Rising Fellow

2025

Exam 7

Study Guide

ADVANCED ESTIMATION OF CLAIMS LIABILITIES

Comprehensive study guide
with original and past CAS problems

Exam 7 Study Guide

2025 Sitting

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Introduction

How to Use This Guide

This guide is intended to **supplement** the Content Outline readings. Although we believe it provides a thorough review of the exam material, the readings provide additional context that is invaluable. Please do NOT skip the Content Outline readings.

Past CAS Exam Problems

Past CAS exam problems & solutions are included for each paper. Note that these questions are solely owned by the CAS. They are included in the online course for student convenience. All past CAS problems are Excel-based and can be downloaded from the online course.

Rounding

Numerous examples are provided throughout the guide. We did ***not round*** any of the intermediate steps in the examples. All calculations were performed with full precision to ensure accuracy, and only the final answers were rounded when necessary. To exactly reproduce the examples, we recommend working them out in Microsoft Excel.

Feedback

We always working to improve the Exam 7 Study Guide and the rest of the Rising Fellow study material. Please send us an email at **exam7@risingfellow.com** if you have feedback about any of the following:

- Sections that are confusing or could be improved
- Errors (ex. formatting, spelling, calculations, grammar, etc.)

Note that errata will be posted on the Rising Fellow website on an as-needed basis.

Blank Pages

Since many students want a printed copy of the study guide, blank pages have been inserted throughout the guide to ensure that all outlines start on odd pages.

Brosius

Outline

I. Introduction

Real-world loss data is subject to the following:

- Random fluctuations
- Systematic distortions

The **least squares method** should be considered whenever **random year-to-year fluctuations** in loss experience are significant.

II. Least Squares Method

Before we dive in, we need to define a few things:

- $x =$ losses to date
- $y =$ losses at a future evaluation

Assuming we have a historical loss triangle, we should have a number of historical (x, y) pairs. The **goal is to predict** y based on x. Let $L(x)$ be the estimate of y, given that we have already observed x.

<u>Link Ratio Method</u>

The link ratio method (i.e., chain-ladder method) **estimates $\boldsymbol{y}$** as follows:

$$\boxed{L(x) = cx}$$

where c is the selected link ratio. For the purposes of this paper, we will assume that c is the volume-weighted average LDF.

Budgeted Loss Method

In some cases, it might make sense to ignore the losses to date. For example:

- When fluctuation in loss experience is extreme
- When past data is not available

In these cases, we could **estimate** y using the budgeted loss method as follows:

$$\boxed{L(x) = k}$$

where k is a constant.

The constant k could be chosen by averaging y over several years, or by multiplying earned premium by an expected loss ratio. For the purposes of this paper, we will assume that k is based on averaging y over several years.

Least Squares Method

The least squares method **estimates** y by fitting a line to the points (x, y) that minimizes the sum of the squares of the residuals. Mathematically:

$$\boxed{L(x) = a + bx}$$

where $b = \frac{\overline{xy} - \bar{x}\bar{y}}{\overline{x^2} - \bar{x}^2}$ and $a = \bar{y} - b\bar{x}$.

Many common methods are **special cases** of the least squares method:

- When $a = 0$, then $L(x) = bx$ (link ratio method)
- When $b = 0$, then $L(x) = a$ (budgeted loss method)
- When $b = 1$, then $L(x) = a + x$ (BF method)

The ability to flex to other methods is a **major advantage** of the least squares method. Later on, we will show that the least squares method is a credibility-weighted average between the link ratio method and the budgeted loss method, highlighting its ability to give more or less weight to the observed value of x as appropriate.

Example: Basic Least Squares Method

An actuary would like to predict AY 2023 losses at 27 months. Given the following information for a small state as of December 31, 2023:

Incurred Losses ($) as of XX Months:

AY	15 mo.	27 mo.	15-27 Link Ratios
2017	19,039	23,279	1.223
2018	33,040	41,560	1.258
2019	14,637	18,937	1.294
2020	2,785	5,185	1.862
2021	51,606	54,206	1.050
2022	5,726	15,726	2.746
2023	40,490		

Due to the volatility in the data and link ratios, we may not want to give full credibility to the high observed loss for 2023 by applying a large link ratio to it. Since the data fluctuations do not appear to be systematic, we should consider using the least squares method.

