Rising Fellow

2025

Exam 7

Cookbook

ADVANCED ESTIMATION OF CLAIMS LIABILITIES

70+ **Step-by-Step Recipes**
to Solve CAS Calculation Problems

Exam 7 Cookbook

2025 Sitting

Rising Fellow

Rising Fellow

Published By:

Rising Fellow
United States, TX, 78006
www.RisingFellow.com

Contact: info@RisingFellow.com

Published in the United States

Table of Contents

Introduction

The purpose of the Exam 7 Cookbook is to prepare you to confidently answer calculation-based problems on exam day without wasting time trying to “think through” a problem-solving approach before typing the solution. This is the same approach I used to help pass my upper-level CAS exams on the first sittings to earn my FCAS.

Since the 2016 sitting, 1,000+ actuaries have used the Exam 5- 9 Cookbooks and Online Courses to help them pass their exams and earn their FCAS. We want to see you be one of them.

Our goal with Rising Fellow is to help you prepare for the exam with less frustration so that you have your best exam sitting yet!

The Structure

The Exam 7 Cookbook goes through the different calculation-based problem-types that I believe are reasonably testable based on the syllabus. By exam day, you should know how to solve each one.

Inside, you’ll find a separate section for each testable problem-type. Each section has the following structure:

Original Practice Problem

Each section has an original practice problem that demonstrates the problem-type. I wrote these based off of the syllabus papers to have a similar difficulty-level and style to what you might see on an exam.

Solution Recipe

The solution recipe solves the practice problem from start to finish and shows the step-by-step approach you should take to answer a similar problem. For each step, you’ll see:

- The description for what to do in the step
- The formula(s) necessary for the step
- The formula(s) translated from symbolic notation to plain-English
- Calculations for the step to solve the example problem

Discussion

Each section includes discussion to add clarity and more context. The discussion also covers underlying concepts that might come up on a part b or part c essay question.

For many problems, I point out potential "twists" that could show up on the exam that would make an exam problem more difficult. Since you've taken actuarial exams up to this point, you know that straightforward exam problems are more the exception than the rule.

CBT Spreadsheet Tips

This new section provides Excel formulas and tips for how to solve a problem more efficiently in the computer-based testing (CBT) PearsonVue spreadsheet environment. There are many types of problems where setting up your solution intelligently and taking advantage of the spreadsheet capabilities such as SUMIF( ), COUNTIF( ), and array formulas, will save you valuable time on the exam.

Source

Each section references the pages in the syllabus reading that you can cross-reference for more information and details. Make sure to check the syllabus section for more context if you get stuck on a problem or to see how the author discusses the concepts.

More Practice

Here, you'll see references to past CAS problems and original Rising Fellow (RF) practice problems. You'll find this helpful especially closer to the exam if there are types of problems that you are struggling with. This section includes references to problems from the 2011-2019 exams, which are the past exams in the current syllabus structure of Exam 7.

Notation and Formulas

One of the big challenges with Exam 7 is that almost every author uses their own unique set of notation for losses, LDFs, premiums, etc. The changing notation between papers makes preparing for the exam more challenging, especially for the following reasons:

- It's harder to see the big picture and draw connections between the main themes in Exam 7 across multiple papers.
- It's more difficult to have an intuitive understanding of the different methods and how they work, which is critical to be able to solve problems on exam day without wasting time.

To help you avoid getting lost in the notation I also show a plain-English version of the formulas in the solution recipe steps. If you're like me and get lost in the symbolic notation in the syllabus papers, you'll find that this feature will save you a lot of frustration.

Below are two examples of what the formulas look like in the solution recipe steps:

Mack Benktander – Benktander Method:

$$\boxed{U_{GB} = C_k + q_k U_{BF}} \qquad U_{GB} = Loss + (1 - \%Paid) \times Ult_{BF}$$

Verrall – Incorporating Expert Opinion in the Chain Ladder Method

$$\boxed{\mathrm{E}\left[C_{i,j}\right] = \left(\lambda_{i,j} - 1\right) \cdot D_{i,j-1}} \qquad \mathrm{E}\left[IncLoss_{AY,k}\right] = \left(LDF_{AY,k} - 1\right) \cdot Loss_{AY,k-1}$$

$$\boxed{\mathrm{Var}\left(C_{i,j}\right) = \varphi \cdot \lambda_{i,j} \cdot \left(\lambda_{i,j} - 1\right) \cdot D_{i,j-1}} \qquad \mathrm{Var}\left(IncLoss_{AY,k}\right) = dispersion \cdot LDF_{AY,k} \cdot \left(LDF_{AY,k} - 1\right) \cdot Loss_{AY,k-1}$$

Note that Mack – Benktander paper uses C to indicate cumulative losses and Verrall uses C to indicate incremental losses. In the plain-English version of the formula I make the distinction between incremental and cumulative losses as well as spell out what the symbols in the formulas ***really*** mean.

I firmly believe you should learn and memorize the formulas in the way that you'll best be able to remember and apply to an exam-day problem. The exam graders want to see that you understand how to apply the different methods, not whether you memorized the specific, symbolic notation from a paper written 20 years ago.

I see no evidence that you would be marked off for writing a formula on the exam with $LDF_{AY,k}$ instead of $\lambda_{i,j}$. In fact, if you look at problem 8 from the 2012 exam, you'll see just that in the sample solutions. Sample solution 1 uses clearer $LDF_{AY,k}$ notation while sample solution 2 uses the Verrall notation, $\lambda_{i,j}$.

How to Best Use the Exam 7 Cookbook

Below is a suggested guide for how you can incorporate the Exam 7 Cookbook in your own study schedule along with the syllabus material and a typical study manual. This is the general approach that I used when I took my fellowship exams.

For each of those exams, I had a main study manual as well as the Exam Cookbook, which I built out while I studied for the exam (but you don't need to waste time doing that part!)

First pass through the syllabus

While you're reading a particular paper in the syllabus and your main study manual to learn the material, use the Exam 8 Cookbook to clearly identify what problem types you need to know from the paper. Study the steps in the solution recipe to learn how to solve the problem types. Make sure to do some practice problems as you go through the syllabus. This will help you learn faster.

Second pass through the syllabus

Review the steps for the problem types and make sure you have an intuitive understanding of how to solve the problems. Start working on the past CAS problems.

The first level of understanding is to be able to follow the recipe and understand the steps and calculations.

The next level of understanding is to be able to recall and apply the steps to solve a problem without relying on study material. During your second pass, focus on building this deeper level of understanding.

