Exam 9 High-Level Summaries

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CAS Exam 9 Online Course

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Clark

Basics of Reinsurance Pricing

Proportional Treaties

Quota Share

- Reinsurer receives flat percentage of premium and assumes flat percentage of loss
- Reinsurer pays cedant a ceding commission to reflect U/W expense differences

Surplus Share

- Reinsurer assumes risk in proportion to the amount that insured value (IV) exceeds retained line, given up to a limit (a multiple of retained line)

Steps to price proportional treaties:

1. Compile historical experience on treaty
2. Exclude catastrophe and shock losses
3. Adjust experience to ultimate level and project to future period (onlevel, develop, trend)
4. Select non-CAT ratio for treaty
5. Load selection for catastrophes with the following options:
    - Average CAT loads based on cedant's projected premium distribution by state
    - Estimate average number of times an occurrence limit might be exhausted in a year (if applicable)
    - Spread historical CAT losses over longer period of time
    - Use a CAT simulation model
6. Estimate combined ratio given ceding commission and other expenses

Special features of proportional treaties

Sliding scale
A percent of premium paid that slides with the actual loss experience, subject to max/min constraints. Since pricing requires an expectation of loss experience, we can use loss ratio probability distributions to estimate the commission.

The **technical ratio** = expected loss ratio + expected commission. Sliding scales can also include **carryforward provisions** that allow the excess loss amount to be included with current year's losses in the estimate of the current year's commission, in order to smooth results over time.

Profit commissions are calculated after the actual loss ratio is known. It takes the reinsurer profit and returns a specified percentage to the cedant.

Loss corridors return assumption of a portion of losses back to the cedant.

Property Per-Risk Excess Treaties

Property per-risk excess treaties provide a limit of coverage in excess of the cedant's retention on a per-risk basis.

Experience rating pricing is done as follows:

1. Collect subject premium and historical losses for several years
2. Adjust subject premium to future level (on-leveling, inflation)
3. Trend large historical losses to determine amount included in layer (if ALAE applies pro-rata with losses, should be added for each loss)
4. Develop excess losses for each historical period
5. Divide trended, developed layered losses by adjusted subject premium to calculate loss costs

Exposure rating models the current risk profile rather than using historical experience. Its steps are:

1. Determine **limits profile**, the distribution of premium by different ranges of insured values
2. Select an exposure curve P which represents amount of loss capped at a given percent p of IV, relative to the total value of the loss
3. Calculate the **exposure factor**

$$\text{Exposure Factor} = \text{P}\left(\frac{\text{Retention} + \text{Limit}}{\text{IV}}\right) - \text{P}\left(\frac{\text{Retention}}{\text{IV}}\right)$$

4. Multiply exposure factor in the treaty layer for each insured value range by the expected loss for the range to get expected treaty losses

Once expected loss costs are estimated using experience and exposure rating models, actuary must reconcile results and select a final expected loss cost

Three issues before completing reconciliation process:

Free cover – when using experience rating it is possible that no losses "trend into" the higher portion of the layer being priced, which would imply no cost for the coverage.

This can be addressed by using experience rating for lowest portion of layer and relativities in exposure rating for the higher portion of the layer.

Credibility

- Expected number of claims (or expected loss dollars based on exposure rating if not available)
- Year-to-year variation in the projected loss cost from each of the historical periods.

Inuring reinsurance means a reinsurance policy applies before another reinsurance policy. The policy applied first **inures** to the benefit of the policy applied second.

- If using experience rating, restate historical experience to be net of inuring reinsurance
- If using exposure rating, apply relevant steps to a risk profile adjusted to reflect inuring reinsurance

Casualty Per-Occurrence Excess Treaties

Three categories

1. Working layer – low attachment point, often retained by cedant
2. Exposed excess – excess layer with attachment point below some policy limits
3. Clash cover – high attachment point above any single policy limit; penetrated due to losses on multiple policies from a single occurrence

Experience rating these policies use the same steps as property per-risk excess treaties, plus:

- ALAE must be captured separately from losses
- For general and auto liability losses, underlying policy limit should be captured
- Workers' comp losses should be captured on undiscounted basis
- Inflation factors should vary by line and be derived from unlimited large losses
- Trended losses must be capped at applicable policy limits
- Determine portion of loss and ALAE in treaty layer:
    - Pro-rata with loss: ALAE in layer is estimated in proportion to losses
    - Included with loss: ALAE is added to loss and treaty limit applied to sum

Excess loss development factors for the industry are available from the Reinsurance Association of America (RAA).

