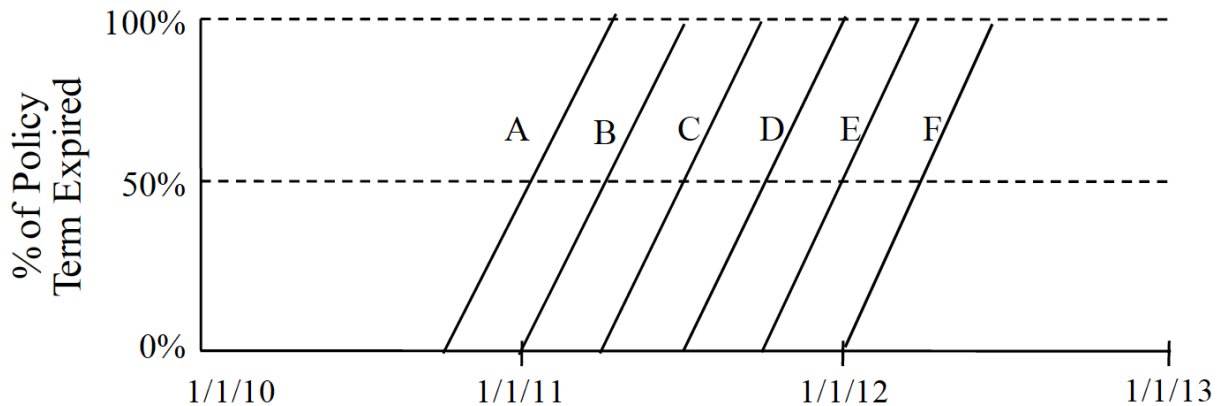
In our example, the policy term for each policy was annual. If the policy term is something other than annual, the process changes slightly.

Example 2

Let's modify our example such that each policy has a six-month term instead of an annual term:

Policy	Effective Date	Expiration Date	Exposure
A	10/01/10	03/31/11	0.50
B	01/01/11	06/30/11	0.50
C	04/01/11	09/30/11	0.50
D	07/01/11	12/31/11	0.50
E	10/01/11	03/31/12	0.50
F	01/01/12	06/30/12	0.50

As we see in the table, the effective dates remain unchanged. However, the expiration dates are six months earlier. In addition, the total exposure for each policy is now 0.50 instead of 1.00. Our picture of each policy changes to the following:



Although each policy is still represented by a diagonal line, they are “steeper” than the original example since each policy has an earlier expiration date. The only meaningful difference in aggregation is the aggregation of CY earned exposures. For example, Policy C no longer contributes to CY 12. Instead, all of Policy C is earned in CY 11. Also, in the original example, 25% of Policy A was earned in CY 10. Now, 50% (3 months out of 6 months) is earned in CY 10. Thus, Policy A contributes 0.25 (50% x 0.50) earned exposure to CY 10 and 0.25 (50% x 0.50) to CY 11. Let's look at the results under **CY aggregation**:

Policy	Eff. Date	Exp. Date	Total Exposure	Written Exposures			Earned Exposures		
				CY 10	CY 11	CY 12	CY 10	CY 11	CY 12
A	10/01/10	03/31/11	0.50	0.50	0.00	0.00	0.25	0.25	0.00
B	01/01/11	06/30/11	0.50	0.00	0.50	0.00	0.00	0.50	0.00
C	04/01/11	09/30/11	0.50	0.00	0.50	0.00	0.00	0.50	0.00
D	07/01/11	12/31/11	0.50	0.00	0.50	0.00	0.00	0.50	0.00
E	10/01/11	03/31/12	0.50	0.00	0.50	0.00	0.00	0.25	0.25
F	01/01/12	06/30/12	0.50	0.00	0.00	0.50	0.00	0.00	0.50
Total			3.00	1.00	2.00	1.00	0.25	2.00	0.75

Here are the results under **PY** aggregation:

Policy	Eff. Date	Exp. Date	Total Exposure	Written Exposures			Earned Exposures		
				PY 10	PY 11	PY 12	PY 10	PY 11	PY 12
A	10/01/10	03/31/11	0.50	0.50	0.00	0.00	0.50	0.00	0.00
B	01/01/11	06/30/11	0.50	0.00	0.50	0.00	0.00	0.50	0.00
C	04/01/11	09/30/11	0.50	0.00	0.50	0.00	0.00	0.50	0.00
D	07/01/11	12/31/11	0.50	0.00	0.50	0.00	0.00	0.50	0.00
E	10/01/11	03/31/12	0.50	0.00	0.50	0.00	0.00	0.50	0.00
F	01/01/12	06/30/12	0.50	0.00	0.00	0.50	0.00	0.00	0.50
Total			3.00	0.50	2.00	0.50	0.50	2.00	0.50

As before, written exposures and earned exposures are identical under PY aggregation since exposures are not split across multiple years for individual policies.

Calculation of Blocks of Exposures

In our example, we converted the total exposure of individual policies into written exposure, earned exposure, etc. However, in many cases, we need to calculate exposures using summarized data.

When doing so, we **assume that all policies are written on the mid-point of the period**. So, if data is summarized by month, we assume that each policy is written on the “15th” of the month.

Assuming policies are evenly written throughout the year, this is a reasonable approximation.

Example 3

The table below calculates in-force exposures and earned exposures using blocks of annual policies:

Written Month	Exposure	Assumed Eff. Date	In-Force Exposures as of:			Earned Exposures in CY:	
			07/01/10	01/01/11	07/01/11	2010	2011
Jan. 2010	240	01/15/10	240	240	0	230	10
Feb. 2010	240	02/15/10	240	240	0	210	30
Mar. 2010	240	03/15/10	240	240	0	190	50
Apr. 2010	240	04/15/10	240	240	0	170	70
May 2010	240	05/15/10	240	240	0	150	90
Jun. 2010	240	06/15/10	240	240	0	130	110
Jul. 2010	240	07/15/10	0	240	240	110	130
Aug. 2010	240	08/15/10	0	240	240	90	150
Sep. 2010	240	09/15/10	0	240	240	70	170
Oct. 2010	240	10/15/10	0	240	240	50	190
Nov. 2010	240	11/15/10	0	240	240	30	210
Dec. 2010	240	12/15/10	0	240	240	10	230
Total	2,880		1,440	2,880	1,440	1,440	1,440

First, let's look at the in-force exposures above:

- For each written month, we assume that the effective date is on the “15th” of the month. Thus, at 07/01/10, we have 1,440 policies in-force written during the months of January 2010 – June 2010. The July policies are not counted since we assume that they are written on 07/15/10, which is after 07/01/10
- At 01/01/11, all 2,880 written policies are in-force. This includes the 240 policies written during January 2010 since the assumed effective date was 01/15/10 (which means the assumed expiration date for these policies is 01/14/11)
- At 07/01/11, the 1,440 policies written from January 2010 – June 2010 have expired. Thus, they are no longer in-force

Next, let's look at the earned exposures above:

- Since we assume an effective date on the “15th” of each month, we can split each month into half-months. Thus, there are 24 half-months in each year
- The policies written in January 2010 earn 23/24 of their total exposures in CY 2010 and 1/24 of their total exposures in CY 2011. Thus, they contribute $(23/24)(240) = 230$ earned exposures to CY 2010 and $(1/24)(240) = 10$ earned exposures to CY 2011
- The policies written in December 2010 earn 1/24 of their total exposures in CY 2010 and 23/24 of their exposures in CY 2011. Thus, they contribute $(1/24)(240) = 10$ earned exposures to CY 2010 and $(23/24)(240) = 230$ earned exposures to CY 2011
- The other months work in a similar way

IV. Exposure Trend

As mentioned in Chapter 1, we want to balance the Fundamental Insurance Equation for the period in which the rates will be in effect. Hence, we want to project the premium, losses, expenses, and exposures to the future.

