

Exam 8

Cookbook

ADVANCED RATEMAKING

70+ Step-by-Step Recipes
to Solve CAS Calculation Problems

Exam 8 Cookbook

2023 Sitting

Rising Fellow



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Introduction

The purpose of the Exam 8 Cookbook is to prepare you to confidently answer calculation-based problems on exam day without wasting time trying to “think through” a problem-solving approach before typing the solution.

Since the 2016 sitting, hundreds of actuaries have used the Exam 7, Exam 8, and Exam 9 Cookbooks to help them pass and get one step closer to their FCAS.

Our goal with Rising Fellow is to help you prepare for the exam with less frustration so that you have your best exam sitting yet!

The Structure

The Exam 8 Cookbook goes through the different calculation-based problem-types that I believe are reasonably testable based on the syllabus. By exam day, you should know how to solve each one.

Inside, you’ll find a separate section for each testable problem-type. Each section has the following structure:

Original Practice Problem

Each section has an original practice problem that demonstrates the problem-type. I wrote these based off of the syllabus papers to have a similar difficulty-level and style to what you might see on an exam.

Solution Recipe

The solution recipe solves the practice problem from start to finish and shows the step-by-step approach you should take to answer a similar problem. For each step, you’ll see:

- The description for what to do in the step
- The formula(s) necessary for the step
- The formula(s) translated from symbolic notation to plain-English
- Calculations for the step to solve the example problem

Discussion

Each section includes discussion to add clarity and more context. The discussion also covers underlying concepts that might come up on a part b or part c essay question.

For many problems, I point out potential “twists” that could show up on the exam that would make an exam problem more difficult. Since you’ve taken actuarial exams up to this point, you know that straightforward exam problems are more the exception than the rule.

CBT Spreadsheet Tips

This new section provides Excel formulas and tips for how to solve a problem more efficiently in the PearsonVue spreadsheet environment. There are many types of problems where setting up your solution intelligently and taking advantage of the spreadsheet capabilities such as SUMIF(), COUNTIF(), and array formulas, will save you valuable time on the exam.

Source

Each section references the pages in the syllabus reading that you can cross-reference for more information and details. Make sure to check the syllabus section for more context if you get stuck on a problem or to see how the author discusses the concepts.

More Practice

Here, you’ll see references to past CAS problems. You’ll find this helpful especially closer to the exam if there are particular types of problems that you are struggling with. This section includes references to CAS problems from the 2011–2019 exams, the years for the current syllabus structure.

How to Best Use the Exam 8 Cookbook

Below is a suggested guide for how you can incorporate the Exam 8 Cookbook in your own study schedule along with the syllabus material and a typical study manual. This is the general approach that I used when I took my fellowship exams.

For each of those exams I had a main study manual as well as the Exam Cookbook, which I built out while I studied for the exam (but you don’t need to waste time doing that part!)

First pass through the syllabus

While you’re reading a particular paper in the syllabus and your main study manual to learn the material, use the Exam 8 Cookbook to clearly identify what problem-types you need to know from the paper. Study the steps in the solution recipe to learn how to solve the problem-types. Make sure to do some practice problems as you go through the syllabus. This will help you learn faster.

Second pass through the syllabus

Review the steps for the problem-types and make sure you have an intuitive understanding of how to solve the problems. Start working the past CAS problems.

The first level of understanding is to be able to follow the recipe and understand the steps and calculations.

The next level of understanding is to be able to recall and apply the steps to solve a problem without relying on study material. During your second pass, focus on building this deeper level of understanding.

Review and Practice Problems (around 6 weeks to 2 weeks before the exam)

At this point you should have a good understanding of the syllabus and how to use the recipe steps to systematically solve the different calculation problems. During this period, you should be doing lots of problems across the syllabus and targeting problem-types that you are finding particularly challenging. By the end of this phase, you might not have all the formulas memorized, but you should know all the steps and how to apply them to solve problems without needing to think too much before beginning to write the solution.