Review and Practice Problems (around 6 weeks to 2 weeks before the exam)

At this point you should have a good understanding of the syllabus and how to use the recipe steps to systematically solve the different calculation problems. During this period, you should be doing lots of problems across the syllabus and targeting problem types that you are finding particularly challenging.
By the end of this phase, you might not have all the formulas memorized, but you should know all the steps and how to apply them to solve problems without needing to think too much before beginning to write the solution.

During this phase, make sure to focus on the types of problems and concepts that you're weak at. This may require some struggle, but struggling with some of the challenging problems will help you master these concepts.

You also should continue building your understanding of the concepts and preparing for essay and more complicated integrative questions. I found it helpful to create flashcards from the papers as well as to re-read sections of the syllabus papers that appear to be likely sources of essay problems.

Final Weeks

In the final weeks, focus on taking practice exams to see problems from the entire syllabus. When taking practice exams, work on your exam strategy to make sure you're able to finish the exam and maximize your points.

Prepare for essay problems in the final weeks by using flashcards to make sure that you know all the details necessary. An approach I found helpful is to say flashcards out loud and to explain the flashcard response in my own words as if I were teaching someone. It sounds weird, but it is a much more efficient way to learn and memorize than simply scanning the front and back of the flashcard.

Prepare for calculation problems by reviewing the recipes in the Exam 7 Cookbook in a similar fashion to how you use flashcards for essay problems. Using this approach on my fellowship exams, I was able to rapidly review the steps and formulas for how to solve each problem-type that might show up on the exam. This was a huge benefit and gave me a lot of confidence going into the exam.

Exam Day

I used the original Exam Cookbooks together with a traditional study manual using the approach above to take my fellowship exams. On exam day, for almost every calculation problem I was able to start writing the solution without wasting time trying to think through how to solve the problem. I had an intuitive understanding of how to solve each of the problems following the step-by-step recipes.

If you follow this approach, you should be able to develop a similar level of understanding and confidence going into the exam room.

Excel Version for Computer-Based Testing Preparation

For each recipe, there is an accompanying Excel version. Make sure to review those so that you know how to solve problems in the spreadsheet format. The CBT Spreadsheet Tips sections and the Excel version showing the formulas and setup for the spreadsheet solution will help you understand how to solve exam problems in the Pearson Vue spreadsheet environment.

Exam-Related Questions

If you purchased the Exam 7 Online Course, **please post your question in the Exam 7 course forum**. We answer exam-related questions through the forum for people in the online course.

Errata

I always hated seeing errors in study manuals when I studied for exams, so I make every effort to ensure the study materials are accurate. Nevertheless, there may still be some errors in the final version, so I keep an updated errata. Please make sure to check it regularly for any fixes.

The link to the errata is below:

https://risingfellow.com/errata

If you find any errors, please send me a message using the contact form on the Errata page so that I can make a correction.

Feedback

I am always working to improve the Exam 7 Cookbook and the rest of the Rising Fellow study material. Please send us an email to exam7@RisingFellow.com if you have feedback about any of the following:

- Recipes or sections that are confusing or could be improved
- New recipes I should include in future versions
- Better ways you've found to solve a problem-type in a spreadsheet
- Any comments or other feedback you have

Reviews

If you find the Exam 7 Cookbook helpful this sitting, please leave us a review and let us know how it helped you prepare for the exam. Other actuaries look at reviews to help decide what study material to buy and it's helpful for us to hear feedback from actuaries like you so that we can better understand what's working and what can be improved.

You can leave us a review by sending us an email to info@RisingFellow.com. Thank you!

Good luck as you start studying and I hope this will be your best sitting yet

Benktander Method

Mack - Benktander

Problem

Given the following information:

Cumulative Paid Losses ($000)

Accident Year	12 Months	24 Months	36 Months	Earned Premium
2021	1,800	2,900	3,350	5,000
2022	2,800	3,600		5,500
2023	2,300			6,000

Selected Loss Development Factors To-Ultimate

12-Ult	24-Ult	36-Ult
1.75	1.25	1.10

- The expected loss ratio is 70%

Calculate the ultimate loss estimate for accident year 2023 using the Benktander method.

Solution Recipe

Method 1 – Using the BF Procedure

1) Calculate the ultimate loss estimate for the BF method (1st iteration of the BF procedure).

$$\boxed{U_{BF} = C_k + q_k U_0}$$

$$Ult_{BF} = Loss + (1 - \%Paid) \times Prem \times ELR$$

$$U_{BF} = 2{,}300 + \left(1 - \frac{1}{1.75}\right) \times 6{,}000 \times 0.7$$

$$= \boxed{4{,}100}$$

2) Calculate the ultimate loss estimate for the Benktander method (2nd iteration of the BF procedure).

$$\boxed{U_{GB} = C_k + q_k U_{BF}}$$

$$U_{GB} = Loss + (1 - \%Paid) \times Ult_{BF}$$

$$U_{GB} = 2{,}300 + \left(1 - \frac{1}{1.75}\right) \times 4{,}100$$

$$= \boxed{4{,}057}$$

Method 2 – Credibility-Weighting the Chain Ladder and Expected Loss Ultimates

1) Calculate the ultimate loss estimate for the Chain Ladder method.

$$\boxed{U_{CL} = \frac{C_k}{p_k}}$$

$$Ult_{CL} = Loss \times CDF$$

$$U_{CL} = 2{,}300 \times 1.75$$

$$= \boxed{4{,}025}$$

2) **Calculate the ultimate loss estimate for the Benktander method. Make sure to use the expected prior loss ultimate (U_0) in the formula.**

$$\boxed{q_k = 1 - \frac{1}{CDF}}$$

$$\boxed{U_{GB} = (1 - q_k^2)U_{CL} + q_k^2 \times U_0}$$

$$Ult_{GB} = [1 - \%Unpaid^2] \times Ult_{CL} + \%Unpaid^2 \times Prem \times ELR$$

$$q_k = 1 - \frac{1}{1.75} = .429$$

$$U_{GB} = (1 - .429^2) \times 4{,}025 + .429^2 \times 6{,}000 \times .7$$
$$= \boxed{4{,}057}$$