For some LOBs, the exposure measure is sensitive to time-related influence. For example, all else being equal, we expect a company's payroll to increase each year due to inflation. In these cases, we should measure the trend in historical exposures over time in order to project future exposure levels. **Two ways to measure these trends** are as follows:

- Internal data, such as workers' compensation payroll
- Industry indices, such as the average wage index

We will discuss exposure trends in more detail in later chapters.

Werner & Modlin

Ch. 5 (Premium)

Outline

As shown in Chapter 1, the Fundamental Insurance Equation is as follows:

$$\text{Premium} = (\text{Losses} + \text{LAE} + \text{UW Expenses}) + \text{UW Profit}$$

The next few chapters will walk through each component of the equation above and explain how we prepare each component for a ratemaking analysis. We begin with premium.

I. Premium Aggregation

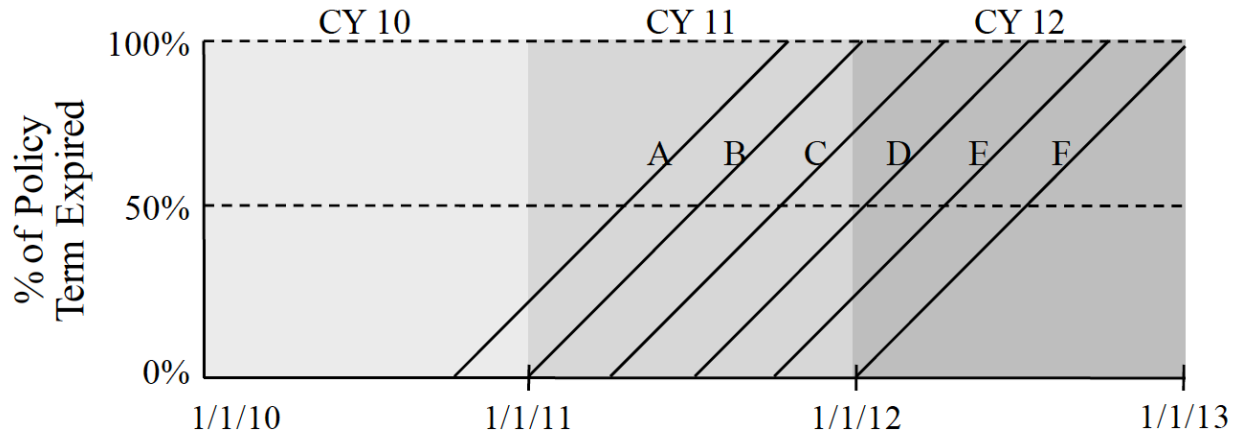
In Chapter 4, we covered exposure aggregation. Premium aggregation **works in the exact same way**. The only difference is that we are calculating written premium, earned premium, unearned premium, and in-force premium instead of written exposures, earned exposures, unearned exposures, and in-force exposures. We will show a brief example to demonstrate CY aggregation of premium.

Example 1

Given the following homeowners policies with annual terms:

Policy	Effective Date	Expiration Date	Premium
A	10/01/10	09/30/11	\$200
B	01/01/11	12/31/11	\$250
C	04/01/11	03/31/12	\$300
D	07/01/11	06/30/12	\$400
E	10/01/11	09/30/12	\$350
F	01/01/12	12/31/12	\$225

These are the same six policies from the Chapter 4 example. The only difference is that we focus on each policy’s premium instead of each policy’s exposure. To aggregate the premium by CY, we consider **the same picture we saw in Chapter 4**:



We aggregate **written premium (WP)** and **earned premium (EP)** in the same way we aggregated written exposures and earned exposures. If a policy is written in CY 10, then its entire premium is contributed to that CY. For earned premium, we use the “even writing” assumption to determine the percentage of each diagonal line that falls within each CY. For example, since policy A is written on 10/01/10, 25% (i.e., 3 months out of 12 months) of the total premium for Policy A is earned in CY 10. The remaining 75% of the total premium for Policy A is written in CY 11. Thus, Policy A contributes \$50 (25% x \$200) of earned premium to CY 10 and \$150 (75% x \$200) to CY 11. Let’s look at the results:

Policy	Eff. Date	Exp. Date	Total Prem.	Written Premium			Earned Premium		
				CY 10	CY 11	CY 12	CY 10	CY 11	CY 12
A	10/01/10	09/30/11	\$200	\$200	\$0	\$0	\$50	\$150	\$0
B	01/01/11	12/31/11	\$250	\$0	\$250	\$0	\$0	\$250	\$0
C	04/01/11	03/31/12	\$300	\$0	\$300	\$0	\$0	\$225	\$75
D	07/01/11	06/30/12	\$400	\$0	\$400	\$0	\$0	\$200	\$200
E	10/01/11	09/30/12	\$350	\$0	\$350	\$0	\$0	\$87.50	\$262.50
F	01/01/12	12/31/12	\$225	\$0	\$0	\$225	\$0	\$0	\$225
Total			\$1,725	\$200	\$1,300	\$225	\$50	\$912.50	\$762.50

For an individual policy, unearned premium = written premium – earned premium at the time being considered. For example, at 12/31/10, Policy A has an unearned premium of \$150 = \$200 – \$50. **For groups of policies**, the formula for unearned premium depends on the method of data aggregation:

- CY Aggregation: $CY \text{ Unearned Premium} = CY \text{ Written Premium} - CY \text{ Earned Premium} + \text{Unearned Premium as of the beginning of the CY}$
- PY Aggregation: $PY \text{ Unearned Premium} = PY \text{ Written Exposures} - PY \text{ Earned Premium}$

Let's see the CY Unearned Premium formula in action:

- CY 10 Unearned Premium at 12/31/10 = \$200 – \$50 + \$0 = **\$150**
- CY 11 Unearned Premium at 12/31/11 = \$1,300 – \$912.50 + **\$150** = **\$537.50**
- CY 12 Unearned Premium at 12/31/12 = \$225 – \$762.50 + **\$537.50** = \$0

As expected, there is no unearned premium at the end of CY 12. Also, if we had applied the “individual policy” formula of WP – EP and summed them up for each CY, we would get the same totals.