Exposure rating casualty per-occurrence excess treaties involves **increased limits factors (ILFs)** for general liability and auto liability.

ILF from limit L to limit U:

$$ILF_{L,U} = \frac{E(x;U)}{E(x;L)}$$

ELF for a limit L:

$$ELF_L = \frac{E(x) - E(x;U)}{E(x)}$$

Exposure factor **for general liability and auto liability**:

$$\text{Exposure Factor} = \frac{E(x;\min(PL, AP + Lim)) - E(x;\min(PL, AP))}{E(x;PL)}$$
$$= \frac{ILF(\min(PL, AP + Lim)) - ILF(\min(PL, AP))}{ILF(PL)}$$

where PL = cedant policy limit, AP = treaty attachment point, Lim = treaty limit.

If treaty includes ALAE in proportion to loss, apply the exposure factor to the subject premium times expected loss and ALAE ratio.

If ALAE is included with loss, we must use a **modified exposure factor**:

$$\text{Exposure Factor} = \frac{E(x;\min(PL,\frac{AP+Lim}{1+e})) - E(x;\min(PL,\frac{AP}{1+e}))}{E(x;PL)}$$

where e = ALAE as a percent of loss (capped at policy limit).

Exposure factor for **workers' compensation:**

$$\text{Exposure Factor} = ELF_{AP} - ELF_{AP+Lim}$$

If an umbrella policy applies on top of the primary policy written by the cedant, we have another modification to the exposure factor:

$$\text{Exposure Factor} = \frac{E(x;\min(UL+PL, UL+AP+Lim)) - E(x;\min(UL+PL, UL+AP))}{E(x;UL+PL) - E(x;UL)}$$

where UL is the limit of the underlying policy and PL is the umbrella policy limit.

If a **drop-down provision** is present:

$$\text{Exposure Factor} = \frac{[E(x;UL+AP+Lim) - E(x;UL+AP)](1-\phi) + [E(x;AP+Lim) - E(x;AP)]\phi}{[E(x;UL+PL) - E(x;UL)](1-\phi) + [E(x;PL)]\phi}$$

An **aggregate annual deductible (AAD)** allows the cedant to retain losses in a layer up until the aggregate losses reach the AAD.

A **swing plan** loads expenses to the actual layer losses and the result is charged back to the cedant, subject to maximum and minimum constraints.

Aggregate Distribution Models

Empirical distributions use historical data

- **Advantage:** easy to calculate
- **Disadvantages:** actual results may differ greatly from history, may not properly reflect changes in mix/growth, may underrepresent true future volatility

Single distribution model assumes aggregate treaty losses follow a known distribution

- **Advantage:** simple to use
- **Disadvantages:** does not allow for loss free scenario, no easy way to reflect impact of changing per occurrence limits on the aggregate losses

Recursive calculation works well for low frequency scenarios (assuming Poisson frequency):

$$A_k = \sum_{i=1}^{k} \left(\frac{\lambda}{k}\right) i S_i A_{k-i}$$

- **Advantages:** simple to work with, provides accurate handling of low frequency scenarios
- **Disadvantages:** more intensive calcs for higher frequencies, can only use a single severity distribution

Collective risk models are more useful for complicated severity distributions. Things to consider:

- Can feel like a "black box"
- Assumes independence between occurrences; frequency and severity distributions are independent
- Could have large error for low frequency scenarios
- Reflects process variance but not full parameter variance

Property Catastrophe Covers

These provide protection for catastrophic events on an occurrence basis. Typically, reinsurance inures to the benefit of the cover. A **reinstatement** can occur when a layer is exhausted, replenishing coverage, in one of two ways:

- **Pro-rata as to amount** (annual premium * pro-rata % * % of layer to be replenished)
- **Pro-rata as to time** (annual premium * pro-rata % * % of policy remaining)

Methods for pricing catastrophe covers include:

- Payback method
    - Estimates years of premium needed to cover a single loss (**payback period**)
    - **Rate on line (ROL)** = 1 / payback period
- Experience rating
    - Not very credible for a single treaty
- Catastrophe models
    - Simulates large number of events and calculates impact to insurer
    - Requires a lot of information (exposure, terms, geography, inuring reinsurance)
    - Though a model, unable to capture every impact from a catastrophe

Covers can be **losses occurring** or **risks attaching**. The latter makes it possible for a reinsurer to pay twice on the same loss event.