<u>Method 3 – Credibility-Weighting the Chain Ladder and BF Reserves</u>

1) **Calculate the reserve loss estimate for the Chain Ladder and BF methods.**

$$\boxed{R_{CL} = \frac{C_k}{p_k} - C_k} \qquad Resv_{CL} = Loss \times (CDF - 1)$$

$$\boxed{R_{BF} = q_k U_0} \qquad Resv_{BF} = (1 - \%Paid) \times Prem \times ELR$$

$$R_{CL} = 2{,}300 \times (1.75 - 1)$$
$$= 1{,}725$$
$$R_{BF} = \left(1 - \frac{1}{1.75}\right) \times 6{,}000 \times 0.7$$
$$= \boxed{1{,}800}$$

2) **Calculate the reserve for the Benktander method. Make sure to use the BF reserve in the formula. Add the loss to the reserve estimate to calculate the ultimate loss estimate for the Benktander method.**

$$\boxed{R_{GB} = (1 - q_k)R_{CL} + q_k \times R_{BF}}$$

$$Resv_{GB} = [1 - \%Unpaid] \times Resv_{CL} + \%Unpaid \times Resv_{BF}$$

$$R_{GB} = (1 - .429) \times 1{,}725 + .429 \times 1{,}800$$
$$= 1{,}757$$
$$U_{GB} = 2{,}300 + 1{,}757$$
$$= \boxed{4{,}057}$$

Discussion

Both Hürlimann and Mack – Benktander cover the Benktander method. This method is very similar to the BF method in that it is a weighting of the Chain Ladder and Expected Loss methods.

Benktander as an Iterated BF Method

The Benktander method is a second iteration of the BF procedure. This is how the iteration works:

1. Start with an ultimate loss estimate, $U^{(m)}$. For $U^{(0)}$, use the expected loss estimate.
2. Apply the BF procedure to get a new loss reserve estimate:

$$\boxed{R^{(m)} = q_k U^{(m)}} \qquad Resv^{(m)} = \%Unpaid \times Ult^{(m)}$$

3. Get a new ultimate loss estimate by adding the losses-to-date to the reserve. This is the starting ultimate for the next iteration:

$$\boxed{U^{(m+1)} = C_k + R^{(m)}} \qquad Ult^{(m+1)} = Loss_k + Resv^{(m)}$$

The ultimate loss estimate ($U^{(m)}$) can be rearranged as a credibility weighting of the Chain Ladder ultimate (U_{CL}) and expected loss ultimate (U_0). This is Method 2 above.

$$\boxed{U^{(m)} = (1 - q_k^m) \times U_{CL} + q_k^m \times U_0}$$

Alternatively, the loss reserve estimate ($R^{(m)}$) can be rearranged as a credibility weighting of the Chain Ladder reserve (R_{CL}) and the BF reserve (R_{BF}). This is Method 3 above.

$$\boxed{R^{(m)} = (1 - q_k^m) \times R_{CL} + q_k^m \times R_{BF}}$$

m	Starting Ultimate ($U^{(m)}$)		New Reserve ($R^{(m)}$)
0	$U_o = Prem \times ELR$	$\xrightarrow{\times \%Unpaid}$	
		$\xleftarrow{+ \text{Loss}}$	$R_{BF} = q_k U_0$
1	$U_{BF} = Loss + R_{BF}$ $U^{(1)} = (1 - q_k^1) \times U_{CL} + q_k^1 \times U_0$		
			$R_{GB} = q_k U_{BF}$ $R^{(1)} = (1 - q_k^1) R_{CL} + q_k^1 \times R_{BF}$
		Benktander	
2	$U_{GB} = Loss + R_{GB}$ $U^{(2)} = (1 - q_k^2) \times U_{CL} + q_k^2 \times U_0$		
			$R^{(2)} = q_k U^{(2)}$ $R^{(2)} = (1 - q_k^2) R_{CL} + q_k^2 \times R_{BF}$
⋮	⋮		⋮
∞	$U^{(\infty)} = U_{CL}$		$R^{(\infty)} = R_{CL}$

Note on the iteration number:
Both Hürlimann and Mack – Benktander show calculations in which the first iteration reserve is labeled R^0, the BF reserve, and the second iteration is labeled R^1, the Benktander reserve. Because of this, the iteration count is a bit confusing.

As the number of iterations increases, the weight on the chain ladder method increases until it converges to the chain ladder method (as $m \to \infty$).

Source

Mack – Benktander – pg. 334-335

More Practice

CAS 2018 – 5	RF Mack Benktander – 1
CAS 2016 – 1	RF Mack Benktander – 2
CAS 2013 – 4	RF Mack Benktander – 3
CAS 2012 – 1	

Credible Loss Ratio Claims Reserves

Hürlimann

Problem

Given the following information:

Incremental Paid Claims ($000)

Accident Year	0-12 Months	12-24 Months	24-36 Months	Earned Premium
2021	180	135	65	450
2022	225	160		475
2023	175			490

Calculate the loss reserve estimate for accident year 2022 for each of the following methods:

i. Benktander loss ratio claims reserve
ii. Neuhaus loss ratio claims reserve
iii. Optimal credible loss ratio claims reserve

Solution Recipe

1) Calculate m_k, the expected incremental loss ratio for each development period.

$$\boxed{m_k = \frac{\sum IncLoss_{i,k}}{\sum Prem_i}}$$

$$m_k = \frac{\sum_{AY=1}^{\#AYs\ in\ dev\ period\ k} IncLoss_{AY,k}}{\sum_{AY=1}^{\#AYs\ in\ dev\ period\ k} Prem_{AY}}$$

$$m_1 = \frac{180+225+175}{450+475+490} = .410$$

$$m_2 = \frac{135+160}{450+475} = .319$$

$$m_3 = \frac{65}{450} = .144$$

2) Calculate the expected loss ratio.

$$\boxed{ELR = \sum m_k}$$

$$Expected\ Loss\ Ratio = \sum E[Inc\ Loss\ Ratios]$$

$$ELR = .410 + .319 + .144$$
$$= \boxed{.873}$$

3) Calculate p_i, the % loss paid for each accident year as of the latest development period.