Like in-force exposures, **in-force premium** is simple to calculate. For a given point in time, we simply add up the total premium for any policy in effect at that time. The only **complication that can arise with in-force premium** is related to mid-term adjustments. Suppose Policy D is adjusted on January 1, 2012, resulting in an increase in the full-term premium from \$400 to \$800. For any date between July 1, 2011 (Policy D's effective date) and December 31, 2011 (the date before the mid-term adjustment), the in-force premium for Policy D is \$400. For any date between January 1, 2012 (Policy D's adjustment date) and June 30, 2012 (Policy D's expiration date), the in-force premium for Policy D is \$800.

The example above showed how to aggregate CY WP, CY EP, CY unearned premium, and in-force premium (note that an “aggregation year” does not apply to in-force premium; in-force premium is calculated the same way regardless of the aggregation method). **We will not show aggregation by PY since it works in the same manner as shown in the Chapter 4 example for exposures.**

Policies Other Than Annual

As shown in Chapter 4, the aggregation concepts work in the same way regardless of the length of the policy. The only difference here is that we are aggregating premium instead of exposures.

One **thing worth noting** is that one must be careful when **comparing in-force premium for portfolios made up of policies with different policy terms**. For example, suppose insurer A and insurer B each write \$10,000,000 in premium per year. Further suppose that insurer A writes annual term policies and insurer B writes semi-annual policies. At any given point in time, the in-force premium for insurer A will be twice that of insurer B even though they have the same annual written premium.

Calculation of Blocks of Policies

Once again, this works in the same way as described in Chapter 4. Using the “even writing” assumption, we treat all policies as if they were written on the “15th” of each month. Thus, policies written in January are assumed to be $(23/24) = 95.833\%$ earned by the end of the year. If the total premium for policies written in January 2010 is \$1,000, then the CY 2010 earned premium for these policies is $\$1,000(0.9583) = \958.33 .

II. Adjustments to Premium

There are **three critical adjustments** that must be made to premium during the ratemaking process:

- 1) Bring historical premium to the current rate level
- 2) Develop premium to ultimate levels (if necessary)
- 3) Project historical premium to the premium level expected in the future

Current Rate Level

During the ratemaking process, we combine several years of historical premium data. Assuming rate changes have occurred over time, that historical premium data will consist of premium from policies written at different rates. Mixing premium written at different rates without proper adjustment can lead to excessive or deficient rates.

For **example**, assume that an actuary combines three years of historical data (2010 – 2012) and that all policies written during that period had a written premium of \$200. Further assume that a 5% rate increase was implemented on July 1, 2012. Thus, the **current rate is \$210 = \$200(1.05)**. Finally, assume that the **true indicated rate is \$220**. What could go wrong in this scenario?

- If the actuary fails to account for the 5% rate increase, they will conclude that the rate needs to be increased by 10% (from \$200 to \$220)
- However, a 10% increase will raise the rate from \$210 (the current rate) to \$231 = \$210(1.10), which is excessive (since it's greater than the true indicated rate)
- If the actuary had restated the historical premium to the current rate level of \$210, they would have concluded that only a 4.8% increase was necessary to raise the rate from \$210 to \$210(1.048) = \$220

Next, we will discuss two methods for bringing historical to the current rate level (also known as **on-leveling the premium**). To fully demonstrate one of the methods, it's helpful to use an actual rating algorithm. Given the following **rating algorithm and rate changes**:

$$\text{Premium} = \text{Exposure} \times \text{Rate per Exposure} \times \text{Class Factor (X, Y, or Z)} + \text{Policy Fee}$$

- July 1, 2010: Base rate was increased, resulting in an overall average rate level increase of 5%
- January 1, 2011: Base rate and policy fee were adjusted, resulting in an overall average rate level increase of 10%
- April 1, 2012: Policy fee and class Y and Z rate relativities were changed resulting in an average rate level decrease of -1%

The following table highlights the changes in the rating algorithm over time:

Rate Level Group	Effective Date	Overall Average Rate Change	Rate Per Exposure	Class Factor			Policy Fee
				X	Y	Z	
1	Initial	--	\$900	1.00	0.60	1.10	\$1,000
2	07/01/10	5.0%	\$950	1.00	0.60	1.10	\$1,000
3	01/01/11	10.0%	\$1,045	1.00	0.60	1.10	\$1,100
4	04/01/12	-1.0%	\$1,045	1.00	0.70	1.05	\$1,090

At first glance, it seems strange that a 5% rate increase changes the rate per exposure from \$900 to \$950. This is a change in rate per exposure of $950/900 - 1 = 5.6\%$. So, how does the 5% rate increase fit into the picture? The overall rate change considers the **average change in the total premium per policy**, which accounts for the rate per exposure, the number of exposures per policy, the class factors, and the policy fee. For example, suppose we have two policies with Class Factor X:

- The total premium before the rate change is $2[(\$900)(1.00) + \$1,000] = \$3,800$
- The total premium after the rate change is $2[(\$950)(1.00) + \$1,000] = \$3,900$
- For these two policies, the rate change is $3,900/3,800 - 1 = 2.6\%$

Thus, when we consider every policy for this insurer and all the applicable class factors, it is the case that a rate per exposure of \$950 leads to an overall average rate change of 5%. This will make more sense when we get to **off-balancing**.

Alrighty, let's dive into our two methods for bringing historical premiums to the current rate level:

- 1) Extension of Exposures
- 2) Parallelogram Method

Extension of Exposures

Under the extension of exposures method, we rerate every policy under the current rating algorithm. This makes intuitive sense. If we rerate every policy under the current rating algorithm, then surely the sum of all the policy premiums represents the total premium under the current rate level.

An **advantage of this method** is that it is the most accurate method for bringing historical premiums to the current rate level.

A **disadvantage of this method** is that it requires the actuary to know the applicable rating characteristics for every policy in the historical period, which is often not available.

Example 2

Assume an actuary wants to adjust the historical premium for Policy Year 2011 to the current rate level. Further suppose a policy was effective on March 1, 2011 and had ten Class Y exposures:

- The actual premium charged was based on the rate effective on January 1, 2011. The total premium based on these rates was $\$7,370 = 10[(\$1,045)(0.60)] + \$1,100$
- The on-level premium is based on the rates currently in effect. The total premium based on current rates is $\$8,405 = 10[(\$1,045)(0.70)] + \$1,090$

The total PY 2011 on-level premium is found by performing the on-level calculation above for every policy written in 2011 and aggregating those premiums.

Parallelogram Method

Given that the information needed to perform the extension of exposures method is not always available, actuaries may choose to approximate the on-level premium using the parallelogram method. Under this method, we **assume that premium is written evenly throughout the period**. We adjust the aggregated historical premium by an **average factor** to put the premium on-level. Since the method differs by policy term, method of aggregation (CY vs. PY), etc., we will look at several examples.