Finite risk refers to property catastrophe covers for which the maximum loss amount is reduced relative to traditional covers. They are multiple year risks with loss sensitive features, reducing downside risk.

A contract is **considered reinsurance** if and only if:

1. Reinsurer assumes significant insurance risk
2. It is reasonably possible the reinsurer may realize a significant loss.

Calculating the Final Price

The **final price** is calculated as follows with the standard pricing formula:

$$Premium = \frac{Loss\ Cost \times (1 + ULAE\%) + Fixed\ Expense}{1 - Variable\ Expense\%}$$

where $Loss\ Cost = Selected\ Loss\ Cost\ \% \times Subject\ Premium$

The formula above does not consider timing of cash flows, risk loads, adjustable features, or profit load provisions. We should consider **target ROE** (and, by extension, allocated surplus).

Bernegger

Swiss Re Exposure Curves and the MBBEFD Distribution Class

Distribution Function and Exposure Curve

The percentage of pure risk premium retained by a cedant is G(d):

$$G(d) = \int_0^d \frac{(1 - F(y))dy}{E(x)}$$

The CDF of x:

$$F(x) = 1 - \frac{G'(x)}{G'(0)}, \text{where } 0 \leq x < 1$$

Where:

- D = cedant's maximum retention under a non-proportional reinsurance treaty
- M = MPL
- X = gross loss, less than or equal to M
- d is the **normalized deductible**, the ratio of cedant's retention to the MPL
- x = X/M, or **normalized loss** – the ratio of gross loss to the MPL
- L(d) = limited expected value of the ratio x
- F(1) = 1

The **probability of a total loss**:

$$p = 1 - F(1^-) = \frac{G'(1)}{G'(0)}$$

If no MPL, divide X or D by M. The resulting p is the probability of a loss greater than the reference loss.

MBBEFD Class of Two-Parameter Exposure Curves

General MBBEFD exposure curve:

$$G(x) = \frac{\ln(a + b^x) - \ln(a + 1)}{\ln(a + b) - \ln(a + 1)}$$

$$F(x) = 1 - \frac{(a + 1)b^x}{a + b^x}$$

Replacing *a* with (g – 1) * b / (1 – gb) results in the re-parameterized exposure curve $G_{b,g}(x)$.
The expected value μ = 1 / G'(0).

Curve Fitting

To fit a MBBEFD curve:

1. Let g = 1 / p
2. Calculate $\mu = 1 / G'(0)$
3. Determine b using the following:
    a. Special cases
        i. If $\mu = 1$, b = 0
        ii. If $\mu = \frac{g-1}{ln(g)g}$, b = 1 / g
        iii. If $\mu = \frac{ln(g)}{g-1}$, b = 1
        iv. If $\mu = \frac{1}{g}$, b = ∞
    b. General case
        i. Must determine b iteratively using $\mu = \frac{ln(gb)(1-b)}{ln(b)(1-gb)}$

The **Swiss Re Y_i exposure curves** (i = 1 to 4) are special curves used by non-proportional property underwriters.

$$G_c(x) = G_{b_c,g_c}(x)$$

$$b_c = e^{3.1-0.15c(1+c)} \qquad g_c = e^{(0.78+0.12c)c}$$

- c = 0 represents distribution of total losses only ($g_c = 1$)
- c = {1.5, 2.0, 3.0, 4.0} coincide with Swiss Re Curves {Y_1, Y_2, Y_3, Y_4}
- c = 5.0 coincides with Lloyd's curve for rating industrial risks

Grossi

Catastrophe Modeling: A New Approach to Managing Risk

Structure of Catastrophe Models

Four basic components (or modules) of a CAT model:

1. **Hazard** – description of catastrophe
2. **Inventory** – describes portfolio at risk
3. **Vulnerability** – determines severity of impact on property
4. **Loss** – determines direct and indirect losses on exposed properties

Use of a Catastrophe Model for Risk Management

Ways CAT models are used:

- Insurers – understand need for reinsurance protection
- Reinsurers – pricing CAT covers
- Capital markets – pricing CAT bonds
- Emergency management agencies – focus attention on concentration of loss

The **exceedance probability** curve is a graphical representation of the probability a certain level of loss will be surpassed in a given period. It helps insurers and reinsurers determine

- size/distribution of potential losses
- risk appetite and prices
- proportion of risk needing to be transferred to reinsurer or capital markets

Three types of exceedance probabilities:

- **Occurrence** (OEP) – probability at least one loss exceeds a given amount
- **Aggregate** (AEP) – probability the sum of all losses exceeds a given amount
- **Conditional** (CEP) – probability the amount of a single event exceeds a specific amount

OEP curve plots losses on x-axis and OEPs on y-axis.