$$\boxed{p_i = \frac{\sum m_k}{ELR}}$$

$$\%Paid_{AY} = \frac{\sum_{k=1}^{\substack{Dev\ Periods\\ to\ date}} m_k}{\sum m_k \text{ for all AYs}}$$

$$p_{2022} = \frac{.410 + .319}{.873}$$
$$= \boxed{.835}$$

4) **Calculate R^{ind} and R^{coll}, the reserve estimates for the individual loss ratio and collective loss ratio methods.**

$$\boxed{q_i = 1 - p_i}$$

$$\%Unpaid_{AY} = 1 - \%Paid_{AY}$$

$$\boxed{R^{ind} = \frac{q_i \times Loss_i}{p_i}}$$

$$R^{ind} = \frac{\%Unpaid_{AY} \times Loss_{AY}}{\%Paid_{AY}}$$

$$\boxed{R^{coll} = q_i \times Prem \times ELR}$$

$$R^{coll} = \%Unpaid_{AY} \times Premium_{AY} \times ELR$$

$$q_{2022} = 1 - .835$$
$$= .165$$
$$R^{ind}_{2022} = \frac{.165 \times 385}{.835}$$
$$= \boxed{76.3}$$
$$R^{coll}_{2022} = .165 \times 475 \times .873$$
$$= \boxed{68.6}$$

5) **Calculate credibility weights, Z_i, for each method.**

Method	Z_i
Benktander(Z_i^{GB})	$\boxed{Z_i^{GB} = p_i}$
Neuhaus(Z_i^{WN})	$\boxed{Z_i^{WN} = p_i \times ELR}$
Optimal(Z_i^{opt})	$\boxed{Z_i^{opt} = \frac{p_i}{p_i + \sqrt{p_i}}}$

$$Z^{GB}_{2022} = \boxed{.835}$$
$$Z^{WN}_{2022} = .835 \times .873$$
$$= \boxed{.729}$$
$$Z^{opt}_{2022} = \frac{.835}{.835 + \sqrt{.835}}$$
$$= \boxed{.477}$$

Assume:
$Var(U_i) = Var(U_i^{BC})$

6) **Calculate the loss reserve estimate as a credibility weighting of R^{ind} and R^{coll}.**

$$\boxed{R_i = Z_i \times R_i^{ind} + (1 - Z_i) \times R_i^{coll}}$$

$$R^{GB} = .835 \times 76.3 + (1 - .835) \times 68.6$$
$$= \boxed{75.0}$$
$$R^{WN} = .729 \times 76.3 + (1 - .729) \times 68.6$$
$$= \boxed{74.2}$$
$$R^{opt} = .477 \times 76.3 + (1 - .477) \times 68.7$$
$$= \boxed{72.3}$$

Discussion

A potential twist to this problem is to use the general version of the optimal credibility formula. The optimal credibility is based on an assumption about the ratio between $Var(U_i)$ and $Var(U_i^{BC})$, f_i. If we assume that the variance of the actual ultimate loss, $Var(U_i)$, is equal to the variance of the burning cost (expected) ultimate loss, $Var(U_i^{BC})$, then $f_i = 1$, and the optimal credibility weight simplifies to the version above.

If an exam problem gives a different assumption (e.g. $Var(U_i)$ is 25% greater than $Var(U_i^{BC})$), then you need to use the general optimal credibility weight formulas to calculate the optimal credibility weight, Z_i^{opt}. See the "Optimal Credibility Weights" recipe for how to do this.

Overall, the Hürlimann method is very similar to the credibility method in Mack - Benktander, which uses the following as the Benktander loss reserve:

$$R_{GB} = (1 - q_k)R_{CL} + q_k \times R_{BF} \quad \rightarrow \quad R_{GB} = p_k R_{CL} + (1 - p_k)R_{BF}$$

As you can see, the Mack - Benktander formula, has the same form as the credibility-weighted formula in Hürlimann:

$$R_{GB} = p_i R^{ind} + (1 - p_i) \times R^{coll}$$

The key difference is that Hürlimann uses expected incremental loss ratios, m_k, to specify the payment pattern (p_i) instead of LDFs. Also, the individual loss ratio reserve is used instead of the chain ladder reserve and the collective loss ratio reserve is used instead of the BF reserve.

Source

See Hürlimann – pg. 82-85 for discussion of the method and pg. 90-91 for the optimal credibility weight. Don't get too sidetracked on all the proofs and notation. Focus on how to apply the method.

More Practice

CAS 2019 – 2	RF Hürlimann – 1
CAS 2019 – 3	RF Hürlimann – 2
CAS 2017 – 1	RF Hürlimann – 3
CAS 2016 – 1	RF Hürlimann – 4
CAS 2015 – 1	
CAS 2013 – 2	

Optimal Credibility Weights

Hürlimann

Problem

Given the following information as of December 31, 2023:

Claims Reserve Estimates

Accident Year	Individual Loss Ratio Claims Reserve	Collective Loss Ratio Claims Reserve
2021	23,900	24,600
2022	126,700	130,300
2023	566,700	577,800

Loss Ratio Payout Factor (p_i)

	12 Months	24 Months	36 Months
p_i	76.8%	93.7%	97.4%

- The variance of the ultimate loss is assumed to be 50% greater than the variance of the burning cost ultimate loss estimate.

Calculate the optimal credibility loss ratio claims reserve estimate for accident year 2023.

Solution Recipe

1) **Calculate the ratio between the variance of the ultimate loss, $Var\ (U_i)$, and the variance of the burning cost ultimate loss estimate, $Var\ (U_i^{BC})$.**

$$\boxed{f_i = \frac{\text{Var}\ (U_i)}{\text{Var}\ (U_i^{BC})}}$$

$$f_i = 1.5$$

2) **Calculate t_i^{opt} based on the variance ratio assumption (step 1) and the % paid loss, p_i.**

$$\boxed{t_i^{opt} = \frac{f_i - 1 + \sqrt{(f_i + 1) \times (f_i - 1 + 2p_i)}}{2}}$$

$$t_{2023}^{opt} = \frac{1.5 - 1 + \sqrt{(1.5 + 1) \times (1.5 - 1 + 2 \times .768)}}{2}$$

$$= \boxed{1.378}$$

3) **Calculate the credibility weights, Z_i^{opt}.**

$$\boxed{Z_i^{opt} = \frac{p_i}{p_i + t_i^{opt}}}$$

$$Z_{2023}^{opt} = \frac{.768}{.768 + 1.378}$$

$$= \boxed{.358}$$

4) **Calculate the loss reserve estimate as a credibility weighting of R^{ind} and R^{coll}.**

$$\boxed{R_i = Z_i \times R_i^{ind} + (1 - Z_i) \times R_i^{coll}}$$

$$R_{2023}^{opt} = .358 \times 566{,}700 + (1 - .358) \times 577{,}800$$
$$= \boxed{\$573{,}828}$$

Discussion

Notice that the normal optimal credibility weight is a special case of the formulas above, where $f_i = 1$ resulting in $t_i^{opt} = \sqrt{p_i}$. According to Hürlimann, using $f_i = 1$ results in the minimum variance of credible loss reserves among $f_i \geq 1$.