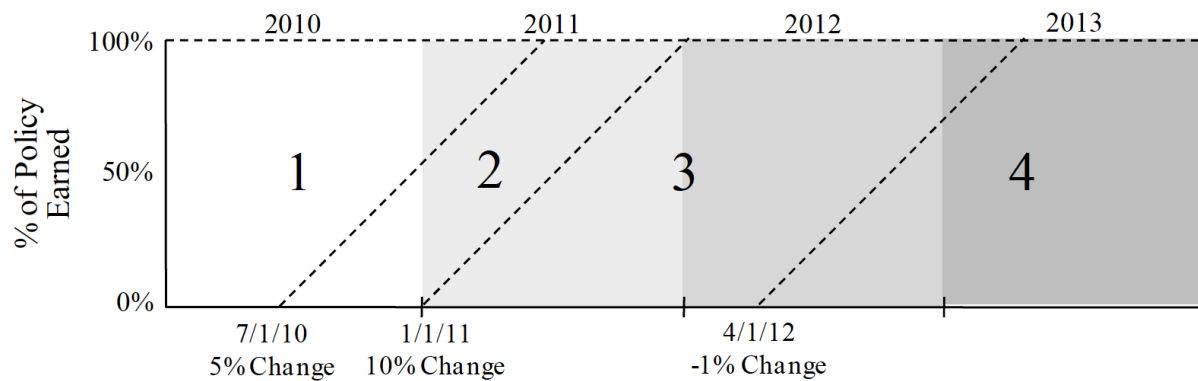
Example 3 (CY EP – Annual Policies)

Step 1: Gather the effective date and overall average rate change for the different rate level groups:

Rate Level Group	Effective Date	Overall Average Rate Change
1	Initial	--
2	07/01/10	5.0%
3	01/01/11	10.0%
4	04/01/12	-1.0%

Step 2: Use geometry to calculate the portion of each CY's EP that corresponds to each unique rate level group:

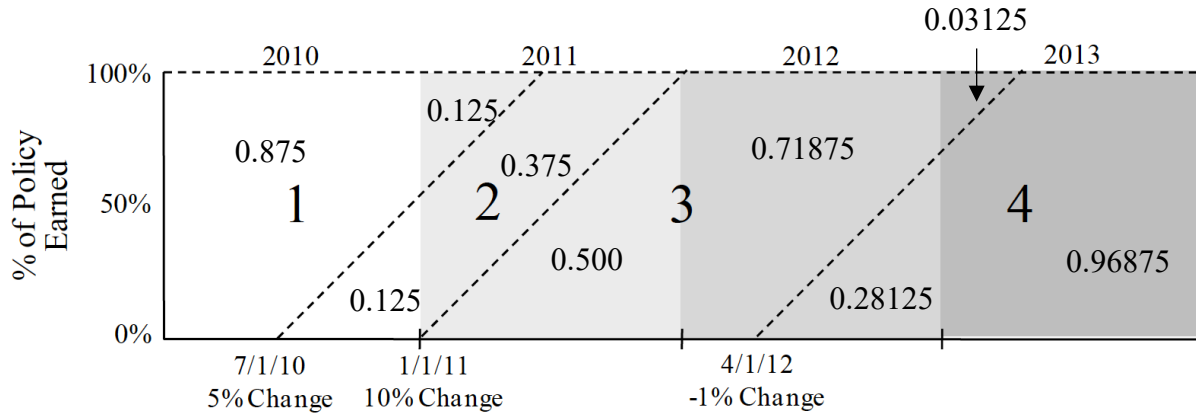
For Step 2, it's extremely helpful to consider the following picture:



How do we interpret the table above?

- Each diagonal line represents a rate change. Since the underlying policies are annual policies, the diagonal lines move from 0% to 100% over a one-year period. We saw a similar picture in Chapter 4
- This allows us to sub-divide the historical experience into rate level groups with the rate level groups representing the different sets of rates that were in effect
- As shown in Chapter 4, we represent each CY as a square. In each CY, the portion before the diagonal line represents the “old” rate level group. The portion after the diagonal line represents the “new” rate level group
- For example, CY 2011 consists of three different rate level groups. Policies written before 07/01/10 were written under Rate Level Group 1. These policies earn a small amount of premium in CY 2011 (the upper left triangle). Policies written between 07/01/10 and 01/01/11 were written under Rate Level Group 2. These policies earn a modest amount of premium in CY 2011 (the trapezoid in the middle). Policies written between 01/01/11 and 01/01/12 earn a large amount (half actually) of premium in CY 2011 (the large triangle after the trapezoid)
- The other CYs are interpreted in the same manner

Now, we calculate the portion of each CY that resides in each Rate Level Group. We do this by calculating the percentage of each square that resides in each Rate Level Group:



The calculations for CY 2011 are as follows:

- Rate Level Group 1 (top left triangle): The area of a triangle is $0.5 \times \text{Base} \times \text{Height}$. If we think of each square as 1×1 , the base of this triangle is 0.50 and the height of this triangle is 0.50. Thus, $\text{Area} = 0.50(0.50)(0.50) = 0.125$
- Rate Level Group 3 (large triangle): $\text{Area} = 0.50(1.00)(0.50) = 0.500$
- Rate Level Group 2 (trapezoid): Since the area of the entire square is 1.00, we can subtract the areas of the triangles to obtain the area of the trapezoid. Thus, $\text{Area} = 1.00 - 0.125 - 0.500 = 0.375$

The calculations for CY 2013 are as follows:

- Rate Level Group 3 (top left triangle): $\text{Area} = 0.50(0.25)(0.25) = 0.03125$. Notice that the base and height of this triangle are both 0.25. This is because the policy was written on 04/01/12 and only the last three months of policy earn out in CY 2013
- Rate Level Group 4 (trapezoid): $\text{Area} = 1.00 - 0.03125 = 0.96875$

Step 3: Determine the **cumulative rate level index** for each rate level group:

This is a simple calculation. Using 1.00 as the rate level index for Rate Level Group 1, the cumulative rate level index of each subsequent group is the prior group's cumulative rate level index multiplied by the rate level index (which equals 1.00 + percentage rate change) for that group. Here are the results:

Rate Level Group	Effective Date	Overall Average Rate Change	Rate Level Index	Cumulative Rate Level Index
1	Initial	--	1.00	1.0000
2	07/01/10	5.0%	1.05	1.0500
3	01/01/11	10.0%	1.10	1.1550
4	04/01/12	-1.0%	0.99	1.1435

In the table above, the rate level index for each rate level group is (1 + rate change). Then, we accumulate these to obtain the cumulative rate level index. For example, the cumulative rate level index for rate level group 3 is $1.1550 = 1.00(1.05)(1.10)$ and the cumulative rate level index for rate level group 4 is $1.1435 = 1.00(1.05)(1.10)(0.99)$.