First, let's calculate the least squares parameters:

Using the LINEST Function

- The known x-values are (19039, 33040, …, 5726)
- The known y-values are (23279, 41560, …, 15726)
- We can use the **LINEST** function in Excel to find the parameters as follows: LINEST(known y values, known x values) = LINEST((23279, 41560, …, 15726), (19039, 33040, …, 5726)) = (0.96781, 5023.70787)
- The first value output by Excel is b and the second value output is a
- Hence, $\boldsymbol{b = 0.96781}$ and $\boldsymbol{a = 6023.70787}$

Using the Least Squares Formulas

On the *exam*, we recommend using the LINEST function to calculate the parameters. However, to demonstrate the least squares formulas, here's how to calculate the parameters by hand:

- $\overline{xy} = \frac{19{,}039(23{,}279)+33{,}040(41{,}560)+\cdots+5{,}726(15{,}726)}{6} = 832{,}562{,}381$
- $\bar{x} = \frac{19{,}039+33{,}040+\cdots+5{,}726}{6} = 21{,}139$
- $\bar{y} = \frac{23{,}279+41{,}560+\cdots+15{,}726}{6} = 26{,}482$
- $\overline{x^2} = \frac{19{,}039^2+33{,}040^2+\cdots+5{,}726^2}{6} = 728{,}681{,}571$
- $\boldsymbol{b} = \frac{\overline{xy}-\bar{x}\bar{y}}{\overline{x^2}-\bar{x}^2} = \frac{832{,}562{,}381-21{,}139(26{,}482)}{728{,}681{,}571-21{,}139^2} = \mathbf{0.96781}$
- $\boldsymbol{a} = \bar{y} - b\bar{x} = 26{,}482 - 0.96781(21{,}139) = \mathbf{6023.70787}$
- These are the same parameters we obtained using the LINEST function

Second, let's estimate the AY 2023 incurred losses at 27 months using the least squares method:

Using the FORECAST Function

- We can use the **FORECAST** function in Excel to estimate y as follows: FORECAST(x, known y values, known x values) = FORECAST(40490, (23279, 41560, …, 15726), (19039, 33040, …, 5726)) = **45210.4966**
- Thus, the estimated AY 2023 incurred losses at 27 months using the least squares method are $45,210.50
- If the only goal is estimate y using the least squares method, then the FORECAST function is useful because it doesn't require the least squares parameters as inputs. However, *we recommend* calculating the parameters to make sure they are sensible (more on this later)

Using the Least Squares Formula

- Recall that $L(x) = a + bx$
- Thus, $L(40{,}490) = 6023.70787 + 0.96781(40{,}490) = \mathbf{45210.4966}$

- This is the same prediction we obtained using the FORECAST function

---

Parameter Estimation Errors

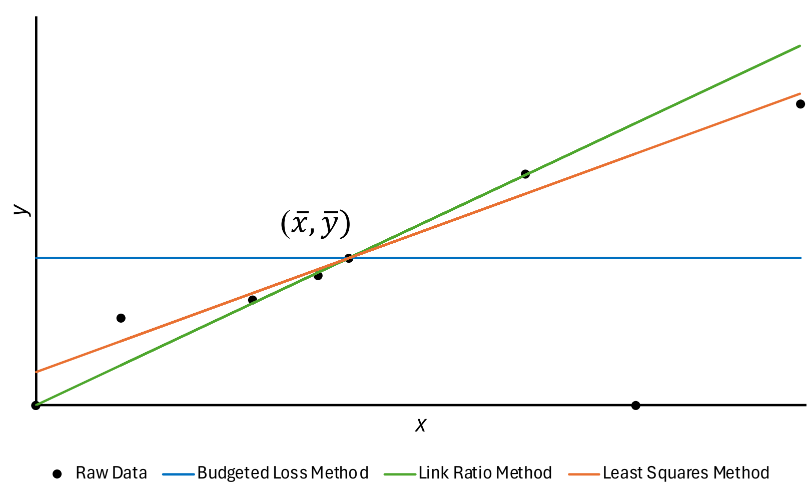
In the example above, there was nothing unusual about the least squares parameters. However, both **significant changes in the nature of the loss experience** and **sampling error** can lead to values of a and b that do not reflect reality:

- When $\boldsymbol{a} < \mathbf{0}$, our estimate of y will be negative for small values of x. In this case, use the **link ratio method**
- When $\boldsymbol{b} < \mathbf{0}$, our estimate of y decreases as x increases. In this case, use the **budgeted loss method**

The parameter estimation errors above are why we recommend always calculating the least squares parameters on the *exam*. In the event that $a < 0$ or $b < 0$, you should estimate y using either the link ratio method or budgeted loss method, respectively.