During this phase, make sure to focus on the types of problems and concepts that you're weak at. This may require some struggle, but struggling with some of the challenging problems will help you master these concepts.

You also should continue building your understanding of the concepts and preparing for essay and more complicated integrative questions. I found it helpful to create flashcards from the papers as well as to re-read sections of the syllabus papers that appear to be likely sources of essay problems.

Final Weeks

In the final weeks, focus on taking practice exams to see problems from the entire syllabus. When taking practice exams, work on your exam strategy to make sure you're able to finish the exam and maximize your points.

Prepare for essay problems in the final weeks by using flashcards to make sure that you know all the details necessary. An approach I found helpful is to say flashcards out loud and to explain the flashcard response in my own words as if I were teaching someone. It sounds weird, but it is a much more efficient way to learn and memorize than simply scanning the front and back of the flashcard.

Prepare for calculation problems by reviewing the recipes in the Exam 8 Cookbook in a similar fashion to how you use flashcards for essay problems. Using this approach on my fellowship exams, I was able to rapidly review the steps and formulas for how to solve each problem-type that might show up on the exam. This was a huge benefit and gave me a lot of confidence going into the exam.

Exam Day

I used the original Exam Cookbooks together with a traditional study manual using the approach above to take my fellowship exams. On exam day, for almost every calculation problem I was able to start writing the solution without wasting time trying to think through how to solve the problem. I had an intuitive understanding of how to solve each of the problems following the step-by-step recipes.

If you follow this approach, you should be able to develop a similar level of understanding and confidence going into the exam room.

Excel Version for Computer-Based Testing Preparation

For each recipe, there is an accompanying Excel version. Make sure to review those so that you know how to solve problems in the spreadsheet format. The CBT Spreadsheet Tips sections and the Excel version showing the formulas and setup for the spreadsheet solution will help you understand how to solve exam problems in the PearsonVue spreadsheet environment.

Errata

I always hated seeing errors in study manuals when I studied for exams, so I make every effort to ensure the study materials are accurate. Nevertheless, there may still be some errors in the final version, so I keep an updated errata. Please make sure to check it regularly for any fixes. The link is below:

<https://risingfellow.com/errata>

If you find any errors, please send me a message using the contact form on the Errata page so that I can make a correction.

Feedback

I am always working to improve the Exam 8 Cookbook and the rest of the Rising Fellow study material. Please send me an email to exam8@RisingFellow.com if you have feedback about any of the following:

- Recipes or sections that are confusing or could be improved
- New recipes I should include in future versions
- Better ways you've found to solve a problem-type in a spreadsheet
- Any comments or other feedback you have

Reviews

If you find the Exam 8 Cookbook helpful this sitting, please leave us a review and let us know how it helped you prepare for the exam. Other actuaries look at reviews to help decide what study material to buy and it's helpful for us to hear feedback from actuaries like you so that we can better understand what's working and what can be improved.

You can leave us a review by sending us an email to info@RisingFellow.com. Thank you!

Good luck as you start studying and I hope this will be your best sitting yet.

Experience Credibility of a Single Exposure

Bailey & Simon

Problem

Given the following information for Class 1 of private passenger auto liability:

Merit Rating Group	Number of Accident-Free Years	Earned Prem. at Present B Rates	Number of Claims Incurred	Earned Car Years
A	3+	1,070,000	1,310	12,800
X	2	70,000	100	810
Y	1	85,000	130	980
B	0	150,000	260	1,720
Total		1,375,000	1,800	16,310

- Calculate the credibility of one exposure with two or more years of accident-free experience.
- Calculate the credibility of one exposure with zero years of accident-free experience.