Derivation and Use of OEP Curve

The events are assumed to be independent Bernoulli random variables.

- Expected loss for event i: $E(L) = p_i L_i$
- **Average annual loss (AAL)** is the sum of expected losses for each event

- OEP for a given level of loss:

$$OEP(L_i) = 1 - \prod_{j=1}^{i-1}(1 - p_j)$$

Insurability of CAT risks

Two conditions to be considered insurable:

1. Ability to identify and quantify frequency/severity
2. Ability to set premiums for each potential customer/class of customers

Factors that influence the rate charged by an insurer:

- Uncertainty of losses
- Supply shortages
- Highly correlated losses
- Adverse selection
- Moral hazard

Insurers try to maximize profit subject to a **survival constraint** (expected probability of insolvency). For n policies charging z dollars of premium, with surplus A:

$$Pr\left(Total\ Loss > (n \times z + A)\right) < p_1$$

Hazard Module

Three elements the hazard module addresses of future events (differing by hazard types):

1. Most likely locations
2. Frequency
3. Severity

Vulnerability Module

This module estimates expected damage for different levels of severity. Two major steps in applying **vulnerability analysis**:

1. Identify and define typical buildings. We can break into classes by most important factors affecting structure
2. Calculate building performance to different intensities (motion, wind). A **damage function** is used to relate damage to event intensity. The uncertainty in the intensity of a particular hazard is given by the **intensity distribution**.

Loss Module

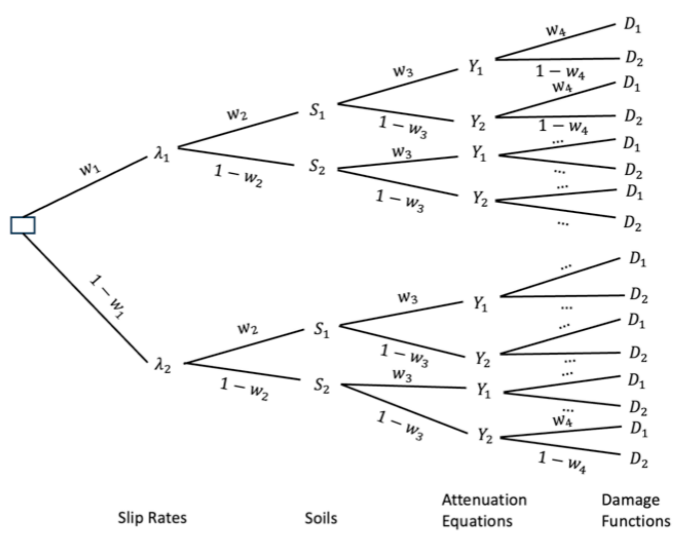
This module uses cost models to estimate damage into monetary loss. Insurers then apply policy terms/conditions to determine insured losses.

Classifications/Sources of Uncertainty

- **Aleatory** – uncertainty due to inherent randomness of natural hazard events (cannot be reduced by more data)
    - Ex: frequency of a hazard occurrence, fragility of a building
- **Epistemic** – uncertainty due to lack of information/knowledge
    - Ex: lack of historical data describing hazard occurrence, lack of accurate data on true market values

Representing and Quantifying Uncertainty

Logic tree approach – Requires simplifying assumptions in which each parameter only has two values:



Simulation methods - Model more complex processes by assuming a distribution for each uncertain parameter. Sample from each parameter distribution and simulate event thousands of times (Monte Carlo).