Step 4: Calculate the **average rate level index** for each CY by taking the weighted average of the cumulative rate indices in Step 3, where the weights are equal to the areas calculated in Step 2:

- CY 2010 average rate level index = $0.875(1.0000) + 0.125(1.0500) = 1.00625$
- CY 2011 average rate level index = $0.125(1.0000) + 0.375(1.0500) + 0.500(1.1550) = 1.09625$
- CY 2012 average rate level index = $0.71875(1.1550) + 0.28125(1.1435) = 1.15177$
- CY 2013 average rate level index = $0.03125(1.1550) + 0.96875(1.1435) = 1.14386$

Step 5: Calculate the on-level factor for each AY as follows:

$$\text{On - Level Factor} = \frac{\text{Current Cumulative Rate Level Index}}{\text{Average Rate Level Index}}$$

Here are the results by CY:

CY	Current Cumulative Rate Level Index	Average Rate Level Index	On-Level Factor
2010	1.1435	1.00625	1.13640
2011	1.1435	1.09625	1.04310
2012	1.1435	1.15177	0.99282
2013	1.1435	1.14386	0.99969

The on-level factors tell us how much we need to change the total CY earned premium to obtain the CY on-level earned premium. For example, the CY 2013 earned premium needs to be multiplied by 0.99969 to restate it at the current rate level.

Step 6: Calculate the on-level earned premium for each CY by multiplying each CY's earned premium by the on-level factor for that CY:

Suppose the CY 2013 earned premium is \$1,000. Then, the CY 2013 on-level premium is $\$1,000(0.99969) = \999.69 .

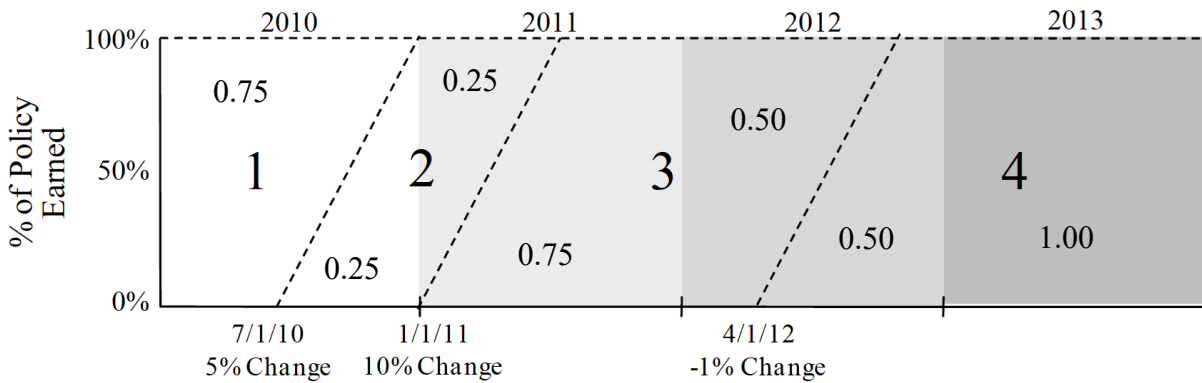
Example 4 (CY EP – Six-Month Policies)

Step 1: Gather the effective date and overall average rate change for the different rate level groups:

Rate Level Group	Effective Date	Overall Average Rate Change
1	Initial	--
2	07/01/10	5.0%
3	01/01/11	10.0%
4	04/01/12	-1.0%

Step 2: Use geometry to calculate the portion of each CY's EP that corresponds to each unique rate level group:

Like Example 3, it's helpful to represent this pictorially:



Comparing the picture above to the one in Example 3, the only difference is the steepness of the diagonal lines. Since these are six-month policies, the lines only extend over a six-month period. The calculations for CY 2011 are as follows:

- Rate Level Group 2 (triangle): The base of this triangle is 0.50 and the height of this triangle is 1.00. Thus, $\text{Area} = 0.50(0.50)(1.00) = 0.25$
- Rate Level Group 3 (trapezoid): $\text{Area} = 1 - 0.25 = 0.75$

The calculations for CY 2012 are as follows:

- Rate Level Group 3 (left trapezoid): The area of a trapezoid is $0.5 \times (\text{Base 1} + \text{Base 2}) \times \text{Height}$. Let's call the bottom of the trapezoid "Base 1" and the top of the trapezoid "Base 2." Then, $\text{Base 1} = 0.25$ (3 months out of 12 months) and $\text{Base 2} = 0.75$ (9 months out of 12 months). The height of the trapezoid is 1.00. Thus, $\text{Area} = 0.50(0.25 + 0.75)(1.00) = 0.50$.
- Rate Level Group 4 (right trapezoid): $\text{Area} = 1.00 - 0.50 = 0.50$

Steps 3 – 6 work in the same manner as Example 3. Here are the results by CY:

CY	Current Cumulative Rate Level Index	Average Rate Level Index	On-Level Factor
2010	1.1435	1.01250	1.12938
2011	1.1435	1.12875	1.01307
2012	1.1435	1.14925	0.99500

2013	1.1435	1.14350	1.00000
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Since semi-annual rate changes earn more quickly, we find that the **on-level adjustment for semi-annual policies is smaller than for annual policies.**

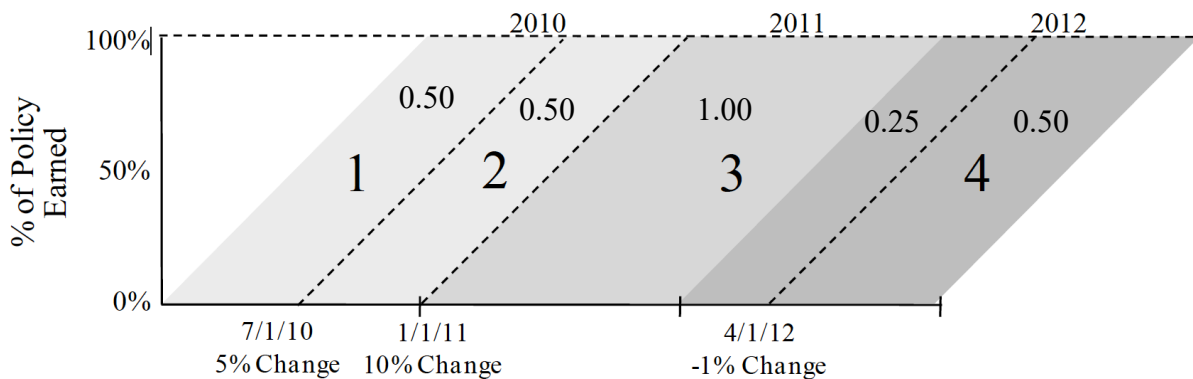
Example 5 (PY EP – Annual Policies)

Step 1: Once again, we gather the effective date and overall average rate change for the different rate level groups:

Rate Level Group	Effective Date	Overall Average Rate Change
1	Initial	--
2	07/01/10	5.0%
3	01/01/11	10.0%
4	04/01/12	-1.0%

Step 2: Use geometry to calculate the portion of each CY’s EP that corresponds to each unique rate level group:

Like Examples 3 and 4, it’s best to create a picture:



How is the picture above different from the calendar year pictures in Examples 3 and 4?