Mean Squared Errors of Collective, Individual, and Optimal Methods

The point of the optimal credibility weights is that they are the weights that minimize the mean squared error (MSE) between the estimated reserve and the true reserve from the actual outcome.

The formulas below give the MSE for the different methods. It's unlikely you'd be asked a question to solve for MSE directly, but it's simple to do it with the following formulas:

$$\boxed{mse\left(R_i^{coll}\right) = E\left[\alpha_i^2(U_i)\right] \times q_i\left(1 + \frac{q_i}{t_i}\right)}$$

$$\boxed{mse\left(R_i^{ind}\right) = E\left[\alpha_i^2(U_i)\right] \times \frac{q_i}{p_i}}$$

$$\boxed{mse\left(R_i^{opt}\right) = E\left[\alpha_i^2(U_i)\right] \times \left(\frac{Z_i^2}{p_i} + \frac{1}{q_i} + \frac{(1 - Z_i)^2}{t_i}\right) \times q_i^2}$$

Source

Hürlimann – pg. 88, 91

More Practice

CAS 2018 – 3 RF Hürlimann – 3

Least Squares Method

Brosius

Problem

Given the following information:

Cumulative Paid Losses ($000)

Accident Year	36 Months	48 Months	60 Months	Earned Premium
2020	260	830	1,240	4,120
2021	840	3,540	3,960	5,350
2022	130	1,860	2,840	6,540
2023	2,160	3,240		7,780
2024	3,610			8,010

- The tail factor from 60 months-to-Ultimate is 1.25

a. Calculate the estimated unpaid losses for accident year 2024 using the Least Squares method.

b. Calculate the credibility weighting on the link ratio method that the Least Squares method uses for accident years 2023 and 2024.

Solution Recipe

Part a – Least Squares Reserve Estimate

1) Convert losses to a cumulative loss ratio triangle and apply the tail factor to develop to ultimate.

$$Loss\ Ratio = \frac{Losses}{Earned\ Premium}$$

$$Loss\ Ratio_{2020,36\ mo} = \frac{260}{4{,}120} = 6.31\%$$

$$Ult\ Loss\ Ratio_{2020} = 30.1\% \times 1.25 = 37.6\%$$

Note:
Converting to loss ratios is only necessary if there's *significant exposure change* over the accident years. In this problem, there's significant Earned Prem growth between 2020 and 2024.

AY	36	48	60	Ult L/R
2020	6.31%	20.1%	30.1%	37.6%
2021	15.7%	66.2%	74.0%	92.5%
2022	1.99%	28.4%	43.4%	54.3%
2023	27.8%	41.6%		
2024	45.1%			

2) Calculate the least squares *a* and *b* parameters for each development period iteratively. Start with the most mature development period. Use undeveloped loss ratios as the 'x' values and ultimate loss ratios as the 'y' values.

$$\boxed{b = \frac{\overline{xy} - \bar{x} \times \bar{y}}{\overline{x^2} - \bar{x}^2}} \quad \boxed{a = \bar{y} - b \times \bar{x}}$$

In Excel

b = SLOPE(Undeveloped Loss Ratio, Ultimate Loss Ratio)

a = INTERCEPT(Undeveloped Loss Ratio, Ultimate Loss Ratio)

Start with the 48 month development period (for accident year 2023):

x (48mo L/R)	y (Ult L/R)
20.1%	37.6%
66.2%	92.5%
28.4%	54.3%

$b =$ SLOPE(48mo Loss Ratios, Ultimate Loss Ratio)

$= \boxed{1.138}$

$a =$ INTERCEPT(48mo Loss Ratios, Ultimate Loss Ratio)

$= \boxed{0.1795}$

3) Calculate the ultimate loss ratio estimate from the loss ratio to-date for each accident year. Start with the oldest accident year without an ultimate loss ratio.

$\boxed{\hat{y} = a + bx}$

$\widehat{Ult\ Loss\ Ratio} = a + b \times Latest\ Loss\ Ratio$

$\hat{y}_{2023} = 0.1795 + 1.138 \times 0.416$

$= \boxed{65.3\%}$

Add the ultimate loss ratios to the triangle (step 1) as an ***additional data points*** for calculating the a and b parameters for the ***next accident year***.

4) Repeat steps 2 and 3 iteratively to calculate the ultimate loss ratios for the remaining accident years.

x (36mo L/R)	y (Ult L/R)
6.31%	37.6%
15.7%	92.5%
1.99%	54.3%
27.8%	**65.3%**

From Step 3

$b =$ SLOPE(36mo Loss Ratios, Ultimate Losses)

$= \boxed{0.9709}$

$a =$ INTERCEPT(36mo Loss Ratios, Ultimate Losses)

$= \boxed{0.4988}$

$\hat{y}_{2024} = .4988 + .9709 \times .451$

$= \boxed{93.6\%}$

AY	Ult Loss Ratio
2023	65.3%
2024	93.6%

5) Calculate unpaid losses using the estimated ultimate loss ratios.

$$Unpaid\ Loss = Earned\ Prem \times Ult\ Loss\ Ratio - Losses$$

$Unpaid_{2023} = 7{,}780 \times 65.3\% - 3{,}240 = \boxed{1{,}843}$

$Unpaid_{2024} = 8{,}010 \times 93.6\% - 3{,}610 = \boxed{3{,}890}$

Part b – Credibility Weighting on Link Ratio Method

6) Calculate the LDF-to-ultimate that the chain ladder (link ratio) method would use for each year using the average ultimate loss ratio and average undeveloped loss ratio.

$$\boxed{c = LDF = \frac{Avg\ Ult\ Loss\ Ratio}{Avg\ Undeveloped\ Loss\ Ratio}}$$

$$LDF_{2023} = \frac{(37.6\% + 92.5\% + 54.3\%)/3}{(6.3\% + 15.7\% + 2.0\%)/3}$$

$$= \boxed{1.607}$$

$$LDF_{2024} = \frac{.6244}{.1294} = \boxed{4.825}$$

7) Calculate the credibility on the link ratio method, *Z*, for each year.

$$\boxed{Z = \frac{b}{c}}$$

$$Cred = \frac{b}{LDF}$$

$$Z_{2023} = \frac{1.138}{1.607} = \boxed{70.8\%}$$

$$Z_{2024} = \frac{.9709}{4.825} = \boxed{20.1\%}$$

Discussion

The key thing to understand is that steps 2 and 3 are an iterative process. We first use 48mo loss ratios as x and ultimate loss ratios as y in order to get the estimated ultimate loss ratio for AY 2023. When we iterate through with 36mo loss ratios as x, we can use our estimated ultimate loss ratio for AY 2023 (65.3%) as an *additional data point*.

