Three Types of Insurance Demand

1. **Risk transfer** – the insured purchases insurance to shift their risk to insurers. They do this by pooling their risks with other insureds

2. **Satisfying demand** – the insured purchases insurance to satisfy statutory, regulatory, or contractual requirements

3. **Risk financing** – the insured purchases insurance to finance uncertain future contingencies in an efficient manner

Identify five forms of insurance services an insurer supplies.

1. Sales
2. Marketing
3. Risk surveys
4. Loss control
5. Risk bearing

Briefly describe two critical functions when managing a risk pool.

1. Controlling access to the pool through UW and pricing

2. Ensuring the pool is solvent by funding risk-bearing assets through the sales of liabilities

Briefly describe three ways in insurers bundle UW, pricing, risk-bearing, claims handling, and customer service when managing a risk pool.

1. Some companies offer all of these services in-house

2. Customer-facing functions are included with a direct writer, but farmed out to independent agents or brokers for other types of writers

3. Managing general agents (MGAs) might only provide UW and pricing services

Parametric Insurance Contracts

- Policies pay based on an objective event outcome, such as hurricane intensity and landfall

- These policies are easy to underwrite since they do not depend on the characteristics of the insured

- These policies are difficult to design in such a way that they eliminate the potential for an insured to profit from a claim

- Under these policies, the insured is exposed to **basis risk**, which is the mismatch between the insured's subject loss and the insurance recovery

Dual-Trigger Insurance Contracts

- Policies pay based on an objective event outcome AND the event causes an economic loss to the insured
- The indemnity payment is a function of the underlying subject loss suffered by the insured
- These policies apply an indemnity function $f()$ to the subject loss L that combines limits, deductibles, etc

Explain why franchise deductibles fail to meet the indemnity function requirements underlying a dual-trigger contract.

The indemnity function $f()$ has the following conditions:

- $0 \leq f(L) \leq L$
- $f(L)$ is a monotonic function of L
- $L - f(L)$ is a monotonic function of L
- f is continuous

Under a franchise deductible d, if $L < d$, then $f(L) = 0$. But, if $L > d$, then $f(L) = L$ instead of $L - d$. Thus, f is not continuous (jump at d) and $L - f(L)$ is not monotone (increases until it hits d, then drops to zero).

Explain why excess of loss covers meet the indemnity function requirements underlying a dual-trigger contract.

The excess of loss indemnity function is $f(L) = (L - a)^+$ for some threshold a.

Clearly, $0 \leq f(L) \leq L$. Also, f is continuous and monotonic since it is zero before a, and then increases from zero after a. Finally, $L - f(L)$ is monotone since it increases to a before a and says there after a.

Describe the general payment order upon liquidation of an insurer.

From highest (i.e., senior) to lowest priority:

1. Receiver expenses
2. Insurance claims from policyholders
3. Debtholders
4. Shareholders

Capital vs. Equity

Capital (i.e., policyholder surplus) equals assets net of liabilities owed to policyholders.

Equity equals assets net of liabilities owed to all parties EXCEPT owners. Equity is also defined as the owner's residual value.

Four Types of Insurer Capital

1. **Common equity** – comprised of capital stock (par value of shares issued), additional paid-in capital (excess of amounts paid-in over par value), and retained earnings. Note that capital stock and additional paid-in capital only apply to stock companies

2. **Reinsurance capital** – a reinsurance transaction can be thought of as the sale of the residual value for a specific portion of the business

3. **Debt capital** – when an insurer cannot issue shares (ex. mutual), it can use surplus notes to raise capital

4. **Preferred equity** – blends characteristics of debt and equity

Six Reasons Why Equity Capital is Expensive

1. Principal-agent problem – investors and management do not have perfectly aligned incentives

2. No independent validation of insurance pricing

3. Equity requires a long-term commitment to a cyclical business

4. Returns are left-skewed (investors prefer right-skewed returns)

5. Regulatory minimum capital standards can force an insurer into supervision before it is technically solvent (could lead to dividend restrictions)

6. Double taxation – insurers' corporate profits and dividend distributions are taxed (essentially taxes investors twice)

Four Ways in Which CAT Bonds Address the High Cost of Equity Capital

1. CAT bonds are not equity and do not have market risk

2. CAT bonds are a diversifying, zero-beta (i.e., independent of the financial market) asset class

3. Catastrophe pricing can be validated

4. There are no principal-agent problems since CAT bond cash flows are contractually defined

Provide a major disadvantage of CAT bonds.

CAT bonds are illiquid and trade in a thin market.

Weighted Average Cost of Capital (WACC)

The WACC combines the cost of all forms of capital into a single figure. It is the weighted average of the various forms of capital.

Briefly describe two theories of capital structures.