Actuarial Principles

- A rate is the estimate of the expected value of future costs
- A rate provides for all costs associated with the transfer of risk
- A rate provides for all costs associated with an individual risk transfer
- A rate is reasonable and not excessive, inadequate or unfairly discriminatory if it is an actuarially sound estimate of the expected value of all future costs associated with an individual risk transfer

Use of Catastrophe Models in Ratemaking

Premium charged for policyholders = AAL + Risk Load + Expense Load

- Risk Load reflects insurer's concern with survival constraint

$$\sigma = \sqrt{\sum_i (L_i^2 p_i) - AAL^2}$$

- Expense load includes administrative costs (LAE, taxes, etc)

Two most critical factors for differentiating risks:

- Structure attributes (ex: construction materials, building codes, building occupancy)
- Location attributes (ex: flood plains, proximity to fault lines or coastline, soil conditions)

Regulation and Catastrophe Modeling

CAT models are troublesome for regulators: they provide scientific approach for quantifying the insurer's risk, but could be viewed as a means to justify higher rates. They are difficult to assess due to the specialized expertise required.

Open Issues for Using Catastrophe Models to Determine Rates

Regulatory acceptance – models are difficult to assess by regulators
Public acceptance – low due to resulting rate increases
Actuarial acceptance – must have a basic understanding of underlying components
Model-to-Model variance – different models can produce vastly different loss estimates

Portfolio Composition and Catastrophe Modeling

Residential vs. commercial policy:

- Residential is usually a single policy for single building
- Commercial is usually a single policy covering facilities in many locations

Three factors in an underwriter's decision to write a new account:

1. Magnitude
2. Correlation with existing portfolio
3. Highest price the client is willing to pay

Most robust way to quantify portfolio risk is **bottom-up**:

- Simulate possible events
- Use a CAT model for each event to calculate ground-up loss
- Allocate each loss to insured, insurer, reinsurer based on policies
- Sum over them to produce OEP curves for each party at the portfolio level

- Calculate important metrics such as AAL for each party at the portfolio level

Loss diagrams can be created to better visualize coverage/retention from various policies.

Special Issues Regarding Portfolio Risk

Three issues to take into account:

1. Data quality
2. Uncertainty modeling
3. Impact of correlation

Mildenhall & Major Ch. 8

Classical Portfolio Pricing Theory

Insurance Demand, Supply, and Contracts

Three types of demand for insurance services:

1. Risk transfer
2. Satisfying demand
3. Risk financing

Supply of insurance services:

- Sales
- Marketing
- Risk surveys
- Loss control
- Risk bearing
- Pricing
- Underwriting
- Customer billing/support
- Investment management services

Two critical functions of managing a **risk pool**:

1. Controlling access to the pool through underwriting and pricing
2. Ensuring the pool is solvent by funding risk-bearing assets through the sale of liabilities

Insurance contracts must be written so that claims are clear and objective, easy to adjust, and discourage fraud. It should pay no more than the subject loss.

Parametric insurance policies pay based on an explicit event outcome. **Dual-trigger** basis for a payout is that an objective event occurs and it causes an economic loss to the insured. The goal is to minimize **basis risk**, a mismatch between subject loss and insurance recovery.

Franchise (disappearing) deductibles:

$$f(L) = 0 \text{ when } L < d$$

$$f(L) = L \text{ when } L > d$$

Insurer Risk Capital

When an insurer fails, it is put into liquidation. Its assets are used to pay out all stakeholders in priority order. **Senior priority** goes to receiver expenses, then policyholder claims, then debtholders, then shareholders.

Capital (policyholder surplus) equals assets net of liabilities owed to policyholders. **Equity** is the value of the owner's residual interest in the firm (assets net of liabilities owed to all parties except owners).

Types of insurer capital:

- Common equity – lowest priority ownership interest
- Reinsurance capital
- Debt capital – **surplus notes**

Why insurer equity capital is expensive:

- Principal-agent problem
- Insurance is not expertizable
- Requires a long-term commitment to a cyclical business
- Returns are left-skewed
- Regulators can force insurers into supervision before it is technically insolvent
- Investor returns may be subject to double taxation

CAT bonds offer lower cost of insurance capital because:

- Not equity and no market risk
- Diversifying, independent of financial market
- Pricing is expertizable
- Limited adverse selection
- No principal-agent problems
- No taxes
- Light regulation
- Defined 3-5 year terms

The **weighted average cost of capital (WACC)** is determined by combining the cost of all forms of capital. The target ROE capital drives the WACC since it is usually the most significant proportion of capital.