- As discussed in Chapter 4, PYs are represented by parallelograms instead of squares!
- The goal is to calculate the area of each Rate Level Group in each separate parallelogram

The calculations for PY 2010 are as follows:

- Rate Level Group 1 (left parallelogram): The area of a parallelogram is Base x Height. For Rate Level Group 1, the Base = 0.5 (6 months out of 12 months) and the Height = 1.00. Thus, the Area = $0.50(1.00) = 0.50$
- Rate Level Group 2 (right parallelogram): Area = $1.00 - 0.50 = 0.50$
- These two smaller parallelograms make up the area of the light-gray parallelogram, which is PY 2010

Since PY 2011 consists solely of Rate Level Group 3, the Area = 1.00. Here are the calculations for PY 2012:

- Rate Level Group 3 (left parallelogram): The Base = 0.25 (3 months out of 12 months) and the Height = 1.00. Thus, the Area = $0.25(1.00) = 0.25$
- Rate Level Group 4 (right parallelogram): Area = $1.00 - 0.20 = 0.75$

Steps 3 – 6 work in the same manner as Examples 3 and 4. Here are the results by PY:

PY	Current Cumulative Rate Level Index	Average Rate Level Index	On-Level Factor
2010	1.1435	1.02500	1.11561
2011	1.1435	1.15500	0.99004
2012	1.1435	1.14638	0.99749
2013	1.1435	1.14350	1.00000

In Examples 3 – 5, each rate change only applied to policies effective on or after that date. Thus, each rate change was represented by a diagonal line. But occasionally, rate changes are in response to law changes that may **mandate the rate change be applied to all policies on or after a specific date, including in-force policies**. Since existing in-force policies are impacted by these types of rate changes, the rate change is **represented as a vertical line rather than a diagonal line**.

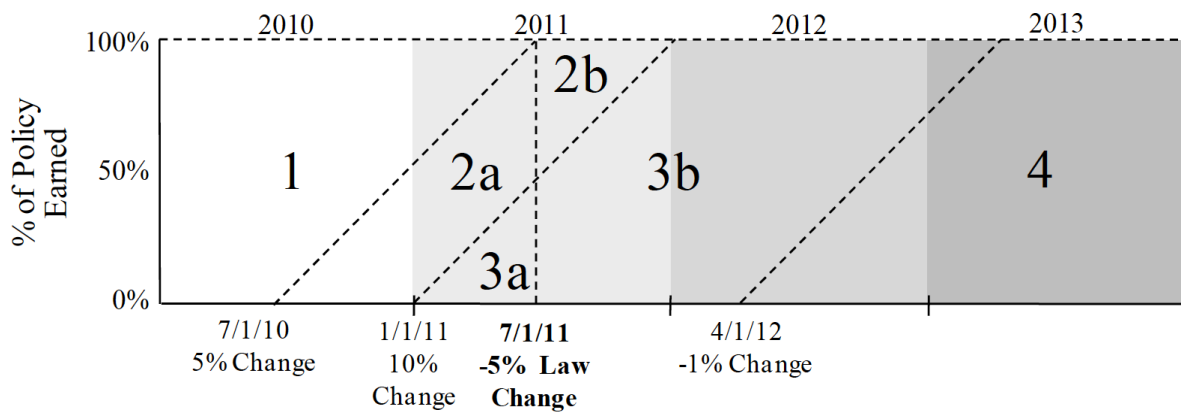
Obviously, this impacts the area calculations.

Example 6 (CY EP – Annual Policies – Including Rate Change Mandated by Law)

Step 1: Once again, we gather the effective date and overall average rate change for the different rate level groups:

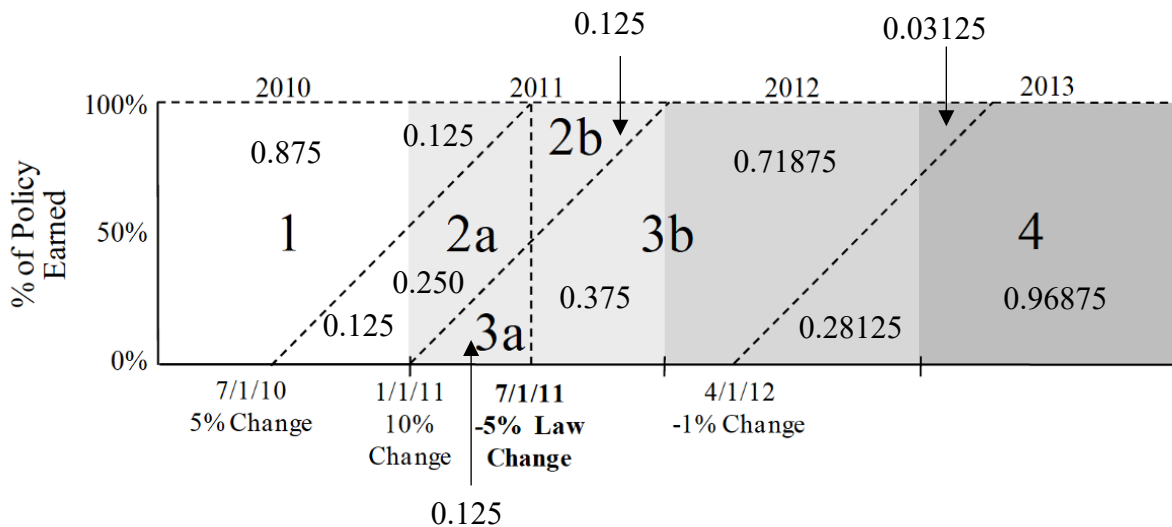
Effective Date	Overall Average Rate Change	Mandated?	Impacted Policies
Initial	--	No	New Policies
07/01/10	5.0%	No	New Policies
01/01/11	10.0%	No	New Policies
07/11/11	-5.0%	Yes	All Policies
04/01/12	-1.0%	No	New Policies

Let's look at the rate changes pictorially:



As we see in the picture above, the -5% rate change mandated by law impacts all policies and is represented by a vertical line. At the same time, the 07/01/10 and 01/01/11 rate changes implemented by the insurer are still earning out. The law change has split Rate Level Groups 2 and 3 into four pieces: 2a, 2b, 3a, and 3b. The “a” rate changes represent life before the mandated rate change of -5%. The “b” rate changes represent life after the mandated rate change.