1. **Trade-off theory** – the debt and equity mix trades off the costs and benefits of each capital type. It compares the expense of equity and the right of debt holders to force bankruptcy

2. **Pecking order theory** – informational asymmetries between management and owners makes equity more expensive, in which case retained earnings are the favorable form of financing

Regulatory Regimes Around the World

1. **Solvency II –** used in the EU; assets are based on market value and liabilities consist of a best estimate plus a risk margin

2. **NAIC SAP –** used in the US; focuses on the balance sheet; assets that cannot be readily converted to cash are considered non-admitted on the balance sheet

3. **GAAP –** used in multiple jurisdictions; loss reserves are booked at the undiscounted best estimate; allows deferral of acquisition expenses

4. **IFRS –** focuses on market value; loss reserves are valued on a discounted basis and include a risk adjustment

5. **Rating agency evaluations –** rating agencies typically use adjusted statutory or GAAP financials in their internal capital models

Top-Down Pricing

This textbook follows **top-down pricing**. This means that it begins by pricing an insurer's total portfolio such that aggregate premium satisfies a **stability criterion**. A stability criterion essentially establishes a constraint.

For example, we may want to price the total portfolio such that the probability of ruin is at most 5%. A bottom-up pricing approach would focus on pricing individual risks rather than the portfolio in total.

Formula for the Total Return on Equity (*TR*) as a Function of UW and Investment Income Returns

$$TR = \frac{Income}{Equity}$$
$$= \frac{U + I}{Q}$$
$$= \frac{U}{P}\left(\frac{P}{Q}\right) + \left(\frac{I}{a}\right)\left(\frac{a}{Q}\right)$$

Where $U =$ underwriting income, $P =$ premium, $I =$ investment income, $Q =$ equity, and $a =$ assets.

The final line above can be written as (Underwriting margin) times (Underwriting leverage) + (Investment return) times (Investment leverage).

Formula for the Total Return on Equity (*TR*) as a Function of Policyholder-Funded Asset Leverage

$$TR = \frac{Income}{Equity}$$

$$= \frac{U + I}{Q}$$

$$= \frac{I}{a} + \frac{R}{Q}\left(\frac{I}{a} + \frac{U}{R}\right)$$

Where $U =$ underwriting income, $R =$ policyholder funds (loss reserves and portion of UEPR), $I =$ investment income, $Q =$ equity, and $a =$ assets.

The final line above shows that the policyholder-fund of $\frac{R}{Q}$ magnifies the net cost of policyholder funds of $\left(\frac{I}{a} + \frac{U}{R}\right)$.

Timing Risk and Amount Risk

Insurance loss payments have both timing risk and amount risk, which may be dependent risks since larger claims often take longer to settle.

Timing risk refers to uncertainty in how and when a claim will be paid out.

Amount risk refers to uncertainty in the amount of the claim.

Assumptions Underlying CAPM and Black-Scholes Financial Models

- **Competitive –** there are many small sellers and buyers and undifferentiated products

- **Perfect –** there are no information or transaction costs, and no bid-ask spread. In addition, everyone can borrow or lend at the same risk-free rate, there are no restrictions on short sales, and there are no taxes

- **Complete –** there are enough securities to replicate any set of future period cash flows by securities trading

- **Arbitrage-free –** there is no potential for a risk-free gain on an initial investment of zero

- **General Equilibrium –** supply equals demand, no trader has an incentive to trade to improve their position, and everyone agrees all products are fairly priced. There is no arbitrage in when prices are in general equilibrium

Myers-Cohn Fair Premium Condition

A premium is fair if "whenever a policy is issued, the resulting equity value equals the equity invested in support of that policy."

If premium increases equity, it is unfair to the policyholder because it transfers wealth from them to investors.

If premium decreases equity, it is unfair to investors because it transfers wealth from them to the policyholder.

Fair rates mean that prices are in equilibrium. Otherwise, investors would have an incentive to write more or less insurance.

Formula for the DCF Premium P

$$P = \frac{L}{1+R_L} + \frac{\tau R_f (P+Q)}{1+R_f} + \frac{\tau P}{1+R_f} - \frac{\tau L}{1+R_L}$$

Where $L =$ expected losses, $P =$ premium, $Q =$ capital invested to support the policy, $R_L =$ risk-adjusted discount rate for losses, $R_f =$ risk-free rate, and $\tau =$ tax rate.

Formula for the DCF Year-End Value V_1

$$V_1 = P\left(1 + R_f\right) + Q\left(1 + R_f\right) - L - \tau(P - L) - \tau R_f(P + Q)$$

Where $L =$ expected losses, $P =$ premium, $Q =$ capital invested to support the policy, $R_L =$ risk-adjusted discount rate for losses, $R_f =$ risk-free rate, and $\tau =$ tax rate.