Two important theories of capital structure:

1. **Trade-off theory** – debt and equity mix trades the costs and benefits of each
2. **Pecking order theory** – informational asymmetries between management and owners makes equity more expensive

Accounting Valuation Standards

Three most important valuation standards:

- Statutory standards (US NAIC, EU Solvency II, APRA)
- Financial standards (GAAP, IFRS)
- Rating agency standards (S&P, AM Best)

Adjustments for accounting must not overlook two important considerations:

1. Accounting conventions have real world consequences
2. Models cannot assume infinite financing

Solvency II uses assets minus liabilities to produce own funds, consisting of

1. Minimum capital requirement (MCR)
2. Solvency capital requirement (SCR)
3. Free assets

NAIC Statutory Accounting (SAP) is a liquidation basis accounting. **Non-admitted assets** consist of illiquid assets or those difficult to convert to cash, and are not counted. Policyholder expenses are earned when written

Generally Accepted Accounting Principles (GAAP) follows SAP treatment of loss reserves, but allows for deferral of acquisition expenses. Reinsurance recoverables are considered an asset.

International Financial Reporting Standards (IFRS) is more market value oriented than GAAP. Loss reserves are discounted including a risk adjustment.

Rating agencies adjust SAP or GAAP financials by removing goodwill and intangible assets and allowing for discounting of loss reserves.

Actuarial PCPs and Classical Risk Theory

A **classical** PCP is a functional that determines premium from a loss distribution. Positive risk loads are included.

Investment Income in Pricing

Total return on equity (TR) from Ferrari:

$$TR = \frac{U}{P} \times \frac{P}{Q} + \frac{I}{a} \times \frac{a}{Q}$$

$$TR = \frac{I}{a} + \frac{R}{Q} \times \left(\frac{I}{a} + \frac{U}{R}\right)$$

- U = underwriting income
- P = Premium
- Q = Equity
- I = Investment income
- a = assets
-

Leverage (R/Q) magnifies the net cost of policyholder provided funds. It is beneficial when:
(I/a + U/R) > 0.

Financial Valuation and Perfect Market Models

Most important assumptions of financial markets: competitive, perfect, complete, arbitrage-free, general equilibrium.

The **Fundamental Theorem of Asset Pricing** says the following are equivalent:

1. Absence of arbitrage
2. Existence of optimal demand for one individual who prefers more to less
3. Existence of a positive linear pricing rule

Two broad classes of general equilibrium models:

1. **Classical** – price fundamental securities (stocks, insurance policies)
2. **Derivative** – price redundant securities like stock options

Discounted Cash Flow (DCF) Model

We assume premium is paid at t = 0 and losses, taxes paid at t = 1. Also:

- P = premium
- R_L = risk-adjusted discount rate (loss)
- R_f = risk-free rate
- L = expected losses
- Q = capital supporting policy
- τ = tax rate

$$P = \frac{L}{1+R_L} + \frac{\tau R_f(P+Q)}{1+R_f} + \frac{\tau P}{1+R_f} - \frac{\tau L}{1+R_L}$$

The year end value:

$$V_1 = P\left(1+R_f\right) + Q\left(1+R_f\right) - L - \tau(P-L) - \tau R_f(P+Q)$$

Return on equity:

$$R = R_f + \frac{L}{Q} \times \frac{R_f - R_L}{1+R_L}(1-\tau)$$

Should investment income credit be at the insurer's anticipated market return or R_f?

Risk-free rate	Anticipated market return
• Policyholders do not share in investment risks and should pay same premium regardless of investment strategy	• Adding investment risk increases probability of default for policyholders and changes expected loss recoveries

The **Internal Rate of Return (IRR)** model is from the investor's standpoint, estimating all cash flows to or from the investor and determining a discount rate (IRR) such that the present value of the cash flows equals zero.

- ϕ = capital-to reserves ratio
- D = market value of liabilities
- R_S = required rate of return

We want to find the IRR which is the discount rate where the NPV of all the cash flows to and from the investor equals zero:

$$\sum \frac{F_t}{(1+IRR)^t} = 0$$

If the IRR ≥ **hurdle rate**, the project should be taken. **Fair premium** is calculated using:

$$P = \frac{L}{1+R_f} + \phi D \frac{\tau R_f + \left(R_S - R_f\right)}{\left(1+R_f\right)(1-\tau)}$$

One **advantage** is that it does not require us to determine a discount rate for each cash flow. A **disadvantage** is that it requires a hurdle rate (required rate of return) for a final decision.