The Least Squares method is a credibility-weighting of the Link Ratio and Budgeted Loss methods with the following formula:

$$\boxed{\hat{y} = Z \times \frac{x}{d} + (1 - Z) \times E[y]}$$

$$\hat{y} = Z \times LDF \times x + (1 - Z) \times \bar{y}$$

AY 2023 Example:

$\hat{y} = .708 \times 1.607 \times .416 + (1 - .708) \times .6148$

$= 65.3\%$ (same as step 3)

When to Use Losses and When to Use Loss Ratios

The least squares method can be used with either actual losses or loss ratios. Brosius recommends using loss ratios *if there is significant premium growth*. If premium increases normally, then using loss ratios is unnecessary. Using loss ratios puts the accident years on a more equal basis for the least squares method.

Potential Problems with Parameter Estimation

Sampling error can sometimes result in negative *a* or *b* values, which can cause nonsensical loss estimates. If this happens, Brosius recommends the following (see Brosius pg. 4 for more):

The intercept is negative (a < 0):

- This causes the estimate of developed losses ($\hat{y}$) to be negative for small values of x.
- **Solution:** Use the link ratio method instead.

$$LDF = \frac{\bar{y}}{\bar{x}} \qquad \hat{y}^{link\ ratio} = LDF \times x$$

The slope is negative (b < 0):

- This causes the estimate of y to decrease as x increases
- **Solution:** Use the budgeted loss method instead, ignoring x.

$$\hat{y}^{budget} = \bar{y}$$

<u>Reviewing the Results</u>

As we go from more recent AYs to more mature AYs, we should usually see the following patterns:

- a *<u>decreases</u>* for more developed years
- c (LDF) *<u>decreases</u>* for more developed years
- Z *<u>increases</u>* for more developed years (more weight on the link ratio method)

This is because, in earlier periods (e.g. at 12 months), the actual loss experience (x) is more volatile and less useful in predicting the future loss (y). Therefore, the intercept (a parameter) is larger, and the slope (b parameter) is smaller due to less credibility (Z).

As the accident years mature, the actual losses have more credibility and we place greater weight on the link ratio method and less weight on the budgeted loss method. The loss experience receives greater weight and we place less weight on the intercept, a. (See pg. 18 in Brosius for more discussion)

These patterns hold for the calculations above, so this is a good additional check that the results are reasonable.

CBT Spreadsheet Tips

On the exam, it's best to use the `SLOPE()` and `INTERCEPT()` formulas to calculate the b and a parameters instead of trying to calculate them with the linear regression formulas. These formulas are available in the Pearson Vue testing environment and will get full credit.

Source

Brosius – pg. 16-18

More Practice

CAS 2018 – 4	RF Brosius – 1
CAS 2017 – 2	RF Brosius – 3
CAS 2016 – 2	RF Brosius – 5
CAS 2012 – 4	RF Brosius – 8
CAS 2011 – 1	

Bayesian Credibility in a Changing System

Brosius

Problem

Prior to a legislative change, a small book of personal auto insurance in State X had the following loss estimates for the upcoming 2025 accident year:

- Expected losses: $35 Million
- Percent of losses reported through 12 months: 70%

A legislative change in the state, effective 1/1/2025, is estimated to impact expected losses and development patterns going forward.

The actuary estimates the following beginning in 2025:

- Expected losses are estimated to fall by: 20%
- Percent of losses reported through 12 months is expected to speed up to: 75%
- The actuary selects the standard deviation of losses to be: $5 Million
- The actuary selects a standard deviation of the percent reported at 12 months to be: 10%

As of December 31, 2025, accident year 2025 reported losses were: $25 Million.

Calculate the estimated accident year 2025 ultimate losses using Bayesian Credibility.

Solution Recipe

1) Identify the necessary inputs: $\boldsymbol{E[Y], \sigma(Y), E\left[\frac{X}{Y}\right], \sigma\left[\frac{X}{Y}\right]}$

$\mathrm{E}[Y]$ → Expected Ultimate Losses	$\mathrm{E}[Y] = 35\mathrm{M} \times (1 - 0.2) = 28\mathrm{M}$
$\sigma(Y)$ → Std Dev of Ultimate Losses	$\sigma(Y) = 5\mathrm{M}$
$\mathrm{E}\left[\frac{X}{Y}\right]$ → Expected %Reported	$\mathrm{E}\left[\frac{X}{Y}\right] = .75$
$\sigma\left(\frac{X}{Y}\right)$ → Std Dev of %Reported	$\sigma\left(\frac{X}{Y}\right) = .10$
X → Random variable for losses (at 12 months)	
Y → Random variable for ultimate losses	

2) **Calculate Z, the credibility weight between the Link Ratio and Budgeted Loss (Expected) methods.**

$$\boxed{VHM = \left(E\left[\frac{X}{Y}\right] \times \sigma(Y)^2\right)^2}$$

$$\boxed{EVPV = \sigma\left(\frac{X}{Y}\right)^2 \times \left(\sigma(Y)^2 + E[Y]^2\right)}$$

$$\boxed{Z = \frac{VHM}{VHM + EVPV}}$$

$$VHM = (.75 \times 5)^2$$
$$= \boxed{14.06}$$
$$EVPV = .10^2 \times [5^2 + 28^2]$$
$$= \boxed{8.09}$$
$$Z = \frac{14.06}{14.06 + 8.09}$$
$$= \boxed{.635}$$

3) **Calculate ultimate losses as a credibility weighting between the Link Ratio ultimate and the Expected ultimate.**

$$\boxed{Ult = Z \times \frac{X}{E\left[\frac{X}{Y}\right]} + (1 - Z) \times E[Y]}$$

$$Ult = Cred \times Ult_{ChainLadder} + (1 - Cred) \times E[Ult]$$

$$Ult = .635 \times \frac{25\text{M}}{.75} + (1 - .635) \times 28\text{M}$$
$$= \boxed{31.4\text{M}}$$

Discussion

A key assumption of the least squares method is that there aren't systematic shifts in the book of business. With this assumption, historical accident year losses can be used to project ultimate losses for undeveloped accident years.

The Bayesian method is appropriate for a new line of business or when there is a significant change in the book of business and the go-forward experience will be different than historical accident years.