Comparing the Methods Graphically

The following plot compares the link ratio, budgeted loss, and least squares methods for the basic least squares example above:



We can summarize the plot as follows:

- The **link ratio method** fits a line through the origin with a slope equal to the selected link ratio
- The **budgeted loss method** fits a horizontal line equal to $\bar{y}$
- The **least squares method** yields a "best fit" line with an intercept of a and a slope of b
- The lines intersect at the point $(\bar{x}, \bar{y})$

III. Hugh White's Question

Question: You are trying to establish the reserve for commercial auto bodily injury and the reported proportion of expected losses as of the statement date for the current accident year period is 8% higher than it should be. Do you:

- Reduce the bulk reserve by a corresponding amount (**budgeted loss method**)
- Leave the bulk reserve at the same percentage level of expected losses (**BF method**)
- Increase the bulk reserve in proportion to the increase of actual reported over expected reported (**link ratio method**)

The point of this question is to show that the three different options might all be reasonable answers to the question. In fact, these three options lie on a continuum of options implied by the least squares estimate $L(x) = a + bx$.

IV. Testing the Least Squares Method Using Loss & Loss Reporting Distributions

The plot of the link ratio, budgeted loss, and least squares methods for the basic least squares example showed a strong fit for the least squares method. However, that could have been pure luck. To truly test the effectiveness of the least squares method, we must know the form of the underlying loss & loss reporting distributions. In this section, we will look at a number of theoretical models.

Simple Model

Given the following information:

- The number of claims incurred each year is a random variable Y which is either 0 or 1 with equal probability
- Th number of claims reported by year-end is a random variable X. If there is a claim, there is a 50% chance it will be reported by year-end. Hence, X is a binomial random variable with $n = y$ and $p = 0.5$

Let $Q(x)$ represent the expected total number of claims, and $R(x)$ represent the expected number of claims outstanding, both given that $X = x$. Mathematically:

$$Q(x) = E[Y|X = x]$$
$$R(x) = E[Y - X|X = x]$$
$$= Q(x) - x$$

To calculate $Q(x)$ and $R(x)$, we need $P(Y = y|X = x) = \frac{P(X = x|Y = y)P(Y=y)}{P(X=x)}$.

The following table calculates the numerator and denominator of the probability above:

	$P(X = x\|Y = y)P(Y = y)$		
x/y	0	1	$P(X = x)$
0	0.50	0.25	0.75
1		0.25	0.25

As an **example calculation**, consider the case where $X = 0$ (zero claims reported by year-end) and $Y = 0$ (zero claims incurred for the year):

- $P(X = 0|Y = 0)P(Y = 0) = \left[\binom{0}{0}(0.5)^0(1 - 0.5)^{(0-0)}\right](0.50) = 0.50$

At this point, we have what we need to calculate $P(Y = y|X = x)$, $Q(x)$, and $R(x)$:

	$P(Y = y \mid X = x)$			
x/y	0	1	$\boldsymbol{Q(x)}$	$\boldsymbol{R(x)}$
0	$0.67 = \frac{0.50}{0.75}$	0.33	$0.33 = 0(0.67) + 1(0.33)$	$0.33 = 0.33 - 0$
1		1.00	1.00	0.00

In this case, $Q(x)$ and $R(x)$ can be written as linear equations:

- $Q(x) = 0.67x + 0.33$
- $R(x) = -0.33x + 0.33$

The best (Bayesian) estimate of y (the number of claims incurred each year), given x (the number of claims reported by year-end), is a line with slope $b = 0.67$ and $a = 0.33$. This relationship is not described by the link ratio, budgeted loss, or BF methods. It is **best described by the least squares method**.