Solution Recipe

Part a – Credibility for an exposure with n+ accident-free years

- Calculate the group modification (Mod) as the relative claim frequency between the merit rating group and the class total. This is the merit rating factor for the group. Use the claim frequency per *Earned Premium at Present B Rates* instead of Earned Car Year.

$$Mod = \frac{\left(\frac{\# Claims_{Group}}{Earned Prem_{Group}} \right)}{\left(\frac{\# Claims_{Class}}{Earned Prem_{Class}} \right)}$$

$$Mod = \frac{\left(\frac{1,310 + 100}{1,070,000 + 70,000} \right)}{\left(\frac{1,800}{1,375,000} \right)} = .945$$

Use groups A and X to get the 2+ AY-free group.

- Calculate the credibility of a single exposure in the group.

$$Z = 1 - Mod$$

$$Z = 1 - .945$$

$$= \boxed{5.5\%}$$

Note:

The formula for Z above is derived from the full Mod credibility formula. Since R is zero for accident-free risks (R = the ratio of actual losses-to-expected losses), the credibility formula simplifies to the form above:

$$Mod = ZR + (1 - Z)$$

$$R = 0 \rightarrow Mod = 1 - Z$$

Part b – Credibility for an exposure with 0 accident-free years

1) Calculate the group modification, same as above.

$$Mod = \frac{\left(\frac{\# Claims_{Group}}{Earned Prem_{Group}} \right)}{\left(\frac{\# Claims_{Class}}{Earned Prem_{Class}} \right)} \quad \left| \quad Mod = \frac{\left(\frac{260}{150,000} \right)}{\left(\frac{1,800}{1,375,000} \right)} = 1.324$$

2) Calculate λ , the overall claims frequency per Earned Car Year for the entire class, not group B.

$$\lambda = \frac{\# Claims_{Class}}{ECY_{Class}} \quad \left| \quad \lambda = \frac{1,800}{16,310} = .110$$

3) Calculate R , the ratio of actual losses to expected losses for group B (with 0 accident-free years) based on the Poisson distribution.

$$R = \frac{1}{1 - \Pr(N=0)} \quad \text{Poisson Distribution:} \quad R = \frac{1}{1 - e^{-\lambda}} = 9.57$$
$$\boxed{R = \frac{1}{1 - e^{-\lambda}}}$$

4) Solve for the credibility using the modification credibility formula.

$$\boxed{Mod = Z \cdot R + (1 - Z)} \quad \left| \quad 1.324 = Z \times 9.57 + (1 - Z)\right.$$
$$\rightarrow \boxed{Z = 3.8\%}$$

Discussion

Our goal is to use the experience of an individual risk to see how much the individual risk differs from the average risk in a rating class. This paper shows that the loss experience of a single exposure is credible and we can use the experience to segment risks within a rating class.

We calculate the Mod as the relative claim frequency between the rating group and the class as a whole using earned premium as the basis. Usually, we prefer to use earned premium in order to avoid the maldistribution of having higher claim frequency territories with more X/Y/B risks and also higher territorial premiums.

Make sure the earned premium is adjusted to the present rates at the same group level. Bailey & Simon adjust to group B rates, but it just matters that the earned premium is at the present rates of the same merit group level.

Using premium only corrects for maldistribution if:

- High frequency territories are high premium territories
- Territorial differentials are proper

Variation of Individual Risks within a Class

If we calculate the ratio of the 3+ year credibility to the annual claim frequency for each class, we can get an idea of how much variation there is of risks within each class. **Classes with higher ratios have a higher variation of individual hazards. This indicates that a class is more broadly defined than other classes.**

If the variation within a class was the same for each class, the credibility (Z) would vary proportionally to the average claim frequency meaning that the Z_{3+} -to-Claim Freq_{Class} ratio would be the same for each class.

From the problem above, we can calculate some of the values for Class 1 (shaded below). The table below summarizes the other classes as well.

In the example below, Class 2 has a higher credibility for experience relative to the overall class claim frequency. This means the class is less narrowly defined and there's more variance in the likelihood of a claim for the insureds within Class 2.