The tricky part with this type of problem will be to properly identify all the inputs necessary to solve the problem. Once you've calculated Z, the credibility weighting, just remember that it's a simple credibility weighting between the chain ladder ultimate and expected ultimate.

Source

Brosius – pg. 14-15

More Practice

CAS 2019 – 1	RF Brosius – 2
CAS 2016 - 2	RF Brosius – 4
CAS 2014 – 1	

Caseload Effect

Brosius

Problem

Insurer ABC writes Motorcycle insurance and recently expanded its book of business into a new state. Management expects the written premium to be $20M in 2024. The reserving actuary makes the following assumptions beginning in 2024:

- Expected written premium $20M
- Expected loss ratio 80%
- 70% of losses are expected to be reported by 12 months of development
- The actuary selects the standard deviation of ultimate losses to be $4M
- The actuary selects the standard deviation of the percent reported at 12 months to be 10%

The actuary assumes that the development ratio varies with caseload and that, <u>*if ultimate losses are 25% higher than expected*</u>, then the percent of losses reported at 12 months will be 65%.

As of December 31, 2024, accident year 2024 reported losses were $13,400,000.

Calculate the estimated accident year 2024 unreported loss reserve using Bayesian Credibility.

Solution Recipe

1) Identify the necessary inputs: $\boldsymbol{E[Y], \sigma(Y), E\left[\frac{X}{Y}\right], \sigma\left[\frac{X}{Y}\right]}$

$\mathrm{E}[Y] \rightarrow$ Expected Ultimate Losses	$\mathrm{E}[Y] = 20\mathrm{M} \times .8 = 16\mathrm{M}$
$\sigma(Y) \rightarrow$ Std Dev of Ultimate Losses	$\sigma(Y) = 4\mathrm{M}$
$\mathrm{E}\left[\frac{X}{Y}\right] \rightarrow$ Expected %Reported	$\mathrm{E}\left[\frac{X}{Y}\right] = .70$
$\sigma\left(\frac{X}{Y}\right) \rightarrow$ Std Dev of %Reported	$\sigma\left(\frac{X}{Y}\right) = .10$
$X \rightarrow$ Random variable for losses (at 12 months)	
$Y \rightarrow$ Random variable for ultimate losses	

2) Calculate $\boldsymbol{Z}$, the credibility weight between the Link Ratio and Budgeted Loss (Expected) methods.

$$\boxed{VHM = \left(E\left[\frac{X}{Y}\right] \times \sigma(Y)\right)^2}$$

$$\boxed{EVPV = \sigma\left(\frac{X}{Y}\right)^2 \times \left(\sigma(Y)^2 + E[Y]^2\right)}$$

$$VHM = (.70 \times 4)^2 = \boxed{7.84}$$

$$EVPV = .10^2 \times [4^2 + 16^2] = \boxed{2.72}$$

$$\boxed{Z = \frac{VHM}{VHM + EVPV}}$$

$$Z = \frac{7.84}{7.84 + 2.72}$$
$$= \boxed{.742}$$

3) **Calculate the parameters that define the development ratio for the modified link ratio method, reflecting how the development ratio varies with caseload.**

$$\boxed{E[X|Y = y] = dy + x_0}$$

$y = 16\text{M}$: $\quad .70 \times 16\text{M} = d \times 16\text{M} + x_0$

$y = 1.25 \times 16\text{M} = 20\text{M}$: $\quad .65 \times 20\text{M} = d \times 20\text{M} + x_0$

$$1.8\text{M} = d \times 4\text{M}$$
$$d = \boxed{0.45}$$
$$x_0 = \boxed{4\text{M}}$$

Note:
For the normal link ratio method, the development ratio is fixed. The caseload effect modifies this to reflect that the expected percent reported at 12 months is ***lower*** than usual if the ultimate losses for the accident year are ***higher*** than expected.

4) **Calculate the ultimate loss estimate as a credibility weighting between the Link Ratio ultimate and the Expected ultimate using the caseload Bayesian credibility formula.**

$$\boxed{\hat{y} = Z \times \frac{x - x_0}{d} + (1 - Z) + \mathrm{E}[Y]}$$

$$\hat{y} = .742 \times \frac{13.4\text{M} - 4\text{M}}{.45} + (1 - .742) \times 16\text{M}$$
$$= \boxed{19.63\text{M}}$$
$$Loss\ Reserve = 19.63\text{M} - 13.4\text{M}$$
$$= \boxed{6.23\text{M}}$$

Discussion

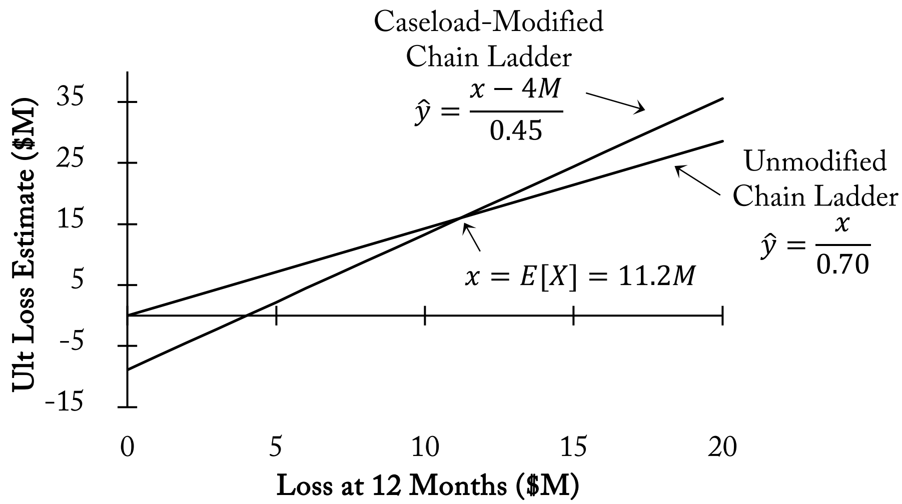
This question is similar to the regular Bayesian credibility method, but the chain ladder estimate is modified for the caseload effect. Instead of using a fixed percent reported, the expected percent reported is lower if ultimate losses (Y) are higher.

The resource constraint piece is the key for this problem type. The idea behind the caseload effect is that the *reported loss percent changes based on the caseload*. When there are a lot of claims (which typically drives higher ultimate losses), the reporting percentage at an earlier development period is smaller. That's what the resource constraint piece is getting at.