Poisson-Binomial Case

Assume that Y is Poisson distributed with mean μ, and where any given claim has probability d of being reported by year-end. In this case, we have the following for $Q(x)$ and $R(x)$:

$$Q(x) = x + \mu(1 - d)$$
$$R(x) = \mu(1 - d)$$

Which method aligns best with the $Q(x)$ and $R(x)$ shown above?

- The link ratio method is NOT optimal since there is no c such that $x + \mu(1 - d) = cx$
- The budgeted loss method is NOT optimal since $Q(x)$ is not a constant
- The **BF method is optimal** since the expected number of outstanding claims, $R(x)$, does not depend on the number of claims already reported

Negative Binomial-Binomial Case

Assume Y is negative binomial distributed with parameters (r, p), and where any given claim has probability d of being reported by year-end. In this case, we have the following for $R(x)$:

$$R(x) = \frac{(1-d)(1-p)}{1-(1-d)(1-p)}(x+r)$$

Except in the trivial case where $d = 1$, this is an **increasing linear function** of x, meaning an **increase in reported claims** leads to an **increase in our estimate of outstanding claims**.

This **does NOT correspond** to any of Hugh White's answers:

- Since $R(x)$ is an increasing function of x, the budgeted loss and BF methods are not optimal
- Since the relationship is not proportional, the link ratio method is also not optimal

Question: Why does an increasing function make intuitive sense in this case?

- **Consider a Poisson** distribution with mean 2. This means that the variance of the Poisson distribution is also 2
- Now **consider a negative binomial** distribution with mean 2. Based on the variance formula for the negative binomial distribution, the variance MUST be greater than 2
- Hence, the negative binomial distribution has more variance than the Poisson distribution with the same mean!
- This means we have less confidence in our prior estimate of expected losses and are more willing to increase our estimated ultimate claim count. Since a Poisson distribution corresponds to "no dependence between outstanding claims and claims already reported," an increase in the estimated ultimate claim count implies an increasing function of x

Fixed Prior Case

Suppose Y is not random, which means there is some value k such that Y is sure to equal to k (ex. single-premium whole life policies). In this case, we have the following for $Q(x)$ and $R(x)$:

$$Q(x) = k$$
$$R(x) = k - x$$

This corresponds to the **budgeted loss method**.

Fixed Reporting Case

Suppose there is a number $d \neq 0$ such that the percentage of claims reported by year-end is always d. In this case, we have the following for $Q(x)$ and $R(x)$:

$$Q(x) = \frac{x}{d}$$
$$R(x) = \frac{x}{d} - x$$

This corresponds to the **link ratio method**.

Non-Linear Case

In each of the previous models, $Q(x)$ was linear in x and was of the form $Q(x) = a + bx$. The following example moves away from linearity.

Given the following information:

- The number of claims incurred each year is a random variable Y that is uniformly distributed on the discrete set, {2,3,4,5,6}
- Th number of claims reported by year-end is a random variable X. If there is a claim, there is a 50% chance it will be reported by year-end. Hence, X is a binomial random variable with $n = y$ and $p = 0.5$

Using the same process we used for the **simple model**, we obtain the following table:

$P(Y = y|X = x)$

x/y	2	3	4	5	6	$Q(x)$	$R(x)$
0	$\frac{16}{31}$	$\frac{8}{31}$	$\frac{4}{31}$	$\frac{2}{31}$	$\frac{1}{31}$	2.839	2.839
1	$\frac{32}{88}$	$\frac{24}{88}$	$\frac{16}{88}$	$\frac{10}{88}$	$\frac{6}{88}$	3.250	2.250
2	$\frac{16}{99}$	$\frac{24}{99}$	$\frac{24}{99}$	$\frac{20}{99}$	$\frac{15}{99}$	3.939	1.939
3		$\frac{8}{64}$	$\frac{16}{64}$	$\frac{20}{64}$	$\frac{20}{64}$	4.813	1.813
4			$\frac{4}{29}$	$\frac{10}{29}$	$\frac{15}{29}$	5.379	1.379
5				$\frac{2}{8}$	$\frac{6}{8}$	5.750	0.750
6					1	6.000	0.000

As the table above shows, $R(x)$ is not linear. However, it is monotonic (the text incorrectly states that it is not monotonic).