Formula for the DCF Return on Equity R

$$R = \frac{V_1 - Q}{Q}$$

$$= R_f + \frac{L}{Q}\left(\frac{R_f - R_L}{1 + R_L}\right)(1 - \tau)$$

Where $L =$ expected losses, $V_1 =$ year-end firm value, $Q =$ capital invested to support the policy, $R_L =$ risk-adjusted discount rate for losses, $R_f =$ risk-free rate, and $\tau =$ tax rate.

Discuss an argument for using the risk-free rate for investment income in the DCF model.

Discuss an argument for using the anticipated market return for investment income in the DCF model.

Proponents of the risk-free rate argue that premium profit margins should not depend on actual investment portfolios, but only on the risk-free rate and systematic underwriting risk. Since policyholders do not share in the investment risks of the insurer, they should pay the same premium regardless of the firm's investment strategy.

Proponents of the anticipated market return argue that adding investment risk increases the probability of default for policyholders and changes their expected loss recoveries. Hence, policyholder premiums should reflect the anticipated market return.

Describe the IRR model.

Provide one advantage and one disadvantage of the IRR model.

The IRR model is from the **investor's viewpoint**. It estimates all cash flows to or from the investor (called "equity flows") and then computes the discount rate (i.e., the IRR) required to produce a present value of zero. If the IRR $\geq$ the investor's hurdle rate, then the project should be undertaken.

An **advantage** of the IRR model is that it doesn't require us to determine discount rates for each cash flow.

A **disadvantage** of the IRR model is that it requires a hurdle rate (i.e., required rate of return) to make the final decision.

Describe the DCF Model.

The DCF model discounts each cash flow in a project at an appropriate risk-adjusted interest rate. Some cash flows are positive (ex. recurring revenue) while others are negative (ex. initial investment). If the total DCF is positive, then the project should be undertaken.

Formula for the IRR Model Premium for a One Period Insurer

$$P = \frac{L}{1 + R_f} + \phi D \left[\frac{\tau R_f + \left(R_S - R_f \right)}{\left(1 + R_f \right)(1 - \tau)} \right]$$

Where ϕ is the constant capital-to-reserves ratio and D is the market value of the liabilities.

Cummins Conclusions on the DCF and IRR Models

1. Ratemaking is prospective, and prospective business should stand alone from legacy business and reserves

2. Investment returns should estimate yields expected during the policy term. In other words, we should use the anticipated return for the firm rather than the risk-free rate

3. Regulation and accounting are relevant only if they affect cash flows

4. It is difficult to determine the appropriate cash flows associated with writing a new policy

5. Both models depend on a liability discount rate, or equivalently, a cost of capital

Formula for the Portfolio CCoC Premium

$$\begin{aligned}\bar{P}_a(X) &= \frac{E[X \wedge a(X)] + \iota a(X)}{1+\iota} \\ &= \nu E[X \wedge a(X)] + \delta a(X)\end{aligned}$$

Where $a(X)$ is a capital risk measure, ι is a constant, $\nu = \frac{1}{1+\iota}$, and $\delta = \frac{\iota}{1+\iota}$. Since policyholder deficits are normally small, it's common to ignore the default adjustment in practice. In this case, we replace $E[X \wedge a(X)]$ with $E[X]$.

Note that Portfolio CCoC pricing assumes:

- Premium = Loss Cost + Cost of Capital
- Cost of Capital = Target Return on Capital × Amount of Capital.

Discuss transaction costs in terms of an insurance market imperfection.

In a perfect market, insureds would purchase insurance directly from investors. However, this doesn't happen due to the existence of underwriting expenses. Underwriting expenses are a form of transaction costs.

In the event that an insured tried to purchase insurance directly from an investor, it's unlikely that the investor would have the underwriting expertise to properly evaluate the insured. As a result, insurance intermediaries and/or insurers are used to perform this task (which obviously has a cost associated with it).

Frictional costs of capital are another insurance market imperfection. Briefly describe three types of frictional costs of capital.

1. **Agency and informational** costs – agency costs refer to the principal-agent problem in which manager incentives do not align with owners. Informational costs refer to an insurer's failure to control adverse selection and moral hazards of insureds due to information asymmetry

2. **Double taxation** – insurers' corporate profits and dividends are taxed

3. **Regulation** – regulation can allow insurer assets to be seized or temporarily controlled by a regulator if the insurer fails minimum capital standards. Regulation can also restrict investment opportunities and payments to investors

Formula for the Portfolio CCoC Premium (including Frictional Costs)

$$\bar{P}_a^{\text{Frictional}(X)} = \frac{E[X \wedge a(X)] + \iota^* a(X)}{1 + \iota^*}$$

Where $(1 + \iota^*) = (1 + \iota)(1 + \tau)$ and τ is a constant frictional cost per period.

Discuss bid-ask spreads in terms of insurance market imperfections.

In addition to transaction costs and frictional costs of capital, there are market frictions such as bid-ask spreads. These spreads are driven by information asymmetry.

For example, underwriters may price in positive loadings even when there is no risk due to uncertainty around adverse selection.