Class	Z_{3+} AY-Free (1)	Claims (2)	Earned Car Years (3)	Class Claim Frequency (λ) (4) = (2)/(3)	Ratio: Z_{3+} -to- Claim Freq _{Class} (5) = (1)/(4)
1	.065	1,800	16,310	0.110	0.587
2	.081	1,760	19,560	0.090	0.900
3	.030	520	11,200	0.046	0.646
4	.049	1,100	13,400	0.082	0.597

$$Mod_{3+AY-Free, Class 1} = \frac{\left(\frac{1,310}{1,070,000}\right)}{\left(\frac{1,800}{1,375,000}\right)} = .935 \quad Z_{3+AY-Free, Class 1} = 1 - .935 = .065$$

A key point here is that greater variation of individual hazards within a class results in greater credibility for experience rating.

Derivation of R for an exposure with 0 accident-free years

$$R = \frac{Actual Losses_{0AY-Free}}{Expected Losses_{0AY-Free}} = \frac{\lambda}{(1 - \Pr(N = 0))} = \frac{1}{1 - \Pr(N = 0)}$$

Source

Bailey & Simon – pg. 159-160 and pg. 164

More Practice

- CAS 2019 – 3
- CAS 2018 – 3
- CAS 2015 – 1
- CAS 2014 – 5

Relative Credibility: Empirical

Bailey & Simon

Problem

Given the following information for a class of private passenger auto liability:

Merit Rating Group	Number of Accident-Free Years	Earned Prem. at Present B Rates	Number of Claims Incurred
A	3+	3,700,000	4,600
X	2	430,000	600
Y	1	500,000	700
B	0	1,320,000	2,260
Total		5,950,000	8,160

Calculate the relative credibility for groups with 1+, 2+, and 3+ years of accident-free experience and interpret the results.

Solution Recipe

- 1) Calculate the relative claim frequency (Mod) for each group relative to the class total.

$$Mod = \frac{\left(\frac{\# Claims_{Group}}{Earned Prem_{Group}} \right)}{\left(\frac{\# Claims_{Class}}{Earned Prem_{Class}} \right)}$$

$$Mod_{1+} = \frac{\left(\frac{700 + 600 + 4,600}{500 + 430 + 3,700} \right)}{\left(\frac{8,160}{5,950} \right)} = .929$$

$$Mod_{2+} = .918$$

$$Mod_{3+} = .907$$

- 2) Calculate the credibility for each group.

$$Z = 1 - Mod$$

$$Z_{1+} = 1 - .929 = 7.1\%$$

$$Z_{2+} = 8.2\%$$

$$Z_{3+} = 9.3\%$$

- 3) Calculate the relative credibility compared to the 1+ years accident-free group.

$$Relative\ Credibility = \frac{Z_{Group}}{Z_{1+}}$$

	1+	2+	3+
Relative Credibility	1.00	1.16	1.32

If the class was stable with no risks leaving or entering and individual insured's chances of an accident constant over time, we'd expect credibility to increase roughly proportionally to the number of years. Since this is not the case (relative credibility of 3+ to 1+ is 1.32, much lower than 3), we conclude that the risks in a group are changing over time.

Discussion

In theory, we would expect the relative credibility to be roughly 1:2:3 for the groups above. This is because we'd expect credibility to vary roughly in proportion to the number of years.

Relative credibilities for the 2+ and 3+ groups may be significantly lower than 2 and 3 for the following reasons:

- Risks may be entering or leaving the class
- An individual insured's probability of an accident changes from one year to the next

The discussion by Hazam also points out that for larger credibilities, we wouldn't expect the relative credibilities to be as close to 1:2:3 as for smaller credibilities because of the credibility formula:

$$Z = \frac{p}{p + K}$$

The number of claims for the group, p , will increase proportionally as the number of years increase, but K is constant. Therefore, the true ratio of $Z_{1+}:Z_{2+}:Z_{3+}$ will be less than 1:2:3.