Intuitively, it makes sense that $R(x)$ should decrease since Y has less variance than a Poisson distribution with the same mean (hence, we have more confidence in our prior estimate of expected losses and are less willing to revise our estimated ultimate claim count).

V. The Linear Approximation (Bayesian Credibility)

In general, it is difficult to compute a pure Bayesian estimate Q since it requires knowledge of the loss and loss reporting processes. This makes it difficult to make assumptions.

As a replacement for the Bayesian estimate, we can use the best linear approximation, which has the following advantages:

- Simpler to compute

- Easier to understand and explain
- Less dependent upon the underlying distribution

Let L be the best linear approximation to Q. Mathematically, L is the linear function that minimizes $E_X[(Q(X) - L(X)]^2$.

Assuming $L(x) = a + bx$, we must minimize $E_X[(Q(X) - a - bX]^2$.

Development Formula 1

Given random variables Y describing ultimate losses and X describing reported losses, the best linear approximation to Q is as follows:

$$\boxed{L(x) = (x - E[X])\frac{Cov(X,Y)}{Var(X)} + E[Y]}$$

This formula provides us with an answer to Mr. White's question:

- If $Cov(X,Y) < Var(X)$, a large reported amount should lead to a **decrease** in the reserve (**budgeted loss method**)
- If $Cov(X,Y) = Var(X)$, a large reported amount should **not affect** the reserve (**BF method**)
- If $Cov(X,Y) > Var(X)$, a large reported amount should lead to an **increase** in the reserve (**link ratio method**)

Example: Development Formula 1

Suppose that $E[Y] = 1{,}000$ (expected ultimate loss) and $E[X] = 500$ (the amount expected to be reported by the end of the year). Then, the expected reserve at the end of the year is 1,000 - 500 = 500.

Now, assume that $x = 750$ (actual amount reported by the end of the year). The following table shows how the reserve changes based on the ratio $\frac{Cov(X,Y)}{Var(X)}$:

$\frac{Cov(X,Y)}{Var(X)}$	**0.90**	**1.00**	**1.10**
Expected Ultimate	1,000	1,000	1,000
Expected Reserve	500	500	500
Revised Ultimate, $L(x)$	1,225	1,250	1,275
Revised Reserve, $L(x) - x$	475	500	525

As an **example calculation**, let $\frac{Cov(X,Y)}{Var(X)} = 0.90$. Then, $L(750) = (750 - 500)(0.90) + 1{,}000 = 1{,}225$ and $L(750) - 750 = 1{,}225 - 750 = 475$. As expected, the higher reported amount led to a decrease in the reserve.

As we can see in the formula for $L(x)$ above, we still have random variables. Instead of making assumptions about the distributions of these random variables, we can use **empirical data** to estimate the means, variance, and covariance from the data:

$$L(x) = (x - \bar{X})\frac{\overline{XY} - \bar{X}\bar{Y}}{\overline{X^2} - \overline{X}^2} + \bar{Y}$$

This formula should look familiar. It's a reconfigured version of the least squares formula!

When is Least Squares Development Appropriate?

As mentioned earlier, the least squares fit **may be appropriate** if year-to-year changes are due largely to **random chance**.

The least squares fit **does NOT make sense** if year-to-year changes in loss experience are due largely to **systematic shifts or distortions** in the book of business.

In the case of systematic distortions, if the data can be adequately adjusted, **we may still be** able to apply the least squares method. Here are a couple of **examples**:

- If studying **incurred loss data**, we can correct for inflation by putting the years on a constant-dollar basis before fitting a line

- If the business expands, we can divide each year's losses by an exposure base to eliminate the distortion

VI. A Credibility Form of the Development Formula

The goal is to reconfigure Development Formula 1 into a credibility weighting system. Let's start with some definitions:

- $E_Y[Var(X|Y)]$ = Expected Value of the Process Variance (*EVPV*). This represents the **variability resulting from the loss reporting process**
- $Var_Y(E[X|Y])$ = Variance of the Hypothetical Mean (*VHM*). This represents the **variability resulting from the loss occurrence process**

<u>Development Formula 2</u>

In order to reconfigure Development Formula 1 into a credibility weighting system, we need an additional assumption regarding d. Suppose there is a real number $d \neq 0$ such that $E[X|Y = y] = dy$ for all y. Then, the best linear approximation to Q is as follows:

$$\boxed{L(x) = Z\frac{x}{d} + (1 - Z)E[Y]}$$

where $Z = \frac{VHM}{VHM+EVPV}$.