Step 2: Use geometry to calculate the portion of each CY's EP that corresponds to each unique rate level group:



The calculations for CY 2011 are as follows:

- Rate Level Group 1: Area = $0.50(0.50)(0.50) = 0.125$
- Rate Level Group 2b: Area = $0.50(0.50)(0.50) = 0.125$
- Rate Level Group 3a: Area = $0.50(0.50)(0.50) = 0.125$
- Rate Level Group 3b: Area = $0.50 - 0.125 = 0.375$ (using the fact that 2b and 3b clearly represent half of CY 2011)
- Rate Level Group 2a: Area = $1.00 - 0.125 - 0.125 - 0.125 - 0.375 = 0.250$

Step 3: Determine the **cumulative rate level index** for each rate level group:

Rate Level Group	Cumulative Rate Level Index
1	1.0000
2a	1.0500
2b	$0.9975 = 1.05(0.95)$
3a	$1.1550 = 1.05(1.10)$
3b	$1.0973 = 1.1550(0.95)$
4	$1.0863 = 1.0973(0.99)$

Notice that 2b and 3b both include the -5% mandated rate change. Then, we obtain the Cumulative Rate Level Index for Rate Level Group 4 by applying the -1% rate change to the 3b cumulative index of 1.0973.

Step 4: Calculate the **average rate level index** for each CY by taking the weighted average of the cumulative rate indices in Step 3, where the weights are equal to the areas calculated in Step 2:

- CY 2010 average rate level index = $0.875(1.0000) + 0.125(1.0500) = 1.00625$
- CY 2011 average rate level index = $0.125(1.0000) + 0.250(1.0500) + 0.125(0.9975) + 0.125(1.1550) + 0.375(1.0973) = 1.06805$
- CY 2012 average rate level index = $0.71875(1.0973) + 0.28125(1.0863) = 1.09421$
- CY 2013 average rate level index = $0.03125(1.0973) + 0.96875(1.0863) = 1.08664$

Step 5: Calculate the on-level factor for each AY as follows:

Here are the results by CY:

CY	Current Cumulative Rate Level Index	Average Rate Level Index	On-Level Factor
2010	1.0863	1.00625	1.07955
2011	1.0863	1.06805	1.01709
2012	1.0863	1.09421	0.99277
2013	1.0863	1.08664	0.99969

Step 6 works in the same manner as shown in the prior examples.

There are a **couple of issues with the Parallelogram Method:**

- 1) It assumes policies are evenly written throughout the year
 - This is not appropriate for all LOBs (ex. boat-owners policies are often purchased in the first half of the year prior to the start of boat season)
 - We can somewhat address uneven writings by performing the Parallelogram Method on more granular periods, such as months or quarters
 - A more accurate way to address uneven writings is by weighting the average Rate Level Indices with the exact distribution of written policies by Rate Level Group (rather than using weights informed by geometric areas)

2) It is applied at the aggregate level

- For rate changes with significant changes by rating classification (ex. age of driver for personal automobile), the Parallelogram Method fails to on-level premium for each classification. In other words, it's only accurate at the aggregate level. For this reason, Extension of Exposures is often used for LOBs with complex rating structures

Premium Development

There are **two common situations** in which the actuary may not know the ultimate amount of premium for the experience period being used in a ratemaking analysis:

- 1) Incomplete Years
- 2) Premium Audits

Incomplete Years

Suppose an actuary includes PY 2009 – 2011 data as of December 31, 2011 in a ratemaking analysis. In this case, the PY 2011 premium is incomplete because the PY is not fully earned until December 31, 2012 (assuming annual policies). Although all of the PY 2011 policies have been written as of December 31, 2011, the total premium could still change between December 31, 2011 and December 31, 2012 due to mid-term adjustments or cancellations. The actuary can estimate the ultimate PY 2011 premium by analyzing historical patterns of premium development to understand the impacts of cancellations and mid-term adjustments on policy year premium.

Premium Audits

Certain LOBs utilize premium audits to determine the final premium. For example, recall that the most common exposure base for workers' compensation coverage is payroll. The final payroll for the policy period is unknown at the policy issue date. The final premium is determined by payroll audits completed three to six months after the policy expires.

Example 7: Given the following for a workers' compensation insurer:

- In 2011, the insurer writes one policy per month

- The estimated premium for each policy is \$500,000. This premium is booked at policy inception
- The premium on every policy develops upward by 8% at the first audit, which occurs six months after the policy expires

Here's how the PY 2011 premium changes over time:

- At 12/31/11: No policy has had an audit. The premium is $\$500,000(12) = \$6,000,000$
- At 12/31/12: The first six policies have had their audit, while the last six policies have not. The premium is $\$500,000(1.08)(6) + \$500,000(6) = \$6,240,000$
- At 12/31/13: All policies have been audited. The premium is $\$500,000(1.08)(6) = \$6,480,000$

The 12-24 premium development factor is $\$6,240,000/\$6,000,000 = 1.04$. The 24-36 premium development factor is $\$6,480,000/\$6,240,000 = 1.0385$. In a future ratemaking analysis, we can use these factors to adjust policy premium at 12 months and 24 months to ultimate.

Note that premium development does NOT typically apply to CY premium since calendar year premium is fixed at the end of the year.

Premium Trend

In Chapter 4, we discussed the concept of exposure trend. In general, exposure trend applies when exposures are inflation-sensitive (such as payroll). This means inflationary pressures impact exposures, which in turn impacts premium.

In addition to inflationary pressures, the average premium level is also impacted by **distributional changes**. When we refer to premium trend, these are the changes we are referencing. **Examples of distributional changes include:**

- Gradual changes in rating characteristics
 - Ex: Homeowners rating structures include amount of insurance (AOI) as a rating variable. In general, AOI automatically increases with inflation. Thus, all else being equal, the average premium level should increase each year as a result

- Coverage changes
 - Ex: An insurer decides to move all existing insureds to a higher deductible. All else being equal, the average premium level should decrease due to the higher deductible. Assuming the insurer moves each insured to the higher deductible upon renewal, the premium level should gradually decrease over the year until all insureds have renewed. The trend should not continue after the renewal period has completed
- Portfolio purchases
 - Ex: A company purchases the portfolio of another company. If the new risks are different from the existing risks, there may be a significant one-time change in the average premium level. This trend should not continue once the books are fully consolidated

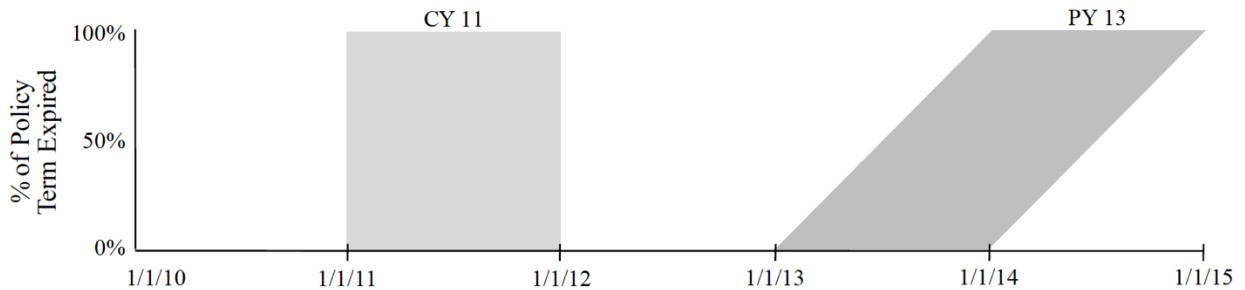
Basic ground rules for a premium trend analysis are as follows:

- **Use average premium per exposure instead of total premium** – A company that is growing (or shrinking) will have increasing (or decreasing) total premium even if there are no distributional changes. The exposure used should be the exposure base underlying the rate
- **Use written premium instead of earned premium** – Since written premium reflects distributional shifts more quickly than earned premium, average written premium is typically used for a premium trend analysis
- **Use quarterly written premium instead of annual written premium** – Assuming the data is available, quarterly data is a better to use since it is more responsive
- **Use on-leveled premium instead of raw premium** – If raw premium is used, it will show significant changes in the premium level corresponding to historical rate changes. If we don't use on-leveled premium, we will have overlapping adjustments

Next, we will look at two examples. The first example uses One-Step Trending while the second example uses Two-Step Trending.