Source

Bailey & Simon – pg. 160 and 163

Hazam – pg. 151

More Practice

CAS 2017 – 3

CAS 2011 – 1

Shifting Parameters: Chi-Squared Test

Mahler

Problem

Given the following claim frequency information for a book of business:

Year	Earned Car Years	Average Frequency	Year	Earned Car Years	Average Frequency	Chi-Squared Distribution		
2005	12,000	8.3%	2013	15,750	9.7%	Degrees of Freedom	.05	.025
2006	13,000	9.4%	2014	16,000	10.2%			
2007	14,500	9.2%	2015	16,000	10.1%	2	5.99	7.38
2008	14,000	9.7%	2016	16,750	9.9%	3	7.81	9.35
2009	13,650	9.5%	2017	17,500	10.3%	4	9.49	11.14
2010	13,800	8.8%	2018	18,000	9.6%	5	11.07	12.83
2011	14,500	9.3%	2019	18,000	10.5%			
2012	15,000	9.4%						

- Assume claim frequency follows a Poisson distribution

The claim frequency parameter is calculated for three-year intervals over the 15-year historical time period.

Evaluate whether the claim frequency is shifting over time using the Chi-squared test at the 5% significance level.

Solution Recipe

- 1) Calculate the overall parameter (frequency) for all years.

$$\lambda = \frac{\sum Exposure \cdot Avg Freq}{\sum Exposure} \quad \left| \quad \lambda = \frac{12,000 \times .083 + 13,000 \times .094 + \dots}{12,000 + 13,000 + \dots} = 9.65\%$$

- 2) Calculate Earned Car Years and the weighted average claim frequency for each three-year interval.

$$\lambda = \frac{\sum Exposure \cdot Avg Freq}{\sum Exposure}$$

Years	Earned Car Years	Average Frequency
2005-2007	39,500	9.0%
2008-2010	41,450	9.3%
2011-2013	45,250	9.5%
2014-2016	48,750	10.1%
2017-2019	53,500	10.1%

3) Calculate the χ^2 test statistic between actual and expected claim counts, based on exposures.

$$\chi^2 = \sum \frac{(\text{actual} - \text{expected})^2}{\text{expected}}$$

Years	Actual (A)	Expected (E)	$= \frac{(A - E)^2}{E}$
2005-2007	3,552	3,810	17.44
2008-2010	3,869	3,998	4.15
2011-2013	4,286	4,364	1.40
2014-2016	4,906	4,702	8.88
2017-2019	5,421	5,160	13.14
Total	22,034	22,034	45.00

$$\text{Actual}_{2005-2007} = 39,500 \times 9.0\% = 3,552$$

$$\text{Expected}_{2005-2007} = 39,500 \times 9.65\% = 3,810$$

4) Compare the χ^2 test statistic to the critical value for the relevant χ^2 distribution (with $n-1$ degrees of freedom) and interpret the test.

$$\text{critical value} = \chi^2_{n-1, \alpha\%}$$

H₀: Frequency parameter is not shifting over time

$$\chi^2_{5-1, 5\%} = 9.49$$

45.00 > 9.49, so we reject the null hypothesis and conclude that frequency is shifting over time.

Discussion

The goal of the Chi-squared test as well as the lagged correlations test is to see if the risk parameters (e.g. average frequency) are shifting over time.

A key point from Mahler is that **when parameters are shifting over time, we should place less credibility on older data and higher credibility on more recent data**. Also, it will be more important to minimize the delays in data that are used for predictions and ratemaking.

CBT Spreadsheet Tips

As a shortcut, you can also calculate χ^2 directly with an array formula without first calculating the helper values of $(\text{Actual} - \text{Expected})^2 / \text{Expected}$. See the Excel version for how this works.

$$\chi^2 = \text{SUM}((\text{Actual Values} - \text{Expected Values})^2 / \text{Expected Values})$$

Important: In Pearson Vue, make sure to press CTRL + SHIFT + ENTER to calculate array formulas.