As we can see from the formula above, L is a credibility weighting of the link ratio estimate $\frac{x}{d}$ and the budgeted loss estimate $E[Y]$:

- If $EVPV = 0$, we give full weight to the link ratio estimate (fixed reporting case)
- If $VHM = 0$, we give full weight to the budgeted loss estimate (fixed prior case)

When **using the least squares version of Development Formula 1**, the credibility weight is as follows:

$$\boxed{Z = bd = \frac{b}{c}}$$

As defined earlier, c must be the **volume-weighted average** for this to work.

Example: Least Squares Method for a Growing Book of Business

An actuary wants to estimate AY 2020 & 2021 ultimate losses using the least squares method. Given the following information for a small book of business as of December 31, 2023:

		Reported Loss ($000) as of XX Months:				
AY	EP ($000)	12 mo.	24 mo.	36 mo.	48 mo.	60 mo.
2017	4,260	102	104	209	650	847
2018	5,563	0	543	1,309	2,443	3,003
2019	7,777	412	2,310	3,083	3,358	4,099
2020	8,871	219	763	1,637	1,423	
2021	10,465	969	4,090	3,801		
2022	11,986	0	3,467			
2023	12,873	932				

- 60-Ult. Tail Factor = 1.10

AY 2020

First, let's divide the reported losses by the earned premium for each year to adjust for the **systematic increase** in the losses:

	Reported Loss Ratio as of XX Months:				
AY	12 mo.	24 mo.	36 mo.	48 mo.	60 mo.
2017	0.024 = $\frac{102}{4,260}$	0.024 = $\frac{104}{4,260}$	0.049	0.153	0.199
2018	0.000 = $\frac{0}{5,563}$	0.098	0.235	0.439	0.540

2019	0.053	0.297	0.396	0.432	0.527
2020	0.025	0.086	0.185	0.160	
2021	0.093	0.391	0.363		
2022	0.000	0.289			
2023	0.072				

By dividing the losses by the earned premium for each year, the AYs are on a more apples-to-apples basis.

Second, let's account for the tail factor:

	Reported Loss Ratio as of XX Months:					
AY	12 mo.	24 mo.	36 mo.	48 mo.	60 mo.	Ult.
2017	0.024	0.024	0.049	0.153	0.199	0.219 = 0.199(1.10)
2018	0.000	0.098	0.235	0.439	0.540	0.594 = 0.540(1.10)
2019	0.053	0.297	0.396	0.432	0.527	0.580 = 0.527(1.10)
2020	0.025	0.086	0.185	0.160		
2021	0.093	0.391	0.363			
2022	0.000	0.289				
2023	0.072					

Third, let's calculate the least squares parameters for AY 2020 to ensure there are no parameter estimation errors:

- Since AY 2020 is 48 months old, the known x-values are (0.153, 0.439, 0.432)
- Since we want to estimate the AY 2020 ultimate loss ratio, the known y-values are (0.219, 0.594, 0.580)
- LINEST((0.219, 0.594, 0.580), (0.153, 0.439, 0.432)) = (1.30145, 0.02007)
- Thus, $b = 1.30145$ and $a = 0.02007$. No parameter estimation errors are present

Fourth, let's estimate the AY 2020 ultimate loss ratio using the least squares method:

- FORECAST(0.160, (0.219, 0.594, 0.580), (0.153, 0.439, 0.432)) = 0.2288

Fifth, let's convert the AY 2020 ultimate loss ratio into an ultimate loss:

- AY 2020 ultimate losses = (AY 2020 ultimate loss ratio)(AY 2020 EP) = (0.2288)(8,871,000) = 2,030,032
- Thus, the AY 2020 ultimate losses are **$2,030,032**