Below is a graphical view of how the caseload effect chain ladder estimate compares to the unmodified chain ladder estimate.

- If x > E[X], the caseload estimate will be higher than the unmodified chain ladder estimate.
- If x < E[X], the caseload estimate will be lower than the unmodified chain ladder estimate.

Unmodified chain ladder estimate vs. caseload-modified chain ladder estimate:



Source

Brosius – pg. 15-16

More Practice

RF Brosius – 9
RF Brosius – 10

Proportional Reinsurance

Friedland

Problem

For a primary insurer, assume the following loss experience both gross of reinsurance and net retained after excess of loss reinsurance for a single line of business:

	Gross ($000)		Net Retained ($000)	
Policy Limit	Earned Premium	Ultimate Loss	Earned Premium	Ultimate Loss
500,000	5,860	4,100	5,630	4,100
1,000,000	9,470	7,800	8,740	7,800
2,000,000	6,320	4,100	6,010	4,100
5,000,000	4,800	8,140	4,550	3,830
10,000,000	2,010	1,500	1,800	1,500

Below are two possible proportional reinsurance treaties for the line of business. Assume only one proportional reinsurance option would apply, not both at the same time.

Option 1: Quota Share Treaty

- Ceded Percentage = 45%
- Excess of Loss reinsurance inures to the benefit of the Quota Share

Option 2: Surplus Share Treaty

- Retained Line = $1,000,000
- Number of Lines = 4
- Excess of Loss reinsurance inures to the benefit of the Surplus Share

Calculate the ceded earned premium and ultimate loss for each proportional reinsurance treaty option and the amount retained after both the excess of loss reinsurance and proportional treaty option are applied.

Solution Recipe

Option 1 - Quota Share Treaty

1) **Apply the ceded percentage to subject premium and losses (gross or net retained premium and losses if other reinsurances inures to the benefit of the treaty) to calculate the amounts ceded. Pay attention if it is a variable quota share or only applies to certain lines or segments.**

$$\boxed{Ceded\ Loss = Subject\ Loss \times \%Ceded}$$

$$\boxed{Ceded\ Prem = Subject\ Prem \times \%Ceded}$$

Note:
The excess of loss reinsurance inures to the benefit of the Quota Share, so the Quota Share applies *after* subtracting out the impact of the Excess of Loss.

	Ceded	
Policy Limit	Earned Premium	Ultimate Loss
500,000	2,534	1,845
1,000,000	3,933	3,510
2,000,000	2,705	1,845
5,000,000	2,048	1,724
10,000,000	810	675

$$Ceded\ Prem_{10M} = 1{,}800\ \times\ 45\% = 810$$

2) **Calculate the retained amount as the gross premium and losses (or net retained premium and losses if other reinsurances are deducted first) minus the amounts ceded to the reinsurance treaty.**

$$Net\ Loss = Subject\ Loss\ - Ceded\ Loss$$

$$Net\ Prem = Subject\ Prem\ - Ceded\ Prem$$

Note:
Here the retained is after both the excess of loss and proportional reinsurance are subtracted.

Retained

Policy Limit	Earned Premium	Ultimate Loss
500,000	3,097	2,255
1,000,000	4,807	4,290
2,000,000	3,306	2,255
5,000,000	2,503	2,107
10,000,000	990	825

$$Retained\ Prem_{10M} = 1{,}800\ - 810 = 990$$

Option 2 - Surplus Share Treaty

3) **Calculate the ceded percentage for each policy limit, based on the Retained Line (RL) and the number of lines reinsured. The maximum reinsured portion is: Retained Line x #Lines**

$$\%Ceded = \frac{\overbrace{Policy\ Limit\ - Retained\ Line}^{\text{Cap at } Retained\ Line \cdot \#Lines}}{Policy\ Limit}$$

Policy Limit	% Ceded
500,000	0%
1,000,000	0%
2,000,000	50%
5,000,000	80%
10,000,000	40%

$$\%Ceded_{10M} = \frac{\text{Min}(10\text{M} - 1\text{M}, 4 \times 1\text{M})}{10\text{M}}$$
$$= 40\%$$

In Excel

Reinsured Portion (numerator) = MAX(MIN(Policy Limit - Retained Line, #Lines * Retained Line) , 0)

4) **Apply the ceded percentage to gross premium and losses (or net retained premium and losses if other reinsurances are deducted first) to calculate the amounts ceded.**

$$\boxed{Ceded\ Loss = Subject\ Loss \times \%Ceded}$$

$$\boxed{Ceded\ Prem = Subject\ Prem \times \%Ceded}$$

Ceded

Policy Limit	Earned Premium	Ultimate Loss
500,000	0	0
1,000,000	0	0
2,000,000	3,005	2,050
5,000,000	3,640	3,064
10,000,000	720	600

$$Ceded\ Prem_{10M} = 1{,}800\ \times\ 40\% = 720$$

5) **Calculate the retained amount as the gross premium and losses (or net retained premium and losses if other reinsurances are deducted first) minus the amounts ceded to the reinsurance treaty.**

$$Net\ Loss = Subject\ Loss\ - Ceded\ Loss$$

$$Net\ Prem = Subject\ Prem\ - Ceded\ Prem$$

Retained

Policy Limit	Earned Premium	Ultimate Loss
500,000	5,630	4,100
1,000,000	8,740	7,800
2,000,000	3,005	2,050
5,000,000	910	766
10,000,000	1,080	900

$$Retained\ Prem_{10M} = 1{,}800\ - 720 = 1{,}080$$

Note:
Here the retained is after *both* the excess of loss and proportional reinsurance is subtracted.

Discussion

The main functions of proportional reinsurance are to manage capital & solvency margin (capital/surplus relief) and to increase capacity. Surplus share reinsurance increases capacity more effectively because it allows the ceding company to cede a smaller portion of smaller risks and a larger portion of larger risks.

Ceding Commission - Usually paid by the reinsurer to the ceding company to reimburse for expenses related to issuing the underlying policies (acquisition & underwriting expenses).

Possible Problem Twists

- You could be given a % retained as opposed to % ceded for a quota share
- Pay attention to whether other reinsurance applies first. If so, use the Net Retained numbers after the first reinsurance applies.

CBT Spreadsheet Tips

Use a nested `MAX( MIN( ) )` formula to calculate the portion of a surplus share ceded correctly to cap it between 0 and #Lines x Retained Line.

Source

Friedland – pg. 10-13

More Practice

RF Friedland – 4
RF Friedland – 8
RF Friedland – 9