Source

Mahler – pg. 235-236

More Practice

CAS 2018 – 1

CAS 2015 – 4

CAS 2012 – 3

Shifting Parameters: Lagged Correlations

Mahler

Problem

Given the following loss ratios between 2010 and 2020 for a line of business:

Year	2020	2019	2018	2017	2016	2015	2014	2013	2012	2011	2010
Loss Ratio	.605	.689	.606	.727	.579	.744	1.066	.977	.944	.906	.764

Using correlations between loss ratios at lags of 1-5 years, test if the loss ratio for this line of business is shifting over time.

Solution Recipe

- 1) Calculate the correlation of the parameter at increasing lag periods (1-year, 2-year, ...).

$$r = \frac{\overline{XY} - \overline{X} \cdot \overline{Y}}{\sigma_X \cdot \sigma_Y}$$

For the first pair (1-year lag):

X = Loss Ratio_t

Y = Loss Ratio_{t-1}

In Excel:

CORREL(X values, Y values)

Lag (years)	Correlation	
1	0.598	← Correlate Years 2011-2020 and Years 2010-2019 (1-year lag)
2	0.325	
3	0.101	← Correlate Years 2013-2020 and Years 2010-2017 (3-year lag)
4	-0.185	
5	0.006	

- 2) Test if the correlations fall as the lag increases. A falling correlation indicates the parameter is shifting over time.

The correlation is relatively high for a 1-year lag but decreases as the lag increases.

This indicates the loss ratio is shifting over time.

Discussion

The key idea here is that if there is significant correlation between years close in time (like the 1-year lagged values here), the recent years can be used to help predict *future values*. We'll see this in the next recipe, "Credibility-Weighted Predictions."

If parameters were stable over time, the correlations wouldn't vary by the length of the lag in years. Higher correlations for years closer together than for years farther apart indicates that the parameters are shifting.

The correlations in this problem are noisy because of the small number of data points. The example in Mahler (pg. 237-238) shows lagged correlations with a smoother trend from high correlations for shorter lags to negligible correlations for longer lags.

An exam problem may give a table of correlations, like in the Mahler example, and ask you to interpret whether parameters are shifting over time.

Source

Mahler – pg. 235, 237-239

Credibility-Weighted Predictions X_{est}

Mahler

Problem

Given the following historical collision claims frequency for a book of Motorcycle policies:

Year	Earned Car Years	Frequency
2014	6,500	0.033
2015	6,760	0.042
2016	7,030	0.035
2017	7,410	0.028
2018	7,630	0.039
2019	7,820	0.032
2020	7,900	0.025
2021	8,120	0.030

- Credibility (Z) 0.4

Calculate the predicted 2022 claims frequency for the book using the following methods:

- Credibility weight of the most recent year and the overall mean frequency
- Equal weight to the three most recent years of data
- Credibility weight of the most recent year and the previous estimate
 - Assume a grand mean of 0.03 is an appropriate a priori estimated value of 2014 frequency
- Credibility weight using varying weights on the data and the grand mean
 - 40% weight on the first prior value, 20% on the second prior value and the rest on the overall mean

Solution Recipe

- 1) Calculate the overall mean using all the years prior to the year being estimated. If there is an exposure basis or earned premium, make sure to use a weighted average.

$$\mu = \frac{\sum Exposure_i \cdot Freq_i}{\sum Exposure_i} \quad \left| \quad \begin{aligned} \mu &= \frac{6,500 \times .033 + 6,760 \times .042 + \dots}{6,500 + 6,760 + \dots} \\ &= .0328 \end{aligned}$$

Note:

Since we're estimating 2022 claims frequency, the overall mean should reflect all years prior to 2022. If we were back testing and estimating the 2018 frequency, the overall mean would include years prior to 2018.

If a problem provides an overall mean or "grand mean", use that for μ .