Lastly, let's demonstrate that the least squares method is a **credibility weighting** of the link ratio method and the budgeted loss method:

- $c = \frac{0.219+0.594+0.580}{0.153+0.439+0.432} = 1.3603$. This is the 48-ultimate volume-weighted average. It is also equal to $\frac{\bar{y}}{\bar{x}}$
- $Z = \frac{b}{c} = \frac{1.30145}{1.3603} = 0.9567$
- Link ratio ultimate loss ratio = $1.3603(0.160) = 0.2182$
- Budgeted loss ultimate loss ratio = $\bar{y} = \frac{0.219+0.594+0.580}{3} = 0.464$
- $L(0.160) = 0.9567(0.2182) + (1 - 0.9567)(0.464) = 0.2288$. This is **equal to the ultimate loss ratio** we calculated in step 4 above

AY 2021

To calculate the AY 2021 ultimate loss ratio, we re-run the least squares method with the AY 2020 estimate included in the y-values.

First, let's calculate the least squares parameters for AY 2021 to ensure there are no parameter estimation errors:

- Since AY 2021 is 36 months old, the known x-values are (0.049, 0.235, 0.396, 0.185)
- Since we want to estimate the AY 2021 ultimate loss ratio, the known y-values are (0.219, 0.594, 0.580, **0.229**). Notice that the AY 2020 estimate of 0.229 has been added to the list of y-values
- LINEST((0.219, 0.594, 0.580, 0.229), (0.049, 0.235, 0.396, 0.185)) = (1.16244, 0.15381)

- Thus, $b = 1.16244$ and $a = 0.15381$. No parameter estimation errors are present

Second, let's estimate the AY 2021 ultimate loss ratio using the least squares method:

- FORECAST(0.363, (0.219, 0.594, 0.580, 0.229), (0.049, 0.235, 0.396, 0.185)) = 0.5760

Third, let's convert the AY 2021 ultimate loss ratio into an ultimate loss:

- AY 2021 ultimate losses = (AY 2021 ultimate loss ratio)(AY 2021 EP) = (0.5760)(10,465,000) = 6,028,028
- Thus, the AY 2021 ultimate losses are **$6,028,028**

To calculate subsequent ultimate loss ratios, we keep adding the new estimates to the known y-values and re-running the least squares method.

In the prior example, we were able to adjust for systematic distortions and apply the least squares method. How do we handle situations where we **CANNOT correct** for year-to-year changes in the loss & loss reporting distributions? Let's look at another example.

Example: Law Change

We wish to develop AY 2023 personal auto losses for a state which has just instituted a strict verbal tort threshold. Given the following information as of December 31, 2023:

- Under the old system, expected losses were $20M. Industry estimates show the reform should save 40% in the first year
- Under the old system, about 62% of incurred losses were reported by year-end. Under the new system, this is expected to rise to 75%

Based on the information above, we expect the following under the new system:

- $E[Y] = 20(1 - 0.40) = 12$ (expected ultimate losses are $12M)

- $E\left[\frac{X}{Y}\right] = 0.75$
- $E[X] = E[Y] \cdot E\left[\frac{X}{Y}\right] = 12(0.75) = 9$ (expected losses reported at year-end are $9M).

  Note that Brosius assumes that $E\left[\frac{X}{Y}\right]$ is independent of Y

Suppose the year-end reported loss is only $6M.

Are the savings from the reform greater than expected (in which case we should reduce our estimate of ultimate loss), or are there temporary reporting delays (in which case we should not reduce our estimate of ultimate loss)?

Neither the least squares method nor the link ratio method works here since they rely on past loss experience, which was based on a different tort system. It may not be appropriate to use the budgeted loss method due to uncertainty surrounding the estimated savings.

In this case, it is best to estimate *EVPV* and *VHM* using Bayesian credibility.