Portfolio Constant Cost of Capital (CCoC) pricing is based on a DCF with no taxes:

$$\bar{P}_a(X) = \frac{E[X \wedge a(X)] + \iota a(X)}{1+\iota} = vE[X \wedge a(X)] + \delta a(X)$$

- a(X) is a capital risk measure
- i is a constant
- v = 1 / (1 + i)
- δ = i / (1 + i)

Insurance Market Imperfections

Three primary kinds of frictional costs of capital:

- Agency and informational costs
- Double taxation
- Regulation

$$\bar{P}_a^{Frictional}(X) = \frac{E[X \wedge a(X)] + \iota^* a(X)}{1+\iota^*}$$

$1 + \iota^* = (1+\iota)(1+\tau)$, where τ is a frictional cost period.

Contract differences:

Short-term	Long-term
• Six- or twelve-month policies	• Policy terms > one year
• Include fixed premium	• Premium uncertainty due to length
• Lack predictable trend in expected losses	• Can have predictable adverse trend in losses

The **equivalence principle** requires that at issue the expected present value of premiums equals the expected present value of benefits plus expenses.

Brehm – Enterprise Risk Model Applications

Enterprise Risk Analysis

Overview

The goal of an enterprise risk model is to understand how assets, liabilities and ongoing underwriting are interrelated in order to create an overall risk profile of the business as a whole.

An Enterprise Risk Model can help with the following strategic decisions:

- Setting capital requirements (to support risk, maintain ratings)
- Identifying sources of significant risk and the cost of capital to support them
- Selecting a reinsurance strategy
- Planning growth
- Managing the asset portfolio
- Valuing companies for M&A

Asset-Liability Management

Asset-Liability Management - The analysis and management of the asset portfolio, reflecting current liabilities, future cash flows, future premium flows and the existing asset and liability portfolios.

Asset-Liability Matching - Setting an investment portfolio to have the same duration as the duration of the liability portfolio to protect firm value against changing interest rates.

Asset-Liability Management (ALM) is a more comprehensive process that looks at the entire picture of the company and considers risks beyond changing interest rates. ALM also considers inflation risk, credit risk and market risk. The insurer can then use this information to hedge risk by adjusting the asset portfolio, purchasing reinsurance or adjusting future underwriting.

Layers of complexity in Asset-Liability Management

- Analysis of the asset portfolio in isolation (mean vs. standard deviation of return)
- Adding fixed liabilities to consideration
  - This creates reinvestment risk if asset duration is shorter than liabilities
  - If asset duration is longer then there's risk if interest rates rise and bond values fall but need to be liquidated to pay liabilities
- Adding variability to the amount and timing to liability cash flows
  - This makes duration matching impossible
- Adding variable cash flows from underwriting
- Adding tax considerations and equity investments further adds complexity

Because of the layers of complexity, an ERM model is needed for Asset-Liability Management because the model needs to account for assets, liabilities and ongoing business operations.

Optimal investment strategies depend on risk & return metrics and risk-return preferences:

- Statutory Accounting metrics - Shows little hedging from duration matching (since bonds are amortized and liabilities are not discounted)
- GAAP Accounting metrics - Shows little hedging from duration matching
- "True Economics" metrics - Duration matching lowers interest rate risk

Asset-Liability Modeling Steps

1. Model Asset classes, existing liabilities and current business operations.
2. Define risk metrics for the analysis (based on income or the balance sheet)
3. Define "return" for the insurer (e.g. ROE, income or terminal value of surplus)
4. Set the time horizon for the analysis (single-year or multi-year)
5. Incorporate relevant constraints (e.g. limits on asset classes or investments from regulators or rating agencies)
6. Run the model with different investment, underwriting and reinsurance strategies, calculating the risk and return metrics for each combination.
7. Create an efficient frontier for risk-return and compare the current portfolio to portfolios on the efficient frontier.
8. Test the effects of different reinsurance structures and asset portfolios on a multi-period model
9. Review simulations where the preferred portfolios performed poorly to see if it's possible to reduce the downside risks

Remember that the overall goal of Asset-Liability Management is to help the insurer make better risk-return decisions, especially with the investment portfolio. ALM is more appropriate than simply looking at the asset and liability durations because it considers all the other risks of the enterprise and the true complexity of how assets, liabilities and ongoing business operations interrelate.