2) Calculate the estimated value X using the appropriate credibility-weighting from Mahler:

Credibility Weight of the Most Recent Year and the Grand Mean

$$\boxed{X_{est} = Z \cdot Y_1 + (1 - Z) \cdot \mu} \quad \left| \quad \begin{aligned} X_{est} &= .4 \times .030 + (1 - .4) \times .0328 \\ &= \boxed{.0317} \end{aligned} \right.$$

Equal Weight to the N Most Recent Years

$$\boxed{X_{est} = \frac{Z}{N} \cdot \sum_{i=1}^N Y_i + (1 - Z) \cdot \mu} \quad \left| \quad \begin{aligned} X_{est} &= \frac{.4}{3} \times (.032 + .025 + .030) + (1 - .4) \times .0328 \\ &= \boxed{.0313} \end{aligned} \right.$$

Credibility Weight of the Most Recent Data and Previous Estimate

$$\boxed{X_{est,i+1} = Z \cdot Y_i + (1 - Z) \cdot X_{est,i}} \quad X_{est,i+1} = Z \cdot Y_i + Z(1 - Z) \cdot Y_{i-1} + Z(1 - Z)^2 \cdot Y_{i-2} + \dots + (1 - Z)^N \cdot \mu$$

$$\begin{aligned} X_{est,2022} &= .4 \times .03 + (1 - .4) \times .0303 \\ &= \boxed{.0302} \end{aligned}$$

Year	Frequency	Estimate	
2014	0.033	0.0300	<- Given in problem
2015	0.042	0.0312	= .4 × .033 + (1 - .4) × .030
2016	0.035	0.0355	= .4 × .042 + (1 - .4) × .0312
2017	0.028	0.0353	= .4 × .035 + (1 - .4) × .0355
2018	0.039	0.0324	
2019	0.032	0.0350	
2020	0.025	0.0338	
2021	0.030	0.0303	
2022		0.0302	

Credibility Weight with Varying Weights

$$\boxed{X_{est} = \sum_{i=1}^N Z_i \cdot Y_i + (1 - \sum Z_i) \cdot \mu} \quad \left| \quad \begin{aligned} X_{est} &= .4 \times .03 + .2 \times .025 + (1 - .4 - .2) \times .0328 \\ &= \boxed{.0301} \end{aligned} \right.$$

Discussion

This section in Mahler shows different ways of how recent years of experience can be used to predict future experience. This is relevant for prospective experience rating. If parameters are shifting over time, we want

to use a credibility-weighting formula that applies more credibility on recent years and less credibility on older years.

Mahler shows several different ways to credibility-weight past experience and the overall mean. In the next recipe, we'll see how to find the optimal credibility weighting (Z) in order to get the best estimate of future experience.

Special Cases of the Credibility Formula

$$X_{est} = Z \cdot Y_1 + (1 - Z)\mu$$

Every Risk is Average:

0% credibility on recent data (except as part of the overall mean)

$$Z = 0 \rightarrow X_{est} = \mu \quad X_{est} = \boxed{0.0328}$$

The Most Recent Year Repeats:

100% credibility on past data

$$Z = 1 \rightarrow X_{est} = Y_1 \quad X_{est} = \boxed{0.0300}$$

CBT Spreadsheet Tips

For the third method, “Credibility Weight of the Most Recent Data and Previous Estimate”, the easiest approach is to set up a table and calculate all the estimates from the beginning instead of attempting a direct calculation of the final estimate.

Source

Mahler – pg. 239-241, 255-256

More Practice

CAS 2018 – 1

Finding Optimal Z: MSE Criteria

Mahler

Problem

Given the following historical loss ratios for a book of business:

Year	Loss Ratio
2011	0.832
2012	0.738
2013	0.825
2014	0.870
2015	0.838
2016	0.736
2017	0.911
2018	0.941
2019	1.207
2020	0.872

- Complement of credibility loss ratio 85%

Predicted loss ratios are estimated as a credibility-weighting of the most recent year and the complement of credibility loss ratio.

Use the least squared error criteria to determine which credibility weight on the most recent year produces the best predicted loss ratio estimates. Select the optimal Z between 0% and 100% in 20% intervals.