Example 8 (One-Step Trending)

An actuary is using CY 2011 earned premium for a ratemaking analysis to estimate the rate need for annual policies that are to be written between January 1, 2013 and December 31, 2013. Pictorially, we have the following:



As we see in the picture above, CY 2011 is represented as a square. The future period refers to all annual policies written during the period of 01/01/13 – 12/31/13. We call this PY 2013 and represent it as a parallelogram.

The final trend factor that we need to apply to the CY 2011 earned premium is $(1 + \text{Selected Annual Trend})^{\text{Trend Period}}$. Let's discuss each component of the formula.

Selected Annual Trend

To select an annual trend for the insurer, we analyze the following data:

Quarter	WP at Current Rate Level (\$)	Written Exposures	Average WP at Current Rate Level (\$)	Annual Change
1Q09	323,189.17	453	713.44	--
2Q09	328,324.81	458	716.87	--
3Q09	333,502.30	463	720.31	--
4Q09	338,721.94	468	723.76	--
1Q10	343,666.70	472	$728.11 = \frac{343,666.70}{472}$	$2.1\% = \frac{728.11}{713.44} - 1$
2Q10	348,696.47	477	731.02	2.0%
3Q10	353,027.03	481	733.94	1.9%
4Q10	358,098.58	485	738.35	2.0%

1Q11	361,754.88	488	741.30	1.8%
2Q11	367,654.15	493	745.75	2.0%
3Q11	372,305.01	497	749.10	2.1%
4Q11	377,253.00	501	753.00	2.0%

The data above follows the basic ground rules from earlier. It uses on-leveled average premium per exposure. Based on the annual changes above, we select an annual trend of 2.0%.

Trend Period

Under one-step trending, we trend from the average written date of premium earned of the historical period to the average written date of premium earned of the future period:

- Historical Period
 - The CY 2011 earned premium contains premium from policies written from 01/02/10 – 12/31/11. Thus, the average written date is 01/01/11. This is the “**trend from**” date
- Future Period
 - Since the future period is PY 2013, defined as all policies written from 01/01/13 – 12/31/13, it’s simpler. The average written date is clearly 06/30/13. This is the “**trend to**” date
- The trend period is calculated as Trend To Date – Trend From Date = 06/30/13 – 01/01/11 = 2.5 years.

The final premium trend factor is $(1 + 0.02)^{2.5} = 1.0508$. Thus, if the historical CY 2011 earned premium was \$1,000,000, the projected premium in the future period in which the new rates would be in effect is $\$1,000,000(1.0508) = \$1,050,800$.

How does the trend period change if we have six-month policies instead of annual policies?

- Historical Period
 - The CY 2011 earned premium contains premium from policies written from 07/02/10 – 12/31/12. Thus, the average written date is 04/01/11

- Future Period
 - The future period is still PY 2013. Thus, the average written date is still 06/30/13
- The trend period is calculated as $06/30/13 - 04/01/11 = 2.25$ years

How does the trend period change if the historical premium is PY 2011 instead of CY 2011?

- Historical Period
 - The PY 2011 earned premium contains premium from policies written from 01/01/11 – 12/31/11. Thus, the average written or trend from date is 07/01/11
- Future Period
 - The future period is still PY 2013. Thus, the average written or trend to date is 06/30/13
- The trend period is calculated as $06/30/13 - 07/01/11 = 2$ years

How does the trend period change if the proposed rates are expected to be in effect for more or less than one year?

Suppose the rates will be in effect for two years, from 1/1/13 – 12/31/14. Then, the “trend to” date for annual policies will be 12/31/13. Assuming the historical period is CY 2011 from the Example 8, the trend period is $12/31/13 - 01/01/11 = 3$ years.

Example 9 (Two-Step Trending)

If the trend in the historical period is significantly different from the expected future trend, then two-step trending is recommended. An example of this is the one-time trend caused by moving all insureds to a higher deductible.

Step 1: Adjust each year’s historical premium to the current trend level by applying the following adjustment factor:

$$\text{Current Premium Trend Factor} = \frac{\text{Latest Average WP at Current Rate Level}}{\text{Historical Average EP at Current Rate Level}}$$

Using the data in Example 8, the latest average WP at current rate level is \$753 from Calendar Quarter 4Q2011. Assuming the average EP for CY 2011 is \$740.00, the current premium trend factor is $1.0176 = 753/740$.

In this case, a factor of 1.0176 is needed to adjust CY 2011 EP to the average written date of 4Q2011. The average written date of 4Q2011 is 11/15/2011, which is simply the middle of the fourth quarter. We call this the **“project from”** date.

As an **alternative to the ratio above**, an actuary may choose to select a current trend using several data points when historical average premium is volatile. Then, the actuary would apply $(1 + \text{Selected Annual Current Trend})^{\text{Trend Period}}$ to the CY 2011 EP to adjust it to the average written date of 4Q2011. For example, suppose an actuary selects a current trend of 2%. The trend period runs from the average written date of CY 2011 (01/01/11) to the average written date of 4Q2011 (11/15/11). Thus, this trend factor would be $(1.02)^{0.875} = 1.0175$.

Step 2: Trend historical average EP at the current trend level to the “trend to” date using a selected annual projected trend:

Due to an upcoming initiative to increase deductibles, the actuary chooses a projected premium trend of -1%. The trend period is calculated as follows:

- Historical Period
 - In Step 1, we derived the factor needed to adjust CY 2011 EP to 11/15/11
- Future Period
 - The future period is still PY 2013. Thus, the average written date (i.e., the “trend to” date) is 06/30/13
- The projected trend period is calculated as $06/30/13 - 11/15/11 = 1.625$ years

The projected premium trend factor is $(1 - 0.01)^{1.625} = 0.9838$. Combining this with the current premium trend factor of 1.0176, the final premium trend factor is $1.0176(0.9838) = 1.0011$. Thus, if the CY 2011 EP at Current Rate Level is \$1,440,788, the Projected EP at Current Rate Level (projected to the period in which rates will be in effect) is $\$1,442,373 = \$1,440,788(1.0011)$.