First, let's estimate the means and standard deviations of the loss Y and the reporting ratio $\frac{X}{Y}$:

- We already have estimates of the means:
  - $E[Y] = 12$
  - $E\left[\frac{X}{Y}\right] = 0.75$
- Discussions with the underwriting and claims team lead to the following estimates of the standard deviations:
  - $\sigma(Y) = 3$
  - $\sigma\left(\frac{X}{Y}\right) = 0.14$

Second, let's calculate *EVPV* and *VHM*:

- Brosius does not provide the derivations of *EVPV* and *VHM* in the paper. Here are the required formulas:

$$EVPV = Var\left(\frac{X}{Y}\right)(Var(Y) + E[Y]^2)$$

$$VHM = \left(E\left[\frac{X}{Y}\right]\right)^2 (Var(Y))$$

- $EVPV = 0.14^2(3^2 + 12^2) = 3$
- $VHM = 0.75^2(3^2) = 5.06$

Third, let's calculate the credibility weight Z:

- $Z = \frac{VHM}{VHM+EVPV} = \frac{5.06}{5.06+3} = 0.628$

Fourth, let's estimate AY 2023 ultimate losses using Bayesian credibility:

- Recall from Development Formula 2 that $L(x) = Z\frac{x}{d} + (1 - Z)E[Y]$
- $L(6) = 0.628\left(\frac{6}{0.75}\right) + (1 - 0.628)(12) = 9.5$
- Thus, the estimate of the AY 2023 ultimate losses is $9.5M

Lastly, let's compare the estimate to the other methods:

- It is larger than the link ratio estimate of $\frac{6}{0.75} = \$8\text{M}$
- It is smaller than the budgeted loss estimate of $E[Y] = \$12\text{M}$
- It is slightly larger than the BF estimate of $x + E[Y]\left(1 - E\left[\frac{X}{Y}\right]\right) = 6 + 12(1 - 0.75) =$ $9M. This makes sense because $b = \frac{0.628}{0.75} < 1$. Conceptually, this means we placed more confidence in the prior estimate $E[Y]$ than if we had used the BF method

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VII. The Caseload Effect

Development Formula 2 assumes that the expected number of claims reported is proportional to the number of claims incurred. Since a claim is more likely to be reported quickly when the caseload is low, we expect the development ratio $\frac{E[X|Y=y]}{y}$ to be a decreasing function of y, not a constant. Development Formula 3 addresses this.

Development Formula 3

Suppose there are real numbers $d \neq 0$ and x_0 such that $E[X|Y=y] = dy + x_0$ for all y. Then, we have the following:

$$\boxed{L(x) = Z\left(\frac{x - x_0}{d}\right) + (1 - Z)E[Y]}$$

where $Z = \frac{VHM}{VHM+EVPV}$.

Regarding the formula above:

- It gives a development ratio of $d + \frac{x_0}{y}$, which decreases as y increases
- It gives $E[X|Y=0] = x_0 > 0$, which doesn't make much sense
- When $x_0 = 0$, we obtain Development Formula 2 as a special case

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Example: Caseload Effect

Given the following information for an accident year:

- Expected losses are $15M
- When the caseload is low ($y = \$5M$), 60% of the claims are expected to be reported by year-end
- When the caseload is high ($y = \$20M$), 37.5% of the claims are expected to be reported by year-end
- The actual amounts of losses reported by year-end are $7.5M
- $Z = 0.25$

Since the amount of losses expected to be reported by year-end depends on y, we should use Development Formula 3.

First, let's set up a system of equations to solve for d and x_0:

$$d + \frac{x_0}{5{,}000{,}000} = 0.600 \longrightarrow 5{,}000{,}000d + x_0 = 3{,}000{,}000$$

$$d + \frac{x_0}{20{,}000{,}000} = 0.375 \longrightarrow 20{,}000{,}000d + x_0 = 7{,}500{,}000$$

This yields $d = 0.3$ and $x_0 = 1{,}500{,}000$.

Second, let's estimate the ultimate losses for the AY:

- $L(7{,}500{,}000) = 0.25\left(\frac{7{,}500{,}000-1{,}500{,}000}{0.3}\right) + (1 - 0.25)(15{,}000{,}000) = 16{,}250{,}000$
- Thus, the estimated ultimate losses are \$16.25M

VIII. Conclusion

Conclusions regarding the least squares method are as follows:

- It is easy to implement and uses easily accessible data
- It works well for developing losses for small states or lines that are subject to serious fluctuations
- It can lead you astray if corrections are not made to account for significant exposure changes or other shifts in loss history
- It is subject to sampling error since parameters are estimated from observed data