Solution Recipe

- 1) Calculate the estimated values for each credibility scenario and set up a table of estimated and actual values for each scenario.

$$X_{est} = Z \cdot Y_1 + (1 - Z)\mu \quad \left| \quad X_{est,2020 \& Z=.2} = .2 \times 1.207 + (1 - .2) \times .85 = .921 \right.$$

Predicted Loss Ratio (X_{est})

Year	Actual Loss Ratio(Y_i)	Z = 0%	20%	40%	60%	80%	100%
2011	0.832						
2012	0.738	0.850	0.846	0.843	0.839	0.836	0.832
2013	0.825	0.850	0.828	0.805	0.783	0.760	0.738
2014	0.87	0.850	0.845	0.840	0.835	0.830	0.825
2015	0.838	0.850	0.854	0.858	0.862	0.866	0.870
2016	0.736	0.850	0.848	0.845	0.843	0.840	0.838
2017	0.911	0.850	0.827	0.804	0.782	0.759	0.736
2018	0.941	0.850	0.862	0.874	0.887	0.899	0.911
2019	1.207	0.850	0.868	0.886	0.905	0.923	0.941
2020	0.872	0.850	0.921	0.993	1.064	1.136	1.207

Note:

We don't calculate an estimated loss ratio for 2011 since the estimate requires a loss ratio from the prior year.

2) Calculate the Mean Squared Error (MSE) for each credibility scenario.

$$MSE = \frac{\sum (Y_i - X_{est,i})^2}{n}$$

$$MSE_{Z=.2} = \frac{(.738 - .846)^2 + (.825 - .828)^2 + \dots}{9} = .01728$$

	0%	20%	40%	60%	80%	100%
MSE	0.01852	0.01728	0.01753	0.01926	0.02246	0.02715

MSE Calculation using Array formulas

The quickest way to calculate MSE is by using an array formula as AVERAGE((Y Values - X Values)^2). This allows you to calculate the MSE directly without needing to set up a whole table of "helper" calculations in the spreadsheet. See the Excel version for this.

Important: In Pearson Vue, make sure to press CTRL + SHIFT + ENTER to calculate array formulas.

3) Select the credibility with the smallest MSE.

Z = 20% has the smallest MSE of the two credibility scenarios and therefore produces better loss ratio estimates according to the least squares error criteria.

Discussion

Even when we know what formula we want to use to calculate the predicted X_{est} values, we need to select a credibility weighting (Z) to use. The goal here is to back test on historical data to find the Z value that minimizes the MSE. This is the credibility weighting we'd use to estimate future loss ratios.

Calculating the Reduction in MSE

Based on the MSE at the extremes (Z = 0 and Z = 1), we can calculate how much the MSE is reduced to using the optimal Z (see Mahler pg. 252).

$$\% \text{ Reduced} = \frac{MSE_{optimal Z}}{\min(MSE_{Z=0}, MSE_{Z=1})}$$

$$\% \text{ Reduced} = \frac{.01728}{\min(.01852, .02715)} = 93\%$$

MSE is reduced to 93% of its previous value.

Using the Average as the Complement

For this problem we're using a selected complement of credibility. One twist to the problem is to use the average loss ratio as the complement instead. With the average loss ratio, make sure to use a running average as opposed to the all-year average (unless otherwise specified).

For example, below are the calculations for $Z = 20\%$. Note the calculation of μ that goes into the X_{est} formula:

$$X_{est,2015} = .2 \times .87 + (1 - .2) \times \frac{.832 + .738 + .825 + .87}{4} =$$

Year	Actual Loss Ratio (Y_i)	μ Running Avg	X_{est} ($Z=20\%$)
2011	0.832		
2012	0.738	0.832	0.832
2013	0.825	0.785	0.776
2014	0.87	0.798	0.804
2015	0.838	0.816	0.827
2016	0.736	0.821	0.824
2017	0.911	0.807	0.792
2018	0.941	0.821	0.839
2019	1.207	0.836	0.857
2020	0.872	0.878	0.943

Source

Mahler – pg. 242-246, 250-252

More Practice

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