Corporate Decision Making with an Enterprise Risk Model

Evolution of Corporate Decision Making

1 - Deterministic Project Analysis
A ***single, deterministic forecast*** is created (for a project, line of business, etc.) and used to project future cash flows. Then, the present value of cash flows or IRR is calculated along with the sensitivity from some of the key variables. This is reviewed to make a decision about whether to pursue the project or not.

Decision makers use their judgment to consider the uncertainties in the decision.

2 - Risk Analysis
First, an insurer sets a forecasted distribution for each of the critical, uncertain variables. Then a model simulates distributions of projected cash flows and the present value of cash flows for each simulation. The result is a range of outcomes that decision makers can review to make a decision.

Decision makers still apply risk judgment intuitively (they'll have expected value and range of the present value of cash flows, but still need to intuitively judge whether it's too risky or not).

3 - Certainty Equivalent
The process is the same as risk analysis, but the risk judgment is quantified with a corporate risk preference or utility function so that risk judgments are consistent.

Decision Making with an Enterprise Risk Model

An ERM model can be used to develop aggregate loss distributions for an insurer as a whole based on the underlying loss distributions from the individual risk sources (e.g. lines of business) and dependencies between risks.

At this point, the following elements are needed to make decisions from the output of the model:

1 - Corporate Risk Tolerance
A corporate risk tolerance is a combination of factors (size of the company, assets, ability and willingness to tolerate volatility). It determines the impact and cost that aggregate losses have on the insurer.

It answers the question, "How much risk are we willing to tolerate?"

2 - Cost of Capital Allocation
Risk Capital - A measure of the firm's total risk-bearing capacity.

For an insurer, it's the cost of capital that's allocated to risk sources and lines of business as opposed to capital that's allocated. This is because capital allocation is theoretical (there's no transfer of capital to a line of business) and each line of business has access to the entire pool of insurer capital.

We can allocate risk capital down to the risk sources (e.g. line of business) and then calculate the cost of capital for the risk source. This is the Return on Risk-Adjusted Capital (RORAC).

$$\boxed{RORAC = \text{Risk-Adjusted Capital} \cdot \text{Hurdle Rate}}$$

3 - Cost-Benefit Analysis (CBA) for Mitigation
After the cost of capital allocation, the insurer uses a cost-benefit analysis to decide whether or not to do a risk mitigation project (e.g. reinsurance). One approach is to use Economic Value Added (EVA):

$$\boxed{EVA = NPV\ \text{Return} - \text{Cost of Capital}}$$

Any risk mitigation project that has a positive EVA should be undertaken. A risk mitigation project with a positive EVA would reduce capital cost (due to a lower risk capital needed) by more than the cost of the project.

Capital Modeling

Insurers need capital to support ongoing underwriting, provide a cushion for reserve deterioration or a decline in assets, and support growth.

Different stakeholders have different demands regarding capital:

- Regulators and rating agencies require enough capital to protect policyholders from default
- Shareholders require that capital is used efficiently

An ERM model can help to set risk capital for the insurer as a whole as well as to allocate capital (or allocate the cost of capital) down to the risks and lines of business.

Setting Required Capital Levels

Potential reference points for setting required capital:

- Set capital so that the probability of default is appropriately remote (default avoidance)
  - Benefit: Focuses on protecting current policyholders
  - Weakness: Insurer may want to focus on avoiding significant partial losses of capital to protect shareholders
- Set capital to support renewal business
  - Example: If renewal business requires 80% of current capital, the reference point for setting capital is a loss of 20% of capital, so you would set capital at $5 \times$TV@R at the probability level you want to protect renewal business.
- Set capital so that an insurer can thrive after a catastrophe

When setting capital levels, it's best to avoid using extreme reference points such as TV@R 99.9 because the model is least reliable in the tail extremes. This is because:

- There's little data in the tail
- Results are sensitive to assumptions about the form of the distributions

Customer reaction from rating levels is one consideration for setting required capital levels:

- Rating increases can slowly increase growth
- Rating decreases can result in a rapid decline in business (customers can quickly leave to competitors)

Practical Models for Setting Capital

Regulators and rating agencies use three different types of tools to measure capital adequacy of insurers:

1 – Leverage Ratios
Leverage ratios (e.g. Net Written Premium-to-Surplus or Net Reserves-to-Surplus) are the simplest way regulators monitor capital adequacy. Leverage ratios are compared to a threshold which is used to trigger regulator attention. In the US, the IRIS ratios are still used to measure capital adequacy.